Problem of the Beam-Beam depolarization effect in super dense colliding beams

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Introduction

Very high density of the longitudinally polarized colliding beams in the projects of Super B and C-Tau Factories makes us to concern about the estimation of beam-beam depolarization (BBD) effect.

BBD mechanism is based on the spin resonant diffusion. Spin-orbit resonances of high order may fall into the footprint of the betatron tune shift caused by the counter beam nonlinear field. Tune shift for a given particle is determined by an square of its betatron oscillation amplitude – “action”. Incoherent chaotic crossing of the spin resonances due to diffusion and damping processes leads, in principle, to depolarization effect.

BBD rate was estimated by A.M. Kondratenko (1974) as applied to conventional storage ring colliders with the traditional interaction region organization and the vertical beam polarization. Main conclusion was:

It is possible to conserve the beam polarization provided that BB effects do not crucially disturb the orbital motion ("no beam blow up" → $\xi < \xi_{cr}$)

This conclusion needs a quantitative verification, on the one hand, wrt the features of Crab Waist IR, and, on the other hand, to the storage ring configuration where longitudinal or horizontal magnetic fields are included to obtain longitudinal polarization.

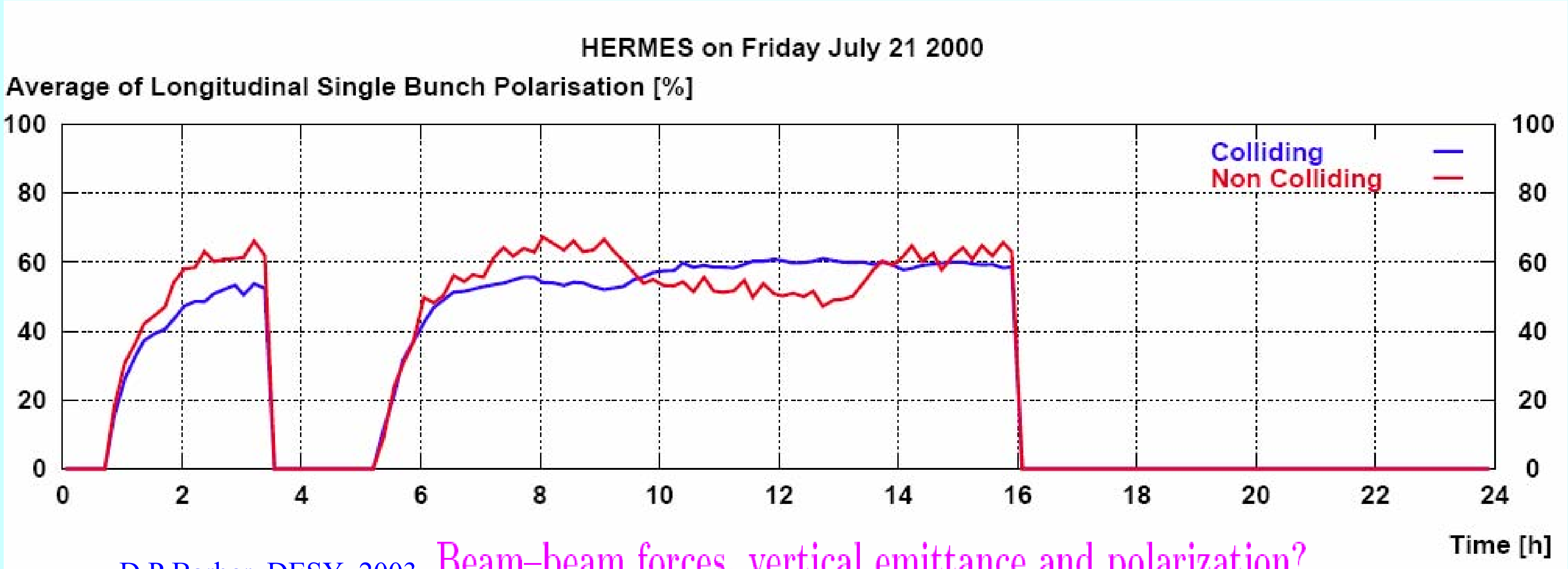
Experimental facts on BBD

- In early experiments at VEPP-2M (0.5 MeV) the comparison of polarizations between the regimes of a single beam and of colliding beams did not reveal any difference (1975). Product of e+ and e- polarizations was measured by the muon production asymmetry.

  The same was repeated at SPEAR (3GeV).

  (*Yu. Shatunov - private communication*)
- In the experiments at VEPP-4 (5 GeV) in 80s the electron and positron beam energy calibrations by the resonant depolarization technique were being performed after taking statistics with colliding beams by the detector MD-1 in a time which exceeded a necessary polarization rise-time. Measured polarization was close to that in the non-colliding beam case.

