Longitudinally polarized electrons in SuperB

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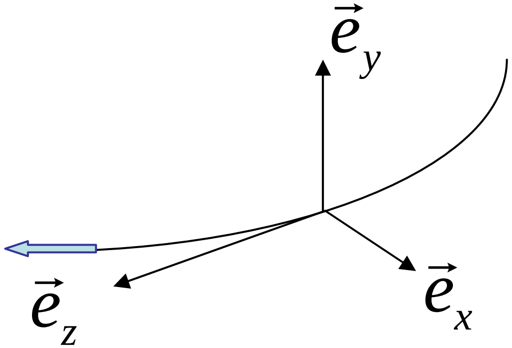
Options for longitudinal spin

1.	Siberian snake (180^0 spin rotator). Spin lies in horizontal plane everywhere. Simplicity, Universality, but depolarization_tau~E^{-7}. Practical limit is E<2.5 GeV for LER bending radii.	E=2 GeV Bl=πBR=21 T*m
2.	90^0 solenoids before FF-bends and after. FF horizontal bends rotate spin around y-axis from x- to z-direction. Spin is upright in arcs. Scheme works only at certain energy. Two insertions are needed to get the spin transparency.	E=4-7 GeV Bl=(π/2)BR
3.	HERA-like 90^0 spin rotator. Combination of transverse bends (J.Buon, K.Steffen). Upright spin in arcs. Advantages: vertical bends are cheaper than solenoids. Spin transparency is garanted. Disadvantage: few meters of the vertical orbit excursion before/after FF.	E=7 GeV

Spin motion description



$$\vec{V} \approx V\left(\vec{e}_z + x'\vec{e}_x + y'\vec{e}_y\right)$$

Orts of the co-moving frame:

$$\vec{a}_1 = \frac{\vec{e}_y \times \vec{V}}{\left|\vec{e}_y \times \vec{V}\right|} \approx \vec{e}_x - x'\vec{e}_z$$

$$\vec{a}_3 = \frac{\vec{V}}{V} \approx \vec{e}_z + x'\vec{e}_x + y'\vec{e}_y$$

$$\vec{a}_2 = \vec{a}_3 \times \vec{a}_1 \approx \vec{e}_y - y'\vec{e}_z$$

Spin perturbations:

$$w_x \approx \nu_0\left(-K_x \frac{\Delta\gamma}{\gamma} - y''\right)$$

$$w_y \approx \nu_0\left(-K_y \frac{\Delta\gamma}{\gamma} + x''\right)$$

$$w_z \approx -(1+a)K_z \frac{\Delta\gamma}{\gamma}$$

$$\nu_0 = \gamma a$$

$$a \approx 1.16 \cdot 10^{-3}$$

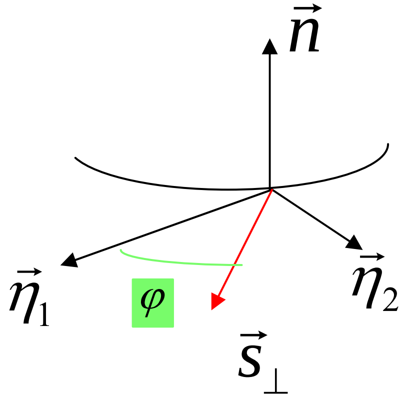
$$K_{x,y,z} = B_{x,y,z} / \left\langle B_y \right\rangle$$

Spin-orbit coupling terms

Chromaticity of spin rotation

Periodic closed spin orbit – always exist!

Derbenev, Kondratenko, Skrinsky, 1970



$$\vec{n}(\theta+2\pi)=\vec{n}(\theta)$$

Real orthogonal vectors $\vec{\eta}_1,\ \vec{\eta}_2$ describe precession of spin around $\vec{n}$. $\vec{\eta}_1\times\vec{\eta}_2=\vec{n}$

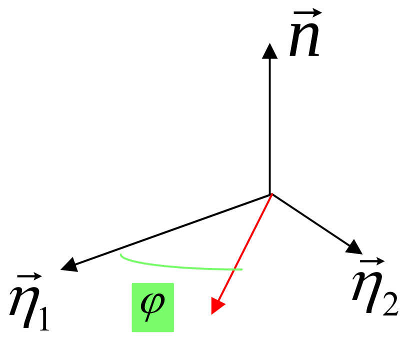
Complex vectors : $\vec{\eta}=\vec{\eta}_1-i\vec{\eta}_2,\ \vec{\eta}^*=\vec{\eta}_1+i\vec{\eta}_2$ are more convenient to use for description of rotation by the angle φ around $\vec{n}$ direction:

$$\vec{\eta}(\theta)=\vec{\eta}(0)e^{i\varphi},\quad \vec{\eta}(\theta)^*=\vec{\eta}(0)^*e^{-i\varphi}$$

$\vec{\eta}(\theta+2\pi)=\vec{\eta}(\theta)e^{i2\pi\nu}$ - precession around $\vec{n}$ by the angle $2\pi\nu$.

