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RIDING THE FEL INSTABILITY (DEDICATED TO ALBERTO RENIERI)

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Abstract

The Free Electron Laser (*FEL*) transforms the kinetic energy of electrons into coherent light. The underlying mechanism occurs through the *FEL* instability, leading to the growth of coherent radiation from noise, characterizing all the generators of coherent radiation from free electrons (Gyrotrons, *CARM* ...). The *FEL* instability shares many features with other instabilities occurring in Plasmas and electron beams, sometimes competing with them. In this paper we give a short description of these analogies, their relevant physical roots and comment on the importance and role of their interplay.

FEL INSTABILITY

The *FEL*, in its modern conception, started in 1976 with the amplification of a CO_2 laser, co-propagating with an electron beam in an undulator magnet [1]. The experiment confirmed what Madey had predicted few years before [2], namely that the electron beam may be induced to lose power, thus enhancing the laser intensity. The FEL small signal gain curve was carefully examined, along with the kinematical conditions under which the laser beam can be amplified or depleted, thus revealing an, albeit inefficient, electron beam acceleration. The following year, an FEL oscillator [3] showed that, under appropriate conditions, the amplification mechanism was sufficient to sustain the laser oscillation in the *IR* and drive the system to complete saturation.

The original conception of *FEL* [2] was a quantum treatment of a process called "stimulated bremsstrahlung" by Madey. The dynamical behavior of *FEL* was successively clarified by Colson [4–6] who showed that all the relevant physics can be described by a classical model based on the use of a pendulum like equation, coupled to those giving the laser field evolution. Many discussions arouse at the time, regarding the nature of the *FEL* itself [7], compared to population inversion devices. However smart the answer was, it could not circumvent the pragmatic observation that the *FEL* is a tool that steels power from a relativistic e-beam, transforming it into coherent "laser-like" power.

The question now becomes what "laser like" means and whether it is true that the *FEL* light is fully coherent. The theoretical treatment developed by Bambini and Renieri [8–12] used means from analytical mechanics to describe the *FEL* problem in terms of a non relativistic Hamiltonian, leading to the pendulum equation and opening the way to the investigation of the *FEL* quantum coherence properties [7,8]. This is the gross picture before the eighties. It became slowly clear that the *FEL* phenomenology had not to be dissimilar from the Physics of travelling wave tubes. The analogy between *FEL* and other generators of light by other free electron devices (Klystron, Gyrotron, *CARM*, ...) was pointed out in [13] (see also ref. [7] for a more recent discussion. For a comparison with conventional laser sources, see [14]).

The common feature emerging from the relevant theoretical picture was that the optical field growth is associated with a dispersion relation leading to a third (or fourth) order *ODE* [15–17]. The roots of the associated characteristic polynomial fix the condition for the onset and rise of the electromagnetic field. Within this context the field grows as the result of the *FEL* instability, with a rise time or gain length, given by the "fast growing" root.

The breakthrough in this direction came from [18] where a new operating condition for the *FEL* was foreseen, thus paving the way for the fourth generation synchrotron radiation devices, the *X*-Ray *FEL*s, characterized by very high brightness, power and by extremely short pulse duration. A proposal to build an *X*-Ray *FEL* using a 15 *GeV* electron beam from the *SLAC* linac [19] led to the design and construction of *LCLS* and other similar systems in Europe and Asia. *X*-Ray *FEL*s give for the first time the possibility of exploring matter with X-rays at the angstrom-femto-second space and time scale characteristic of atomic systems, thus opening a new window on the exploration of matter and its dynamical processes.

We have pointed out that the *FEL* small signal dynamics is the result of an instability, the further interplay with the e-beam produces a heating, determining the increase of the energy spread, which eventually induces saturation [20]. This is a complex mechanism, associated with Landau damping, and will be further discussed in the next sections of the paper.

STORAGE RING *FEL*, RENIERI LIMIT AND SAW-TOOTH INSTABILITY

In the previous section we have mentioned that the *FEL* growth is the result of an instability, counteracted by the self induced beam energy spread. Many *FEL* oscillators have been proposed and operated in the past [21–28] in an electron storage ring, with the intent of providing

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radiation in the *IR* to *UV* with relatively large average power.