- 1996: high proton currents at HERA ===> first indications of beam--beam effect on 27 GeV electron/positron polarization (*home page by D.P. Barber)*. It is hard to treat all HERA results, appeared from 1996, as having a single meaning. Nevertheless, in “no beam blow up” case the longitudinal polarization of colliding bunches was shown to stay closely approximated from that of non-colliding ones.



D.P.Barber, DESY, 2003

Beam–beam forces, vertical emittance and polarization?

Processes affecting polarization

- Radiative polarization and depolarization (SR)
- IBS without spin flip*
- IBS with spin – flip**
- Residual gas scattering**
- BBD (to be estimated)

* increases the depolarization rate in the same extent that IBS does wrt the energy diffusion ordinarily determined by SR

** estimated as negligible

Relaxation rate
$$\frac{1}{\tau_r} = \sum_i \frac{1}{\tau_i}$$

Equilibrium degree
$$\frac{\tau_{dep} P_0}{\tau_{pol} + \tau_{dep}}$$

Temporal hierarchy for Super B Factory

Radiation Spin Relaxation Time τ_r	Touschek Lifetime τ_{IBS}	Luminosity Lifetime τ_{lum}	Effective Lifetime τ_l
$\geq$ 10 min *	7 min	3.35 min	2.3 min

* from I. Koop's Talk

Polarized particles are injected into Super B or CTau rings from a linac in the Trickle Injection regime. Role of BBD is determined by its place occupied in the temporal hierarchy.

Polarization in Trickle Injection Regime *simple approach*

ΔN — an injected portion of particles per bunch

P_0 — an injected bunch polarization

$\tau = 1/f_i$ — an injection cycle period

λ_l — a particle loss rate

$N_1 = \dfrac{\Delta N}{1 - e^{-\lambda_l \tau}}$ — a maximal particle number per bunch in cycle

$k_i = \dfrac{\Delta N}{N_1} = 1 - e^{-\lambda_l \tau}$ — a piling coefficient ($k_i = \lambda_l \tau << 1,\ \lambda_l = k_i / \tau = k_i f_i$)

λ_{BBD} — a depolarization rate due to Beam-Beam

$P_1 = P_0 \dfrac{1 - e^{-\lambda_l \tau}}{1 - e^{-(\lambda_l + \lambda_{BBD})\tau}}$ — a maximal polarization in cycle

$P_1 \to P_0 \quad \text{if} \quad \lambda_l >> \lambda_{BBD}$

Valid if $\lambda_{BBD} >> 1/\tau_p$, τ_p is a radiative polarization relaxation rate

In our case, the loss rate parameter λ_l plays a role of scale rate !

BBD Theory Elements

$\mathrm{Q+mQ_x+nQ_y=k+\varepsilon_k}$; k is integer; $\varepsilon_\mathrm{k} << 1$

Q=spin tune $\left(=\nu=\dfrac{\mathrm{E[MeV]}}{440.65}\right.$ in traditional storage rings)

$Q \neq \nu$, generally, in rings with longitudinal polarization inserts

Single fast crossing $\rightarrow$ a small spin $\vec{n}$-projection change

Crossing $\rightarrow$ due to diffusion and synchrotron oscillations

$w_k = 2\nu \left\langle H_x F^{\nu_k} e^{i\nu_k \tilde{K}_z} \right\rangle$, the spin perturbation harmonic

Non-correlated successive crossings $\rightarrow$

equation for average rate of the spin $\vec{n}$-projection change:

$$\overline{\frac{dS_n}{dt}} = -\pi\kappa \frac{|w_k|^2}{|\varepsilon_k^{'}|} S_n, \quad \kappa = I_r^{'} f(I_r),\ \varepsilon_k = \varepsilon_k(I) = 0 \text{ at } I = I_r$$

BBD rate is $\lambda = \tau^{-1} = \pi \left[\dfrac{|w_k|^2}{d\varepsilon / dI} f(I) \right] \Bigg|_{I=I_r} = \pi \left\langle \left| w_k \right|^2 \delta(\varepsilon) \right\rangle$

Total effect of a set of resonances $\tau_{BBD}^{-1} = \pi \sum_k \left\langle \left| w_k^2 \right| \delta(\nu - \nu_k) \right\rangle$

Criterion for spin-orbit resonances $\nu+k_z\nu_z=k$

Maximal nubmer $|k_z|$ of "working" resonance is found from the condition ($A.M.Kondratenko$, 1974)

$$\lambda_{BBD}^{(k)} \approx \frac{4}{\pi}\cdot\frac{Nr_e}{\gamma}\cdot\frac{\nu^2}{\beta_z^*}\cdot|F^\nu|^2\cdot A\cdot\left[\frac{<a_z^2>}{<a_z^2>+z_0^2+z_0\sqrt{z_0^2+2<a_z^2>}}\right]^{|k_z|} \le \lambda_r$$