ν - is a spin tune. In the flat machine and without solenoids $\nu=\nu_0=\gamma a$.

Calculation of spin orbit distortions



$$\Delta\vec{n}(\theta) = \mathrm{Re}\left(i\vec{\eta}(\theta)^* \int_{-\infty}^{\theta} \vec{\mathrm{w}}\,\vec{\eta}\,d\theta \right)$$

$$\int_{-\infty}^{\theta} \vec{\mathrm{w}}\,\vec{\eta}\,d\theta = \sum_{\mathrm{i}} \frac{\int_{\theta\text{-}2\pi}^{\theta} \vec{\mathrm{w}}_{\mathrm{i}}\vec{\eta}\,d\theta}{1-e^{-i2\pi(\nu+\nu_i)}} = \sum_{\mathrm{i}} \frac{\int_{\theta}^{\theta+2\pi} \vec{\mathrm{w}}_{\mathrm{i}}\vec{\eta}\,d\theta}{e^{i2\pi(\nu+\nu_i)}-1}$$

ν - is a spin tune

$\nu_{\mathrm{i}} = 0, \pm\nu_x, \pm\nu_y$ transverse motion frequencies, $\mathrm{w}_{\mathrm{i}}(\theta+2\pi) = \mathrm{w}_{\mathrm{i}}(\theta)e^{i2\pi\nu_i}$

Calculation of a spin-orbit coupling vector

$$\Delta\vec{n}(\theta) = \mathrm{Re}\left(i\vec{\eta}(\theta)^* \int_{-\infty}^{\theta} \vec{\mathrm{w}}\,\vec{\eta}\,d\theta \right)$$

$\vec{d} \equiv \Delta\vec{n}(\theta) / \left(\dfrac{\Delta\gamma}{\gamma} \right)$ - spin-orbit coupling vector

$$\vec{d} = \vec{d}_{\gamma} + \vec{d}_{\beta}$$

$\vec{d}_{\gamma}$ - direct dependence of $\vec{\mathrm{n}}$ on the energy

$\vec{d}_{\beta}$ - contribution from the betatron motion

$\vec{d}_{\gamma}=0$ - zero spin chromaticity!

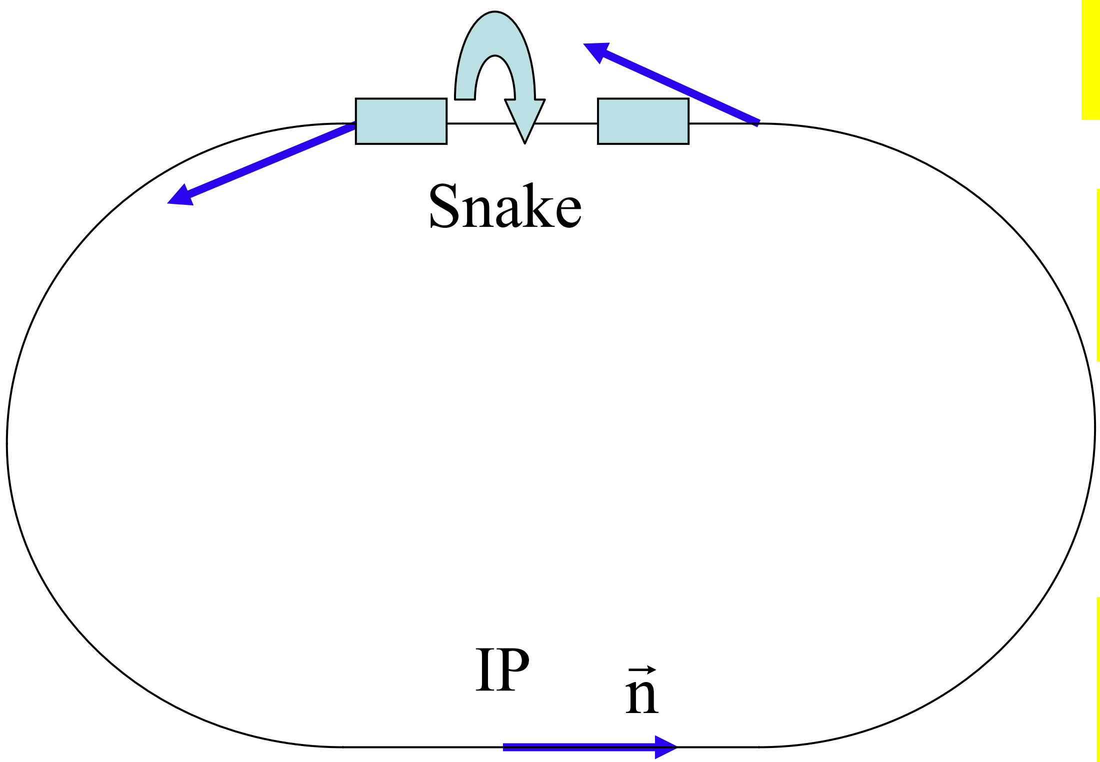
$\vec{d}_{\beta}=0$ - spin transparency!

