COLLECTIVE BEAM-BEAM INSTABILITIES OF BUNCHES WITH TUNESPREADS

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Near sum resonances the tunespreads can result in Landau anti-damping of resonant coherent oscillations (Ya. S. Derbenev, N. S. Dikansky, All Union PAC, Moscow 1970, v.2, 391).

Coherent oscillations of colliding bunches become unstable, when the incoherent oscillation tunes approach **the sum resonances:** e.g. $\nu_x \simeq n/(2m)$.

We test if incoherent tunespreads of colliding bunches also result in Landau anti-damping of coherent beam-beam oscillations.

More details: D.V. Pestrikov, Nucl. Instr. and Meth. A 588/3, p. 336, 2008.

Two identical, short, counter-moving relativistic electron and positron bunches pass separate storage rings with identical lattices and interact head-on at a single interaction point (IP). We assume a zero dispersion function at the IP.

Incoherent horizontal oscillations of particles:

$$X = \sqrt{J\beta}\cos\psi,\ \ P_x = -p\sqrt{J/\beta}\sin\psi.$$

Here, $I = pJ/2$ and ψ ($\psi(\theta + 2\pi) = \psi(\theta) + 2\pi\nu_x$) are the action-phase variables, $\Pi = 2\pi R_0$ is the perimeter of the closed orbit, $s = R_0\theta$ is the path along the closed orbit, $p = \gamma Mc$ is the reference particle momentum.

We use the model where:

Colliding bunches are very flat:

$$f^{(1,2)}(I_y, x, \psi, \theta) = \delta(I_y) f^{(1,2)}(x, \psi, \theta).$$

The bunches execute coherent oscillations only in the horizontal plane:

$$f^{(1,2)}(x, \psi, \theta) = \frac{e^{-x}}{p\epsilon} + \sum_{m=-\infty}^{\infty} f_m^{(1,2)}(x, \theta) e^{im\psi}, \;\; I = xp\epsilon.$$

In the first approximation of the perturbation theory and ν_x near $n/(2m)$ the combinations $f_m^{(\pm)} = f_m^{(1)} \pm f_m^{(2)}$

$$f_m^{(\pm)} = e^{-x} p_m^{(\pm)}(x) \frac{x^{m/2}}{i^m} \sum_{n=-\infty}^{\infty} \frac{e^{-i(\nu+n)\theta}}{\nu + n - m\nu_x(x)}$$

obey ($m > 0$; slow modes are: $\nu \simeq m\nu_x \simeq n - m\nu_x$):

$$\boxed{\frac{d}{dx}\left(x^{m+1}\frac{dp_m^{(\pm)}(x)}{dx}\right) = \pm\frac{2\delta(x)}{z_1^2 - \delta^2(x)}e^{-x}x^m p_m^{(\pm)}(x),}$$

with boundary conditions:

$$p_m^{(\pm)}(z_1, 0) = 1,\ \frac{dp_m^{(\pm)}(z_1, 0)}{dx} = \pm\frac{2\delta(0)}{(m+1)(z_1^2 - \delta^2(0))}.$$

Dispersion equations of the problem:

$$\boxed{p_m^{(\pm)}(z_1, \infty) = 0.}$$

Here, $\xi = Ne^2/(2\pi pc\epsilon)$ (and e.g. $B = 2\pi\xi < 1$)

$$z_1 = \frac{1}{m\xi}\left(\nu - \frac{n}{2}\right),\ \delta(x) = \frac{1}{\xi}\left(\nu_x(x) - \frac{n}{2m}\right),$$