$<a_z^2>$ an averaged square of vertical oscillation amplitude

z_0 a vertical beam size

$|F^\nu|_{I.P.}$ Spin Response Function (SRF) shows the spin perturbation amlification/reduction resulting from oscillations excited over the ring by a kick in I.P. (if $F^\nu \to 0$ then $\lambda_{BBD}^{(k)} \to 0$ for any k)

A a factor depending on amplitude distribution

$\lambda_r = \max\{\lambda_{BLT}, 1/\tau_p, \ldots\}$ a reference rate $\sim \lambda_l$

Generally, $\lambda_{BBD} \sim \sum_k \lambda_{BBD}^{(k)}$, a sum over all working resonances

Estimated max{k_z}=3 for cases under consideration

General Features of BBD at the Factory with Longitudinal Polarization and Crab Waist IR

1. In the main, the dangerous spin resonances of high order in betatron tunes are unavoidable due to elongated BB foot print
2. Very strong collective field of the counter bunch due to very small beam transverse sizes
3. Large Piwinski's angle → length of interaction is reduced (positive fact!)
4. Spin perturbations from BB impact in two planes
5. Spin Response Factor (finally increasing or decreasing BBD) determined by excited in IP oscillations depends on the chosen rotator scheme

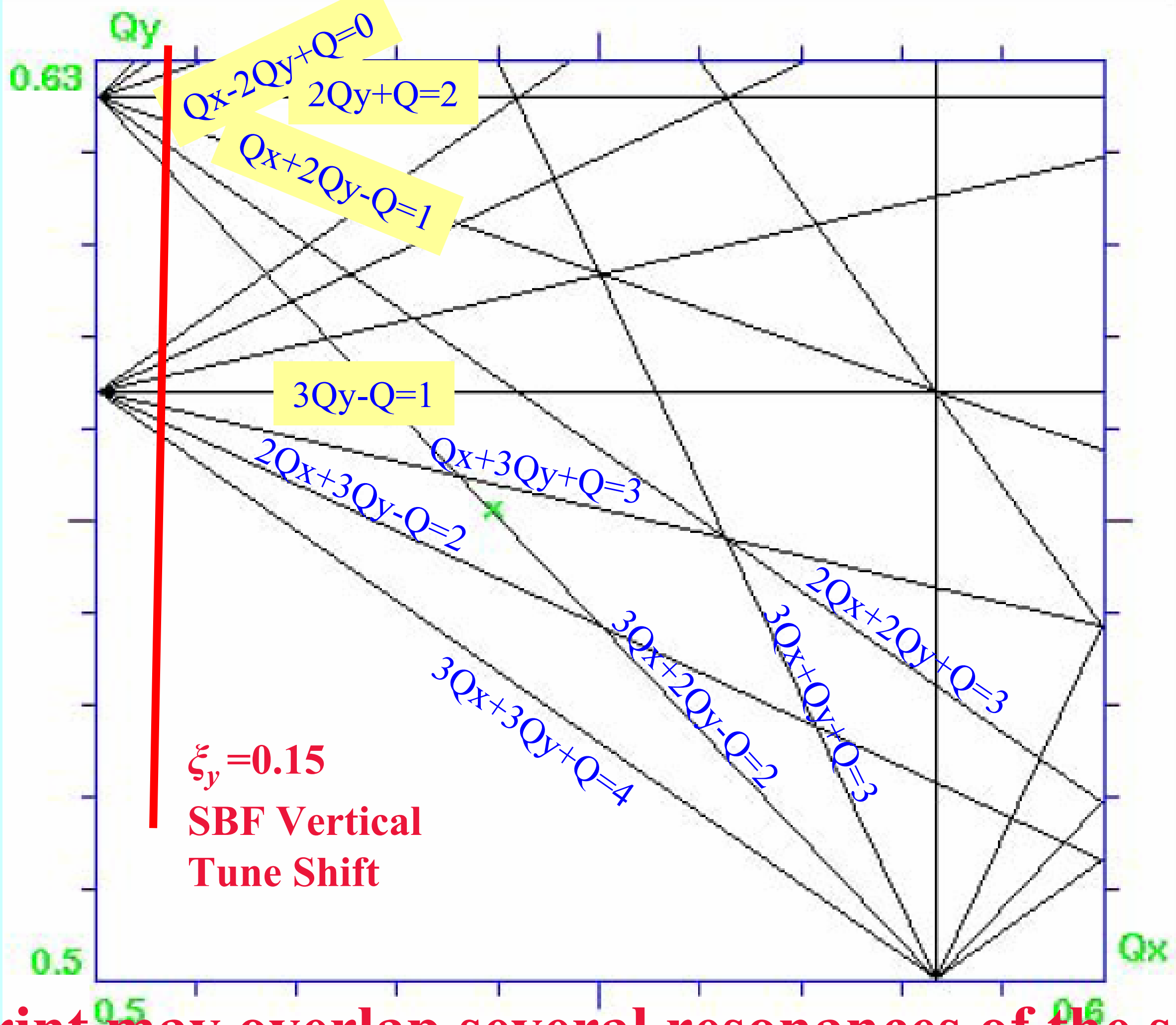
Notice!