Polarization Scenario for E=2 GeV

- Polarized electron source.
- Acceleration in a linac.
- About $5*10^{10}$ electrons/pulse at about 40 Hz are needed to compensate particle losses caused by luminosity and by the Touschek effect. Estimation of a lifetime τ~100 s.
- Depolarization time much longer (τ~4000 s at E=2 GeV).
- Establish the closed spin orbit by placing Siberian Snake in the straight section opposite to IP (option for the LER).
- Spin at IP is directed longitudinally at any energy! Spin tune is half integer in case of full Snake and fractional with the Partial Snake.
- Rotation of spin by 180^0 around z-axis is provided by the solenoid field integral Bl=πBR =21 Tm for E=2 GeV.

Spin orbit in presence of Siberian Snake

Derbenev, Kondratenko, Skrinsky, 1977



Snake rotates spin by 180^0 around z-axis

In arcs spin lies in the horizontal plane

At IP the spin is directed longitudinally (exactly!)

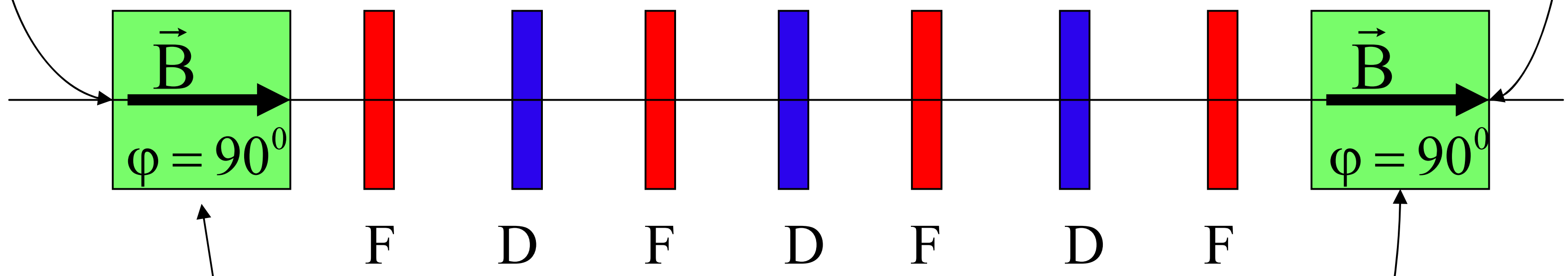
With a partial snake at a magic energy spin is directed also longitudinally at IP as well at the snake's location

180^0 Spin Rotator for Siberian Snake

Decoupling Optics: $T_x = -T_y$

Litvinenko, Zholentz, 1980

$$T_x = -T_y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ - for the spin transparency!