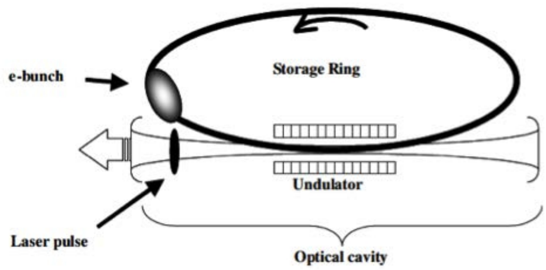

Figure 1: *FEL* Storage Ring Oscillator: the electron and a synchronous laser bunch overlap after any ring turn.

The relevant dynamics can be illustrated by the following steps (see Fig. 1):

a) The electron beam is circulated many times inside the optical cavity, synchronized to the radiation pulse. At each interaction, it transfers energy to the radiation and increases its own energy spread.

b) The *FEL* intensity increase continues until the induced energy spread reduces the gain below the cavity losses. The threshold energy spread determining the laser switching off is of the order of

$$\langle \Delta E \rangle \leq \frac{E}{4N} \tag{1}$$

whit N number of undulator periods.

c) The laser process stops and radiation in the storage ring reduces the energy spread. After a damping time τ_s, when the beam original condition have been restored by the ordinary damping mechanisms, the laser process starts again. The average laser power is therefore given by

$$P_L \leq \frac{1}{4N}, \qquad \frac{E}{\tau_s} = \frac{1}{4N} P_s \tag{2}$$

This is the Renieri limit [29, 30] and states that the laser power of a storage ring *FEL* does not exceed a fraction of the power P_s lost in the whole machine via synchrotron radiation. It has been confirmed by the theory, simulation and experiment [31, 32]. It is a very important contribution to the Physics of *FEL*'s operating in a storage ring.

However the *FEL* instability and radiation damping are not the only physical mechanisms at work in a storage ring, which is a complex environment. Electrons move inside the vacuum chamber, where various instrumentation items and *RF* cavities are present. The interaction of the electron with these devices generates additional electromagnetic fields, called "wake-fields". They interact with the beam and can induce further instabilities. One of them is the saw-tooth instability, manifesting itself through an anomalous increase of the beam energy spread and bunch length. The literature in the field is quite impressive (see ref. [33–43] for a partial list), where authoritative account (analytical and numerical) of the relevant phenomenology have been presented. The anomalous energy spread increase can limit the *FEL* power below the value foreseen by the Renieri theory.

We report below an outline of the relevant theoretical studies.

i) Understanding of the relevant phenomenology, development of scaling relations aimed at clarifying the existence of a threshold current above which an anomalous spread appears (Kheil-Schnell, Boussard, ...).

ii) Mode coupling and instability (Sacherer, Laclare, ...).

iii) Fokker-Planck analysis and turbulent mode coupling (Renieri).

iv) Integral equation treatment (Wang, Krinsky, Pellegrini, ...) which provided the analytical basis for the Boussard criterion.

With the advent of *FEL* the question was raised about the interplay between microwave and *FEL* instabilities. It was noted that they induced qualitatively analogous behavior on the beam energy spread, characterized by a fast blow up and then by a damping. The understanding of such an interplay occurred using arguments not dissimilar from those leading to the Renieri limit and goes as it follows.

The Boussard criterion states the existence of a threshold beam current (I_{th}) above which the energy spread exceeds the natural value, the parameter $\delta^2 = \frac{I}{I_{th}}$ can be used to fix the associated amount of energy spread increase with respect to the natural counterpart $\sigma_A^2 = \delta^2 \sigma_0^2$. If the *FEL* induced energy spread exceeds σ_A, the instability can be switched off (see Figs. 2-3) and the corresponding *FEL* threshold intensity is [44–46]

$$P^* \simeq \chi \frac{P_s}{4N}, \qquad \chi = 1.673 \frac{\delta^{\frac{2}{3}} - 1}{g_0} \mu^2, \qquad \mu = 4N\sigma_0 \tag{3}$$

where g_0 is the small signal gain coefficient.