In some following slides the “y” index usually marking the vertical axis is replaced by “z” index which marks the same. In this case the letter “y” meets the longitudinal axis. Anyway, you can easily distinct the cases in a sense.

Sorry!

Spin Resonances Q+mQx+nQy=k at Q=0.75, |m|+|n| ≤ 6

Dangerous Resonances due to Beam-Beam



Central Arc Longitudinal Polarization Scheme for SBF LER 4 GeV

Modulation resonances with synchrotron frequency are not shown !

BB footprint may overlap several resonances of the same order!

Synchrotron modulation resonances

$Q+mQ_x+nQ_y+j\nu_\gamma=k$

ν_γ, synchrotron tune

Correlated periodical crossings due to modulation $(\nu^2/\nu_\gamma^3 \ll \tau_r f_0) \rightarrow$ isolated resonances

Relative depolarization rate at 1st side band=

$$=\frac{I_1\left(\dfrac{\sigma_\nu^2}{\nu_\gamma^2}\right)}{I_0\left(\dfrac{\sigma_\nu^2}{\nu_\gamma^2}\right)} \approx \frac{1}{50} \quad \text{(VEPP-4M, 1.85 GeV)}$$

$\sigma_\nu^2 = 2\nu^2\sigma_\gamma^2$, instant spin tune spread in a beam

σ_γ, energy spread

τ_r, radiative polarization time

ν, spin precession frequency

Notice: in contrast to Alternative Schemes, Siberian Snake fundamentally eliminates modulation resonances!

BB acts on longitudinal polarization in two planes

$a = \sigma_x,\ \ b = \sigma_z,\ \ c = \sigma_y = \sigma_{long}$, the Gaussian beam sizes

$$E_x(x,0,0) = x \cdot \frac{2\gamma e N}{\sqrt{\pi}} \int_0^\infty \frac{e^{-\frac{x^2}{2a^2+t}}}{(2a^2+t)^{3/2}\sqrt{(2b^2+t)(2\gamma^2 c^2+t)}}\,dt$$

$$H_z(x,0,0) = E_x(x,0,0)$$

$$E_z(0,z,0) = z \cdot \frac{2\gamma e N}{\sqrt{\pi}} \int_0^\infty \frac{e^{-\frac{z^2}{2a^2+t}}}{(2b^2+t)^{3/2}\sqrt{(2a^2+t)(2\gamma^2 c^2+t)}}\,dt$$

$$H_x(0,z,0) = E_z(0,z,0)$$

Ratio between the squares of vertical and horizontal strengths affecting the spin vector in the counter beam's EM field:

$$\left(\frac{\acute{I}_{x}(0,b,0)}{H_z(a,0,0)}\right)^2 \approx 1.4$$

$E = 4.1\ GeV,\ \ \sigma_x$=5.7 μm, σ_z=0.035 μm, σ_y=1 cm (SBF-like case)

$\nu\chi_{x,z} \propto \nu H_{z,x}$, the spin rotation angle in the horizontal/vertical plane

Oscillations in both planes are important!

Strengths of vertical and horizontal kicks in I.P. are neighbours.

Definition of Spin Response Function for BBD

Known formula for traditional storage rings (*Ya. Derbenev, A. Kondratenko, A. Skrinsky*, 1979) :

$$F^{\nu}(\theta)=F^{\nu}\left(\theta+\frac{2\pi}{m}\right)=\frac{\nu e^{i\nu\theta}}{2}\left\{\frac{1}{1-e^{i\frac{2\pi}{m}(\nu+\nu_z)}}\cdot f_z\int\limits_{\theta-2\pi/m}^{\theta}\overline{f}_z{}'Ke^{-i\nu\tilde{K}}d\theta'-\frac{1}{1-e^{i\frac{2\pi}{m}(\nu-\nu_z)}}\cdot\overline{f}_z\int\limits_{\theta-2\pi/m}^{\theta}f_z{}'Ke^{i\nu\tilde{K}}d\theta'-\right\}$$

$F^{\nu}\propto\sqrt{\beta_z}$, $\quad |F^{\nu}|\geq 0$, $\quad$ grows at $\nu\pm\nu_z=m\cdot k$, $\quad$ non-monotonic with energy

This definition is not valid in the case of rings with longitudinal polarization inserts!