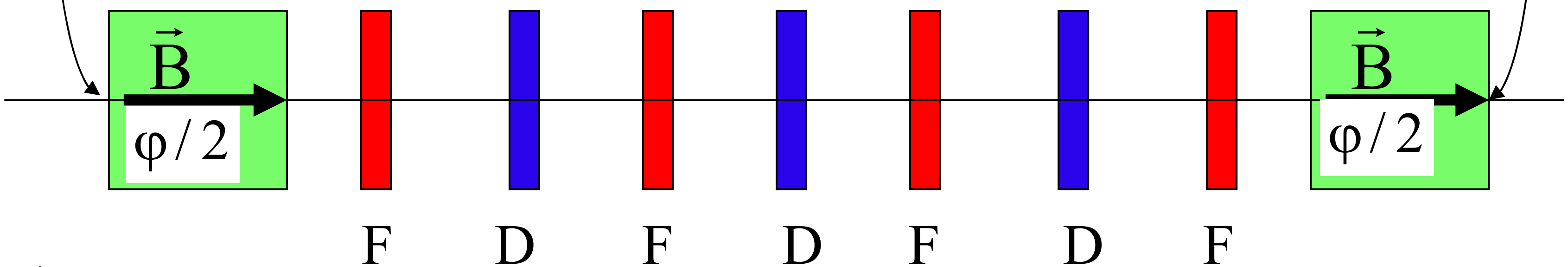
All quads are not skewed!

Each of two solenoids provide 90^0 spin rotation

Spin Rotator for partial Snake

For decoupling again should be $T_x = -T_y$

$$T_x = \begin{pmatrix} -\cos\varphi & -2r\sin\varphi \\ (2r)^{-1}\sin\varphi & -\cos\varphi \end{pmatrix}$$ for the spin transparency!



$r = pc/eB$

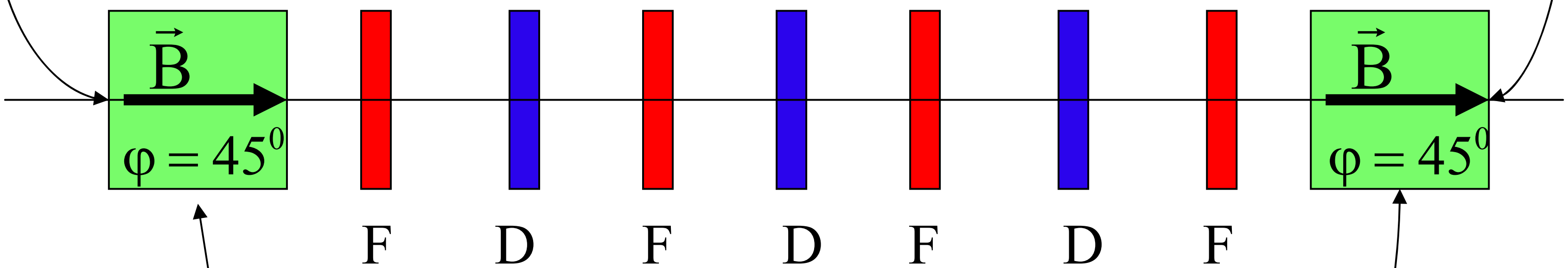
Two solenoids provide spin rotation by $\varphi \le 180^0$

All quads don't need to be skewed!

90^0 Spin Rotator optics

Decoupling condition: $T_x = -T_y$

$$T_x = -T_y = \begin{pmatrix} 0 & -2r \\ (2r)^{-1} & 0 \end{pmatrix}$$ for the spin transparency!



$$r = pc / eB$$

Each of two solenoids provide 45^0 spin rotation

Depolarization time in presence of Siberian Snake

$$\tau_p^{-1} = \frac{5\sqrt{3}}{8}\lambda_e r_e c\gamma^5 \left\langle \frac{1 - \frac{2}{9}(\vec{n}\vec{v})^2 + \frac{11}{18}\vec{d}^2}{|r|^3} \right\rangle$$

$\vec{d} = \gamma \dfrac{\partial \vec{n}}{\partial \gamma}$ is the spin – orbit coupling vector

$$\left\langle \vec{d}^2 \right\rangle_{\min} = \frac{\pi^2}{3}\nu^2$$

Betatron oscillations could increase |d|!
Spin transparency for the snake is desirable.

For $E = 2\,\mathrm{GeV}$ $(\nu = \gamma a = 4.54)$, $r = 20\,\mathrm{m}$, $\tau_p = 4000\,\mathrm{s} >> \tau_{life}$