The *SR-FEL* dynamics has displayed so many interesting complex phenomenon, with implications going well beyond the mere generation of coherent radiation. The non linear effects in accelerators and the associated instabilities make the study of the relevant dynamics very much interesting. The study of the competition of different instabilities has opened an entire new world. We learned how they may contribute to control each other, we understood how the *FEL* may contribute to the regulation of saw-tooth, head tail [47] instabilities, Touscheck beam lifetime ... The relevant studies dynamics taught us how the whole system (in

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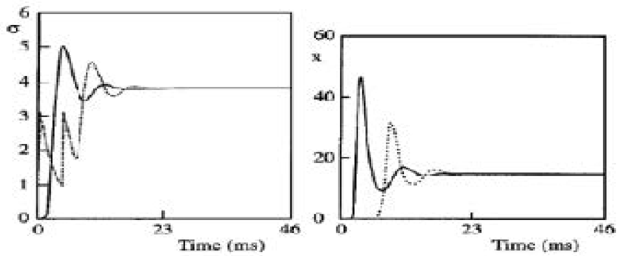

Figure 2: Induced energy spread in storage ring *FEL* without (solid line) and with (dash line) saw tooth instability. Same for the laser intensity.

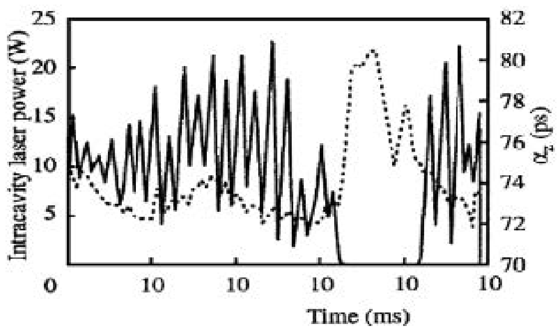

Figure 3: Experimental results (SUPERACO) solid line induced energy spread *FEL* on(solid) Saw-tooth contribution (dash), when *FEL* is off the saw tooth contribution blows up.

Storage Rings and other accelerator as well) should be considered from a unitary point of view, and within this context the *FEL* appears as one of the intrinsic feedback mechanisms contributing to the electron-beam equilibrium with the accelerator environment [48]. Alberto Renieri has been a pioneering and decisive contributor to these studies.

CONCLUDING COMMENTS

In the previous sections we insisted on regarding the *FEL* as the result of an instability, we have commented on the role of control acted on the accelerator environment by its competition with other types of instabilities, hence the title of this paper. The importance of the seminal ideas put forward in the previous comments reflects in more recent researches [49, 50] foreseeing the use of a Storage Ring to drive a short wavelength *SASE FEL* operation. The role of laser heater in suppressing or mitigating the coherent synchrotron instability [51, 52] can be framed within the same context.

Let us however go back to the *FEL* power growth and to its route to saturation. The small signal *FEL* dynamics is ruled by a Volterra integro-differential equation with a gain memory kernel reported below [53]

$$\frac{d}{d\tau}a = i\pi g_0 \int_0^{\tau} \tau' e^{-i\nu\tau'} a(\tau - \tau') d\tau'. \qquad (4)$$

It can be reduced to the third order *ODE*

$$\begin{aligned} &\left(\hat{D}_\tau^3 + 2i\nu\hat{D}_\tau^2 - \nu^2\hat{D}_\tau\right) a(\tau) = i\pi g_0\, a(\tau), \\ &a\,|_{\tau=0} = a_0, \qquad \hat{D}_\tau\, a\,|_{\tau=0} = 0, \qquad \hat{D}_\tau^2\, a\,|_{\tau=0} = 0. \end{aligned} \qquad (5)$$