Generally, find a change of transverse to $\vec{n}$-axis spin vector component

$(\overrightarrow{\delta S_{\perp}})^2=(\delta S_x)^2+(\delta S_z)^2$, reduced to I.P. $\quad (\vec{n}_{I.P.}=\overrightarrow{e_y})$

$\overrightarrow{\delta S_{\perp}}$ is accumulated over many turns as result of particle oscillations excited by a single kick in I.P.

Most simply to define SRF in cases of pure spin-vertical or spin-horizontal orbit resonances:

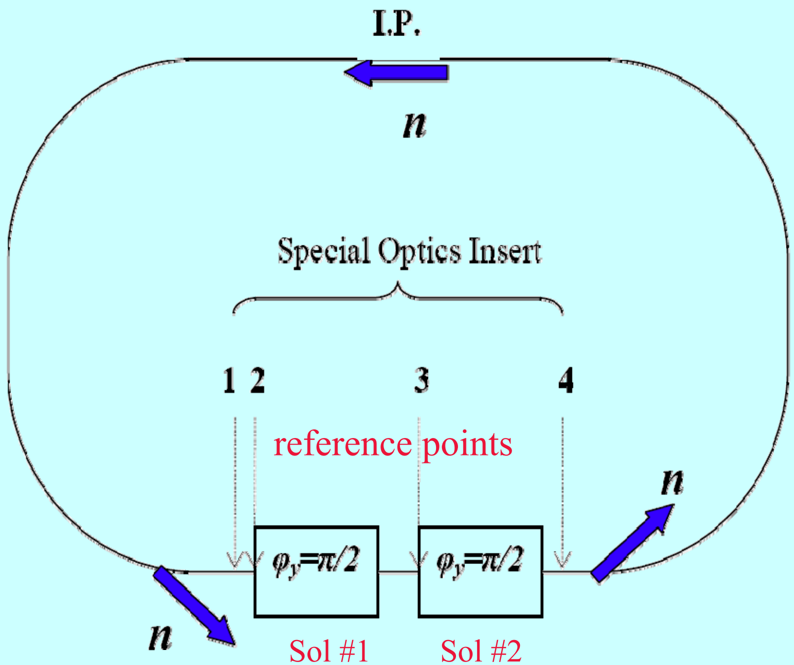
$|F^{\nu}|^2_{I.P.}=\dfrac{(\overrightarrow{\delta S_{\perp}})^2}{(\nu\chi)^2}$, $\quad \chi=\chi_{x,z}$, the deflection angle in $x(z)$ plane depending on particle transverse position

$$\overrightarrow{\delta S_{\perp}}=\sum_{k}^{N}\left[\overrightarrow{\delta S_{\perp}}^{(k)}\cos 2\pi\nu_0(N-k)+\vec{n}\times\overrightarrow{\delta S_{\perp}}^{(k)}\sin 2\pi\nu_0(N-k)\right]$$, a sum over turns

$\overrightarrow{\delta S_{\perp}}^{(k)}$, the spin perturbation at k - th turn found from solution of equation $\dfrac{d\vec{S}}{d\theta}=(\vec{\omega}+\delta\vec{\omega})\times\vec{S}$

$\delta\vec{\omega}=\delta\vec{\omega}(x,x',z,z')$, $\quad$ the precession frequency perturbation

Siberian Snake SRF for vertical kick in I.P.



$$\begin{pmatrix} z_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} c_2 & d_2 \\ a_3 & b_3 \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_1' \end{pmatrix}$$

Effective spin tune $\nu_0 = \dfrac{1}{2}$

In points 2 and 3 the fringing field of solenoids ends

$$|F^{\nu}|^2_{I.P.} = \frac{(\delta S_{\perp})^2}{(\nu\chi_z)^2} = \frac{(\delta S_x)^2 + (\delta S_y)^2}{(\nu\chi_z)^2}$$