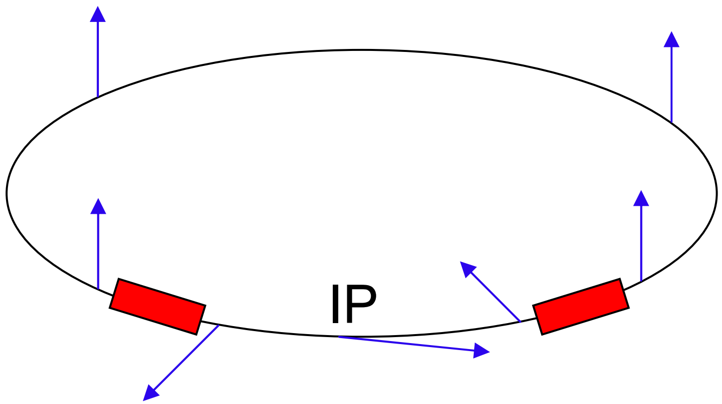
Equilibrium selfpolarization degree $\zeta \propto \vec{b}\vec{n} = 0$!!! (Here $\vec{b} = \vec{B}/B$)

Different polarization options

1. Single snake (full or partial): $\left\langle \vec{d}_\gamma^2 \right\rangle = \pi^2 \nu_0^2 \left(\frac{1}{\sin^2 \varphi} - \frac{2}{3} \right)$

Full snake is preferable due to much lower $\left\langle \vec{d}^2 \right\rangle$

2. Single insertion with two 90^0 rotators and 180^0 rotation around the vertical axis in between: $\left\langle \vec{d}^2 \right\rangle_{min} = \frac{\pi^2 \nu_0^2}{4}$

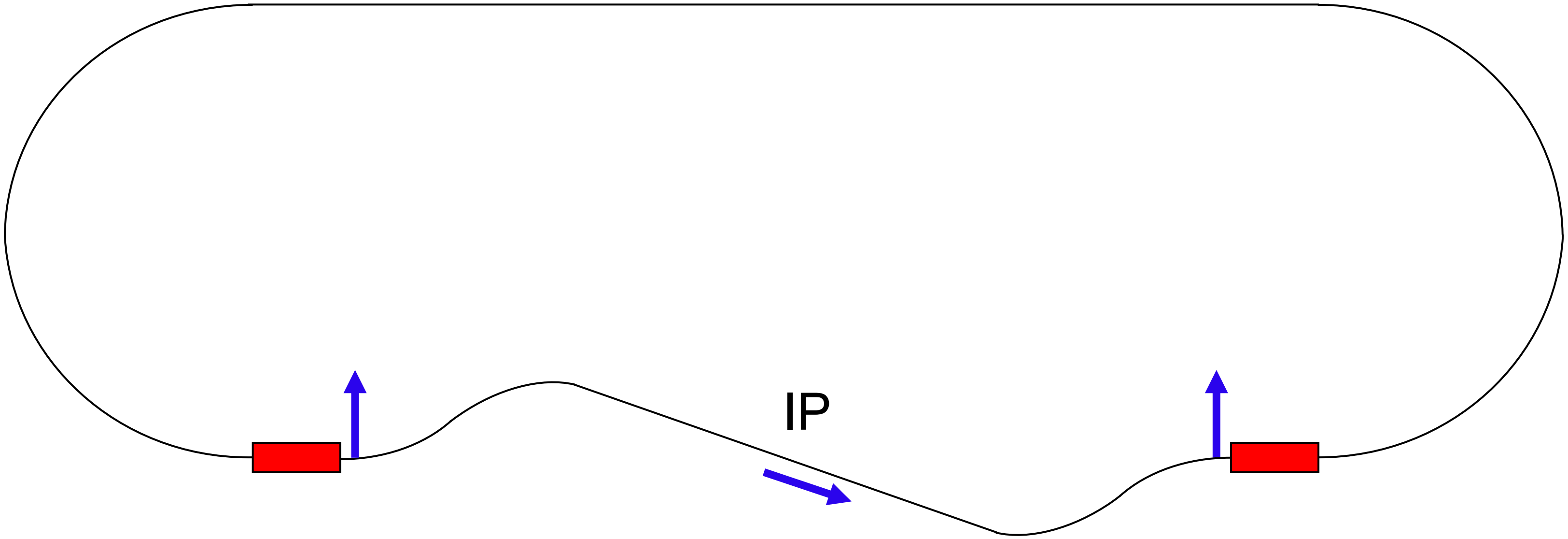


Bend in between two spin rotators:
39.6^0 for E=2 GeV
19.8^0 for E=4 GeV, 11.3^0 for E=7 GeV

Two such insertions placed in sequence compensate spin chromaticity of each other!