Its solution depends on tat of the characteristic third order polynomial. The explicit form of the dimensionless field amplitude writes [54]

$$\begin{aligned} a(\tau) &= \frac{a_0}{3(\nu + p + q)} e^{-\frac{2}{3}i\nu\tau} \cdot \\ &\cdot \left\{ (-\nu + p + q) e^{-\frac{i}{3}(p+q)\tau} + 2(2\nu + p + q) e^{\frac{i}{6}(p+q)\tau} \cdot \right. \\ &\cdot \left. \left[\cosh\left(\frac{\sqrt{3}}{6}(p-q)\tau\right) + i\frac{\sqrt{3}\nu}{p-q} \sinh\left(\frac{\sqrt{3}}{6}(p-q)\tau\right) \right] \right\}, \\ p &= \left[\frac{1}{2}\left(r + \sqrt{d}\right)\right]^{\frac{1}{3}}, \qquad q = \left[\frac{1}{2}\left(r - \sqrt{d}\right)\right]^{\frac{1}{3}}, \\ r &= 27\pi g_0 - 2\nu^3, \qquad d = 27\pi g_0(27\pi g_0 - 4\nu^3), \\ \tau &= \frac{Nz}{\lambda_u}, \qquad \nu \equiv detuning\ parameter. \end{aligned} \qquad (6)$$

The fast growing term is contained in the contributions associated with the hyperbolic functions. A practical issue is that of embedding the above formulae to get a simple relation yielding either the power growth $P(z)$ (including the saturation) and the induced energy spread $\sigma_i(z)$ along the undulator coordinate z (Fig. 4)

$$\begin{aligned} P_1(z) &= P_0 \frac{A(z)}{1 + \frac{P_0}{P_{F,1}}(A(z) - 1)}, \\ A(z) &= \frac{1}{9}\left[3 + 2\cosh\left(\frac{z}{L_g}\right) + 4\cos\left(\frac{\sqrt{3}}{2}\frac{z}{L_g}\right)\cosh\left(\frac{z}{2L_g}\right)\right], \\ &P_0 \equiv Input\ seed\ power, \qquad L_g \equiv Gain\ lenght, \\ &P_F = \sqrt{2}\,\rho\, P_E, \qquad \rho \equiv Fel\ Pierce\ parameter, \\ &P_E \equiv e{-}beam\ power \end{aligned} \qquad (7)$$

and

$$\begin{aligned} \sigma_i(z) &= 3C\sqrt{\frac{A(z)}{1 + 9B(A(z) - 1)}}, \qquad C = \frac{1}{2}\sqrt{\frac{\rho P_0}{P_E}}, \\ B &\simeq \frac{1.24}{9}\frac{P_0}{P_F}, \qquad \sigma_{i,F} \simeq \frac{C}{\sqrt{B}} \simeq 1.6\,\rho. \end{aligned} \qquad (8)$$

The derivation of the previous relations can be framed within different analytical models, Ginzburg-Landau, asymptotic solution of pendulum field equations, wise combination of numerical and scaling relations [55].

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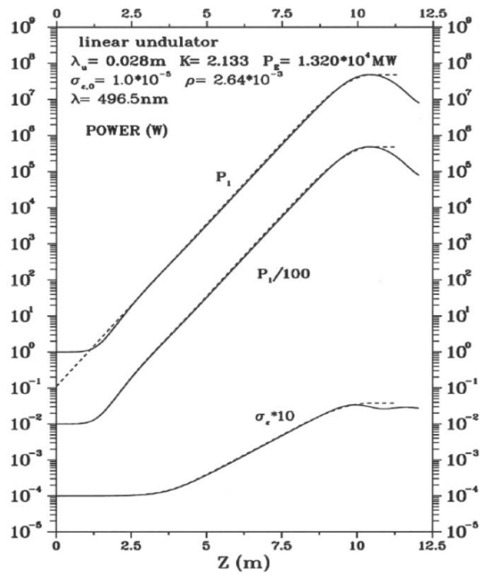

Figure 4: Power and energy spread evolution.
Upper curves - continuous line: 1-D simulation, dashed line fast growing root only.
Middle curves – continuous line: 1-D simulation, dashed line: Eq. (7).
Lower curves - continuous line: 1-D simulation, dashed line: Eq. (8).

Whatever the procedure is, they represent the result of a non common ability of combining physical intuition and analytical means to provide results of practical interest. Alberto mastered all these aspects and left an important lesson for all of us as man and scientist.

Aa a final comment we like to mention other two distinguished members of the "ENEA-Frascati-*FEL* school": Franco Ciocci and Amalia Torre, they passed away on the eve of last year, their work, their contributions and their smiling presences will remain forever with us.

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