Particularly, in a case $\nu = \dfrac{E[\mathrm{MeV}]}{440.65} = \dfrac{2k+1}{2}$:

$$\frac{\delta S_x}{\nu\chi} = \frac{\pi}{4\cos\pi\nu_z}\{\sqrt{\beta_z^*\beta_{z,1}}(c_2 + a_3)\sin(\pi\nu_z - \mu_{z,1}) -$$

$$-\sqrt{\frac{\beta_z^*}{\beta_{z,1}}}(b_3 + d_2)\cdot[\cos(\pi\nu_z - \mu_{z,1}) - \alpha_{z,1}\sin(\pi\nu_z - \mu_{z,1})]\}$$

$$\mu_{z,1} = \int_0^{\theta_1}\frac{d\theta}{\beta_z}, \quad \alpha_{z,1} = -\frac{1}{2}\frac{d\beta_z}{d\theta}\bigg|_{\theta_1}$$

$$\frac{\delta S_y}{\nu\chi} \approx \frac{\nu}{2\cos\pi\nu_z}\{-\int_0^{\theta_1} g\cdot K\cdot\sin\nu\Psi d\theta + \int_{\theta 4}^{2\pi} g\cdot K\cdot\sin\nu\Psi d\theta\},$$

$$g(\theta) = \cos(\pi\nu_z - \mu_z) - \alpha_z\sin(\pi\nu_z - \mu_z)$$

$$\Psi = \int_0^{\theta} K d\theta', \qquad \Psi(\theta_1) = \Psi(\theta_4) = \pi$$

Siberian Snake SRF for vertical kick in I.P. (2)

Attempt of numerical estimate

$$\left|\frac{\delta S_x}{\nu\chi}\right| = \frac{\pi(b_3 + d_2)}{4\cos\pi\nu_z}\sqrt{\frac{\beta_z^*}{\beta_{z1}}},$$

$b_3 \approx 2.3,\ d_2 \approx 1.0$ (from 1983 BINP Project)

$\{\nu_z\} \approx 0.55,$

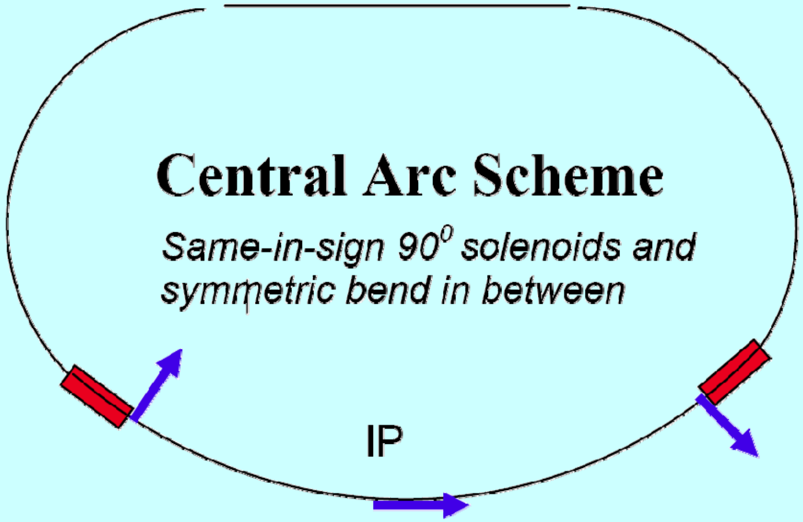
$\beta_z^* = 700\ \mu\text{m},\ \beta_{z1} = 200\ \text{cm},$

$$\left|\frac{\delta S_x}{\nu\chi}\right| \sim 0.3$$

$\left|\frac{\delta S_z}{\nu\chi}\right|$ may be of the order up to $\sim\left|\frac{\nu}{\cos\pi\nu_z}\cdot\frac{1}{(\nu_z - \nu)}\right|$

$|F^\nu|^2 = \frac{|\delta S_x|^2 + |\delta S_z|^2}{(\nu\chi)^2}$ can be >1, in principle, depending on energy and focusing parameters in I.P., arcs and the solenoid insert

…Similar “hand-made” calculation of the SRF has been performed also for one of the SBF longitudinal polarization schemes based on using solenoids and restoration of the vertical polarization in arcs (a talk by I. Koop). It gives a possibility to estimate the BBD rate for the actual SBF variant.



“Hand-made” estimations of BBD are useful but confined by the cases of spin-orbit resonances which are 1D in orbital motion (either vertical or horizontal). In more complicated cases the BBD simulation is needed.

Account of interaction length reduction

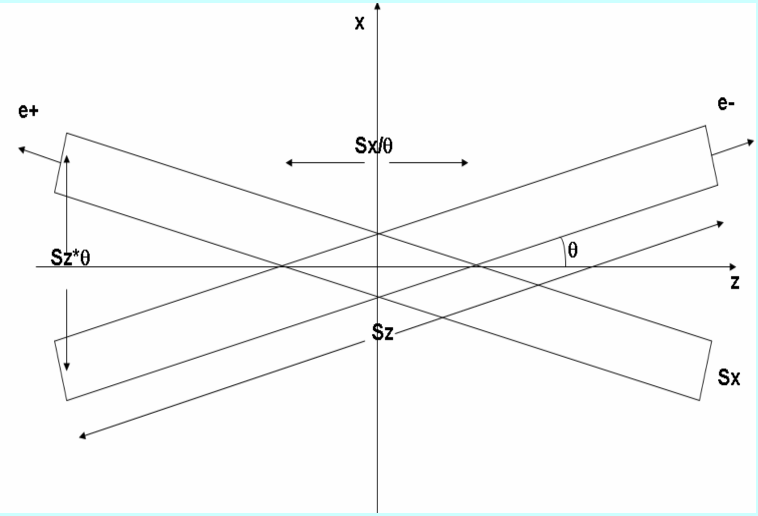
$w_k = 2\nu \left\langle H_x F^{\nu_k} e^{i\nu_k \tilde{K}_z} \right\rangle$, the spin harmonic