Different polarization options, cont'd

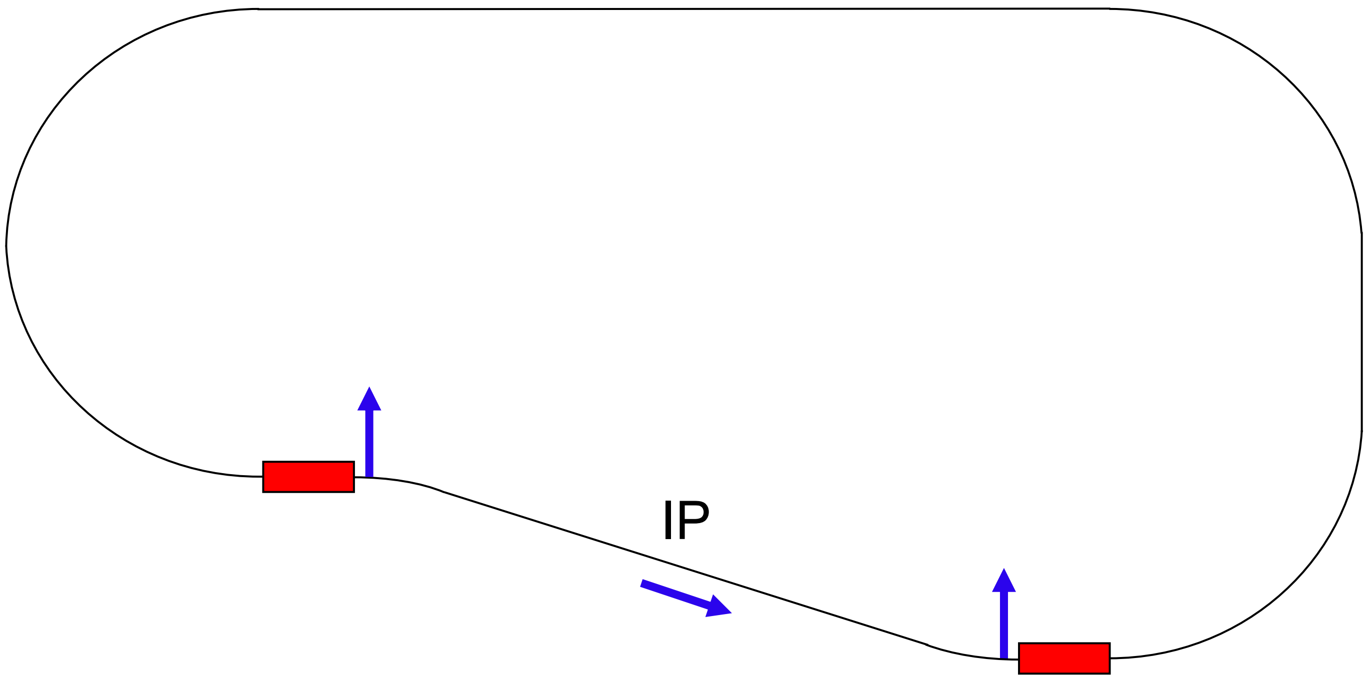
3a. Single insertion with +/-90^0 solenoids and anti-symmetric bend in between



Advantage: Spin direction is achromatic in arcs. $< d^2 >_{arcs} = 0$.
Disadvantage: Extra bends are needed.

Different polarization options, cont'd

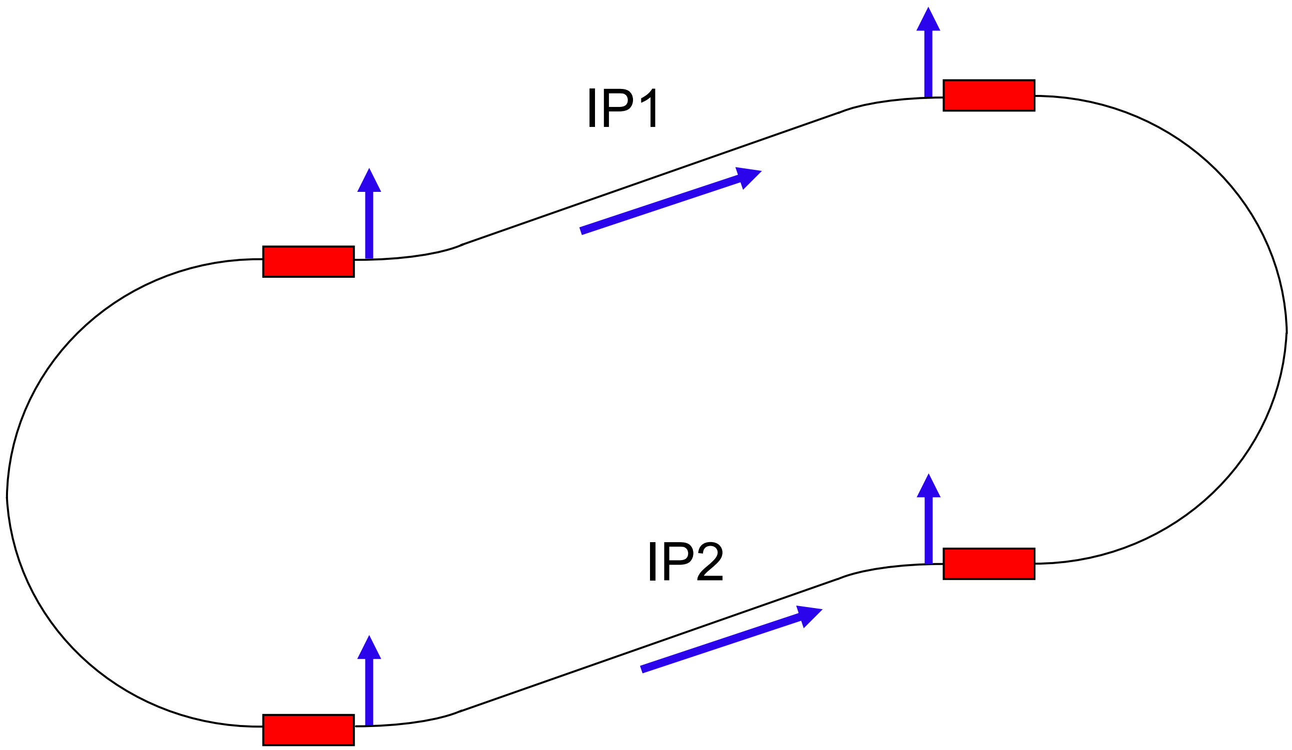
3b. +/-90^0 solenoids and anti-symmetric bend in between (not in scale!).



Advantage: Less number of bends between spin rotators compared to 3a. version.
Spin direction is achromatic in arcs. $< d^2 >_{arcs} = 0$.

Different polarization options, cont'd

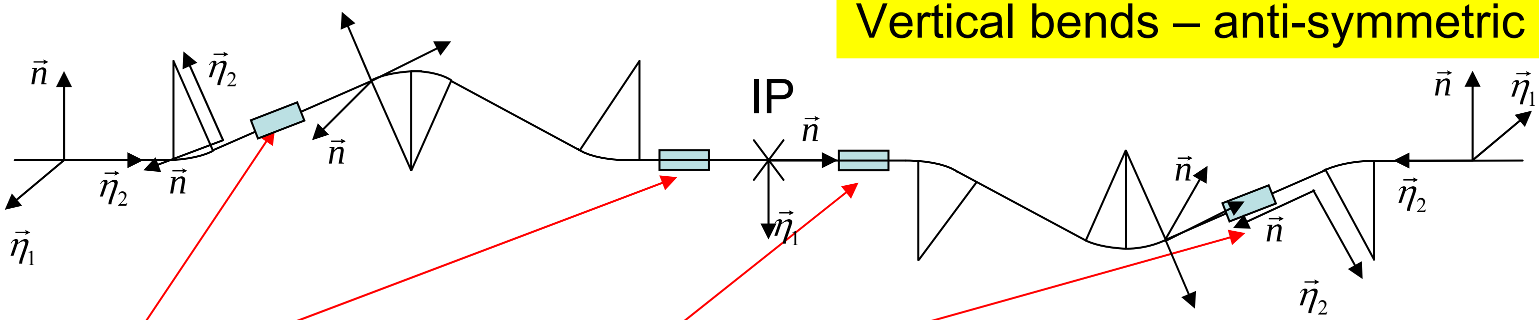
3c. Two identical insertions with +/-90^0 solenoids and anti-symmetric bends in between.



Advantage: Spin direction is achromatic in arcs. $< d^2 >_{arcs} = 0$.
Two arcs become identical to each other. Second IP presented as option.
Disadvantage: Extra bends are needed.

Transverse bends 90^0 spin rotator

Vertical bends – anti-symmetric



Horizontal bends - all positive

All vertical and horizontal bends are equal to 5.66^0 at 7 GeV (90^0 for spin). They are achromatic being divided in two half-bends and lenses in between.

After two first bends x-plane becomes inclined by 97.5 mrad. Could be rolled back by weak solenoid with Bl=0.455 T*m.