$\nu = \nu_{\mathrm{k}} = k + k_z \nu_z + k_x \nu_x$, the spin resonance condition

$$\left| w_k \right| \propto \frac{N_e r_e \nu \left| F^{\nu_k} \right|}{\gamma V_b} \cdot l_i,$$

V_b, the counter bunch volume

$$l_i = \begin{cases} \sim \sigma_l, & \text{the beam length (usually)} \\ \sim \dfrac{\sigma_x}{\theta}, & \text{the interaction length (Crab Waist)} \end{cases}$$



For the resonance $\nu_{\mathrm{k}} = k + k_z \nu_z$, the BBD rate is in order $\lambda_{\mathrm{BBD}} = \dfrac{1}{\tau_{BBD}} \sim \dfrac{\pi \left\langle \left| w_k \right|^2 \right\rangle}{\sigma_b}$, $\quad \sigma_b = | k_z \cdot \Delta \nu_z |$

$$\Delta \nu_z = \frac{2 N_e r_e \beta_z^*}{\pi \gamma \sigma_z^* (\sigma_x^* + \sigma_z^*)} \propto \frac{N_e r_e}{\gamma V_b} \cdot l_i, \quad \text{the tune shift}$$

As result, $\lambda_{\mathrm{BBD}} \propto \dfrac{N_e r_e \nu^2 \left| F^{\nu_k} \right|^2}{\gamma} \cdot \dfrac{l_i}{\sigma_l}$, $\quad \dfrac{l_i}{\sigma_l} \approx \dfrac{\sigma_x}{\theta \sigma_l} = \dfrac{2}{\Psi_p} \approx 0.02$ for SBF (Ψ_p - Pivinski's angle)

The estimate of λ_{BBD} by Kondratenko's formula is reduced as $\dfrac{\sigma_x}{\theta \sigma_l}$

Preliminary estimates of BBD rate

Collider	E MeV	β^*_x cm	β^*_y cm	σ^*_x μm	σ^*_y μm	σ_l cm	N 10^{10}	Crossing Angle mrad	$\lvert F^{\nu}\rvert^2_{1.F.}$	$1/\lambda_{BBD}$ min
Super B	4100	2	0.02	4	2×10^{-2}	0.6	3.52	34	1(?)	200
C-Tau	2000	2	0.076	14	0.28	1	7	34	1(?)	30
VEPP-4M	1890	70	4	290	4	4.5	2	0	0.08	2500

$Q_s \pm 3Q_y = k,$ the spin resonance type

So, what's next?

- Development of the beam-beam depolarization simulation code as applied to the storage ring collider with Longitudinal Polarization and Crab Waist IR

  (for instance, based on LIFETRAC by D.Shatilov-

  a concept is under development)
- Study of beam-beam depolarization effects in longitudinal polarization scheme in SBF and C-Tau projects using jointly analytical and simulation approaches
- Go on experimental testing BBD at the active collider with transverse beam polarization (at present, such a machine is unique →VEPP-4M)
- …

BBD Test at VEPP-4M

Partial problem:

Experimental verification of the theoretical prediction for the BBD spin resonance $\nu \pm 3\nu_z = k$ influence. (No influence!)

Method:

Measurement of the beam polarization relaxation time

in the conditions of e-e+ collisions “on/off” at E=1.89 GeV

($\nu = 4.29$) and the proper betatron working point ($\nu_z \approx 0.57$)

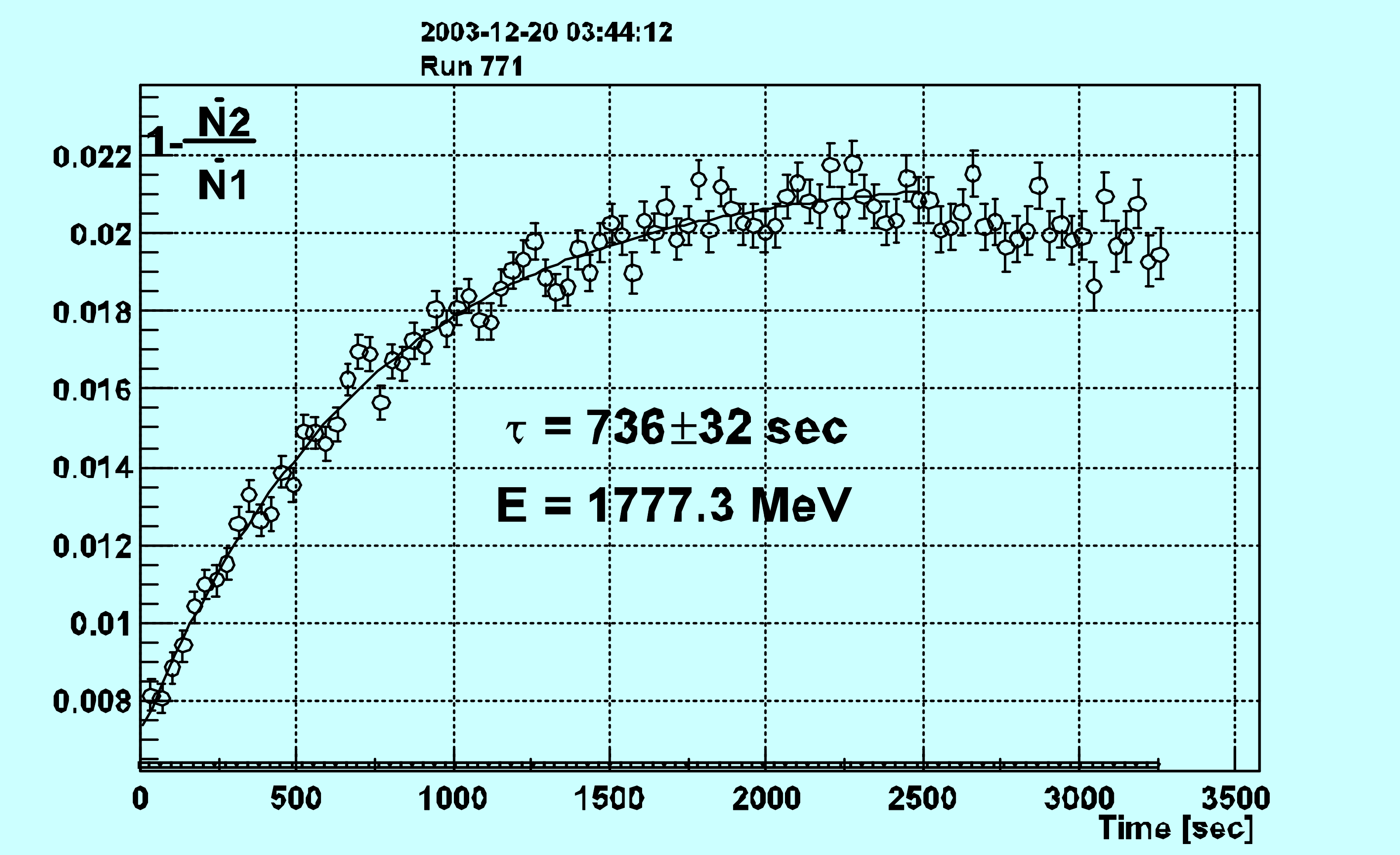
Polarization Measurement:

Functioning system for detecting Touschek particles used for

the absolute beam energy calibration by RD technique

Depolarization process near the integer spin resonance ν=4 observed in Touschek scattering rate

(in the experiment at VEPP-4M at the energy of Tau production threshold)



Summary-1

- Known experimental facts on BBD are optimistic. But no comparisons with theory. No evidence of notable BBD.
- The scale parameter for BBD rate in the high luminosity factory projects can be the inverse beam lifetime.
- In crude guess the maximal order of the BB related spin resonances giving concern equals 3.
- Large Piwinski angle gives a significant reduction of the BBD rate estimate elaborated for head-on BB.
- Preliminary estimates: it seems, BBD is yet not a crucial factor restricting possibilities of the projects.

Summary-2

- BB footprint may overlap at once several spin resonances of the same order due to large vertical tune shift
- Simultaneous account of several relevant resonances can make BBD faster ~ in several times. And still this rate will be notably lower than a critical lever (in the SBF case).
- To refine estimates one must calculate SRF for real parameters. Method to calculate SRF in I.P. for solenoid-based longitudinal polarization schemes has been developed.
- Be careful regarding the C-Tau project (in present status) for which an expected Beam Life Time is relatively not small ~10-20 mins ($1/\lambda_{BBD} \leq 30$ mins).
- Try to minimize SRF in I.P in design project. This helps to kill just all BB related spin resonances.
- Development of BBD simulation code is needed to clarify most complicated cases.

Acknowledgements

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Thank you for attention!