Advantages: Dipoles are cheaper than solenoids; Spin transparent solution.

Disadvantages: Few meters orbit bump in vertical direction. Influence on circumference.

Comparison of Lifetimes

Beam bremsstrahlung cross – section :

$\sigma_{\text{Loss}} \approx 1.8 \cdot 10^{-25}\ \text{cm}^2$ - estimated by E.Paoloni

For $L = 10^{36}\ \text{cm}^{-2}\text{s}^{-1} \quad \dot{N} \approx 1.8 \cdot 10^{11}\ \text{s}^{-1}$

$$\tau_{\text{Lum}} = \frac{2.4 \cdot 10^{14}}{1.8 \cdot 10^{11}(\text{s}^{-1})} = 1300\ \text{s}$$

$$\tau_{\text{Touschek}} = 100\ \text{s}\,?$$

$$\tau_p = 4000\ \text{s}$$

$$\tau_p / \tau_{\text{beam}} \approx 40$$

Polarization equilibrium.

Polarization degree of electrons from a gun: $\varsigma_{beam} = 0.9$

Asymmetric FF bends (d_{arc}=0!). Spin relaxation time: $\tau_p = 3500$ s

Equilibrium polarization by SR: $\varsigma_p = 0.12$ (4 GeV), $\varsigma_p = 0.06$ (7 GeV)

Average polarization (taking into account some depolarization in a ring, mainly by wigglers): $\tau_{beam} = 3\,\text{min}$.

$$\varsigma = \varsigma_{beam} \frac{\tau_p}{\tau_{beam} + \tau_p} + \varsigma_p \frac{\tau_{beam}}{\tau_{beam} + \tau_p}$$

Finally average polarization with asymmetric FF bends:

$\varsigma = 87\%$ (4 GeV), $\varsigma = 84\%$ (7 GeV)

What one could gain having two polarized beams?

$\zeta = \dfrac{\zeta_1 + \zeta_2}{1 + \zeta_1 \zeta_2}$ - event rate asymmetry

An example: $\zeta = 0.995$ for $\zeta_1 = \zeta_2 = 0.9$

$\dfrac{L}{L_0} = 1 + \zeta_1 \zeta_2 = 1.81$ – gain in luminosity compared to the unpolarized beams

Polarized positrons? Difficult task!

Decoupling Insertion between two Solenoids

$$M_{Sol} = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} \cdot \begin{pmatrix} I \cdot \cos(\varphi) & I \cdot \sin(\varphi) \\ -I \cdot \sin(\varphi) & I \cdot \cos(\varphi) \end{pmatrix}$$

$$M_{Sol} \cdot \begin{pmatrix} T_x & 0 \\ 0 & T_y \end{pmatrix} \cdot M_{Sol} = ???$$

$$\text{For } T_x = -T_y \;\rightarrow$$

$$\begin{pmatrix} I \cdot \cos(\varphi) & I \cdot \sin(\varphi) \\ -I \cdot \sin(\varphi) & I \cdot \cos(\varphi) \end{pmatrix} \cdot \begin{pmatrix} T & 0 \\ 0 & -T \end{pmatrix} \cdot \begin{pmatrix} I \cdot \cos(\varphi) & I \cdot \sin(\varphi) \\ -I \cdot \sin(\varphi) & I \cdot \cos(\varphi) \end{pmatrix} =$$

$$= \begin{pmatrix} T & 0 \\ 0 & -T \end{pmatrix} \;\rightarrow\; M_{Sol} \cdot \begin{pmatrix} T & 0 \\ 0 & -T \end{pmatrix} \cdot M_{Sol} = \begin{pmatrix} ATA & 0 \\ 0 & -ATA \end{pmatrix}$$

Polarization Measurements

Compton scattering of circular polarized light on longitudinally polarized electrons – high asymmetry!

Conclusion

- High degree $\zeta > 90\%$ electron polarization is achievable from a gun.

- Siberian Snake concept is applicable below 2.5 GeV. It provides the longitudinal spin direction at IP. Could be made spin transparent. But, unavoidably any snake is spin chromatic!

- Partial Snake concept works at magic energies: 1.76, 2.2, 2.64, … GeV. Saves the needed longitudinal field integral.

- At 7 GeV the HERA-like spin rotator approach looks most favorable . Spin transparent solution with all positive horizontal bends! But vertical bends contribute substantially to the vertical emittance growth.

- In both scenarios the average polarization of the circulated electron beam could reach $\zeta > 80\%$