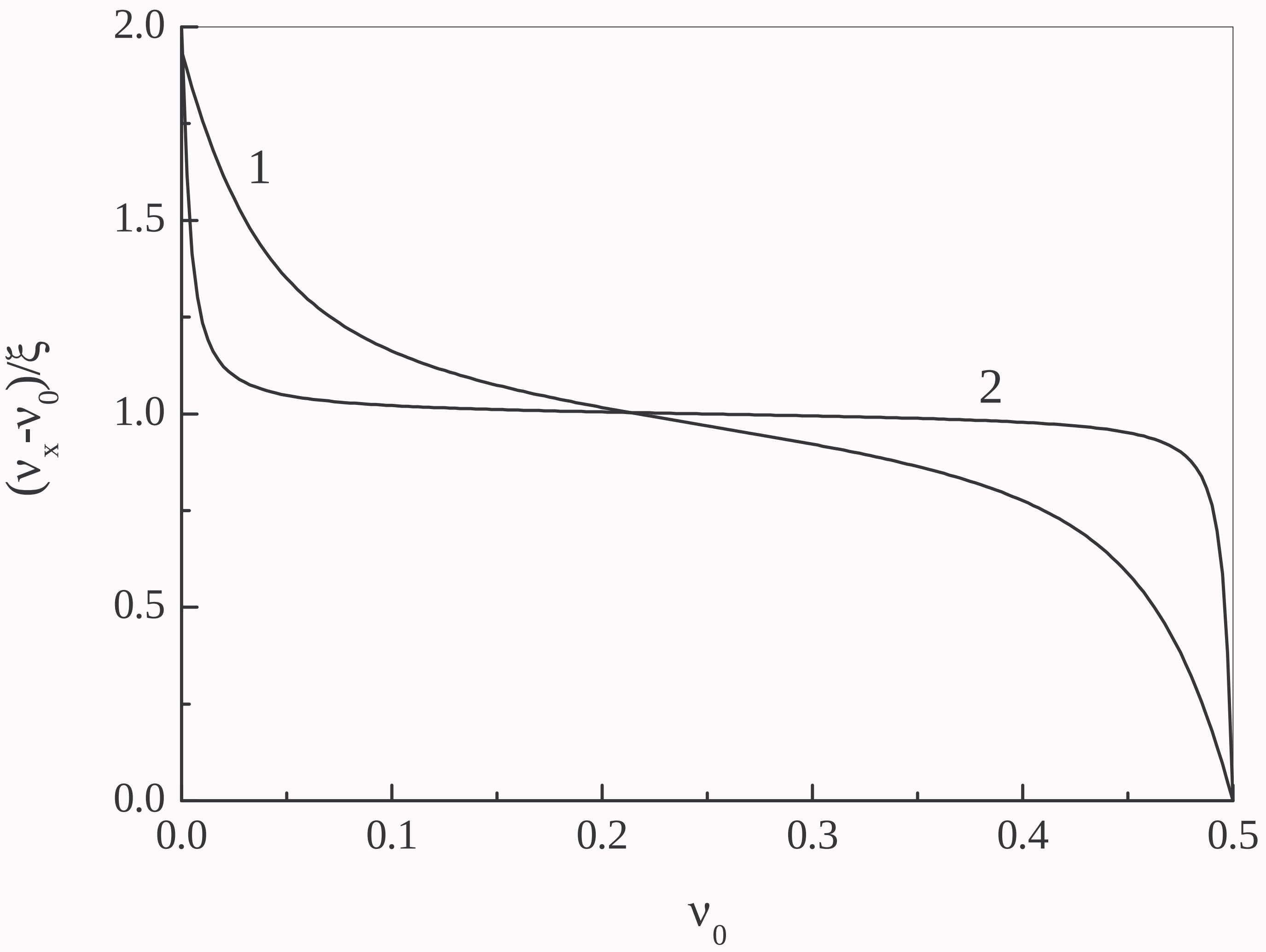
We look for unstable modes $z_1 = ir$ with largest r.

Calculating increments, widths of stopbands of such modes and their positions in ν_x, we take into account self-consistent variations of the oscillation tunes and of β-functions by the beam-beam interactions (but, ignore flip-flop).

Using simulations, we find that in our model

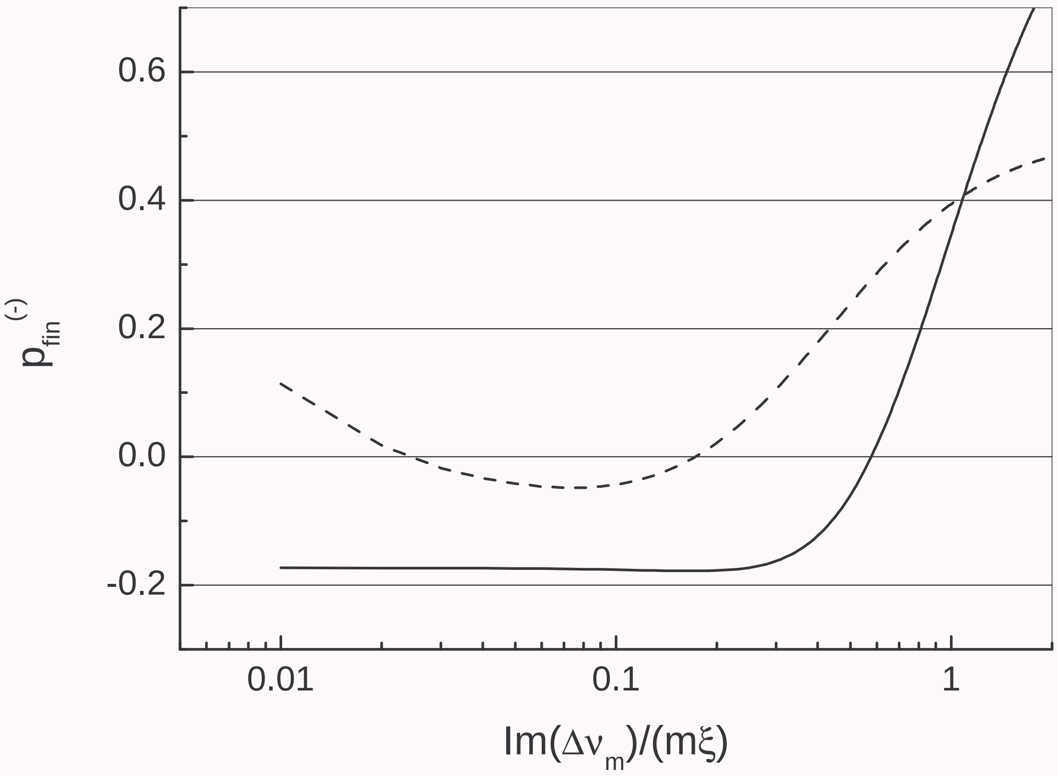
$$\nu_x(x) = \nu_x - \Delta\nu_x(0)\left(1 - \frac{1 - e^{-x}}{x}\right),$$

$\Delta\nu_x(0) = \nu_x - \nu_0$ is the linear beam-beam tuneshift.

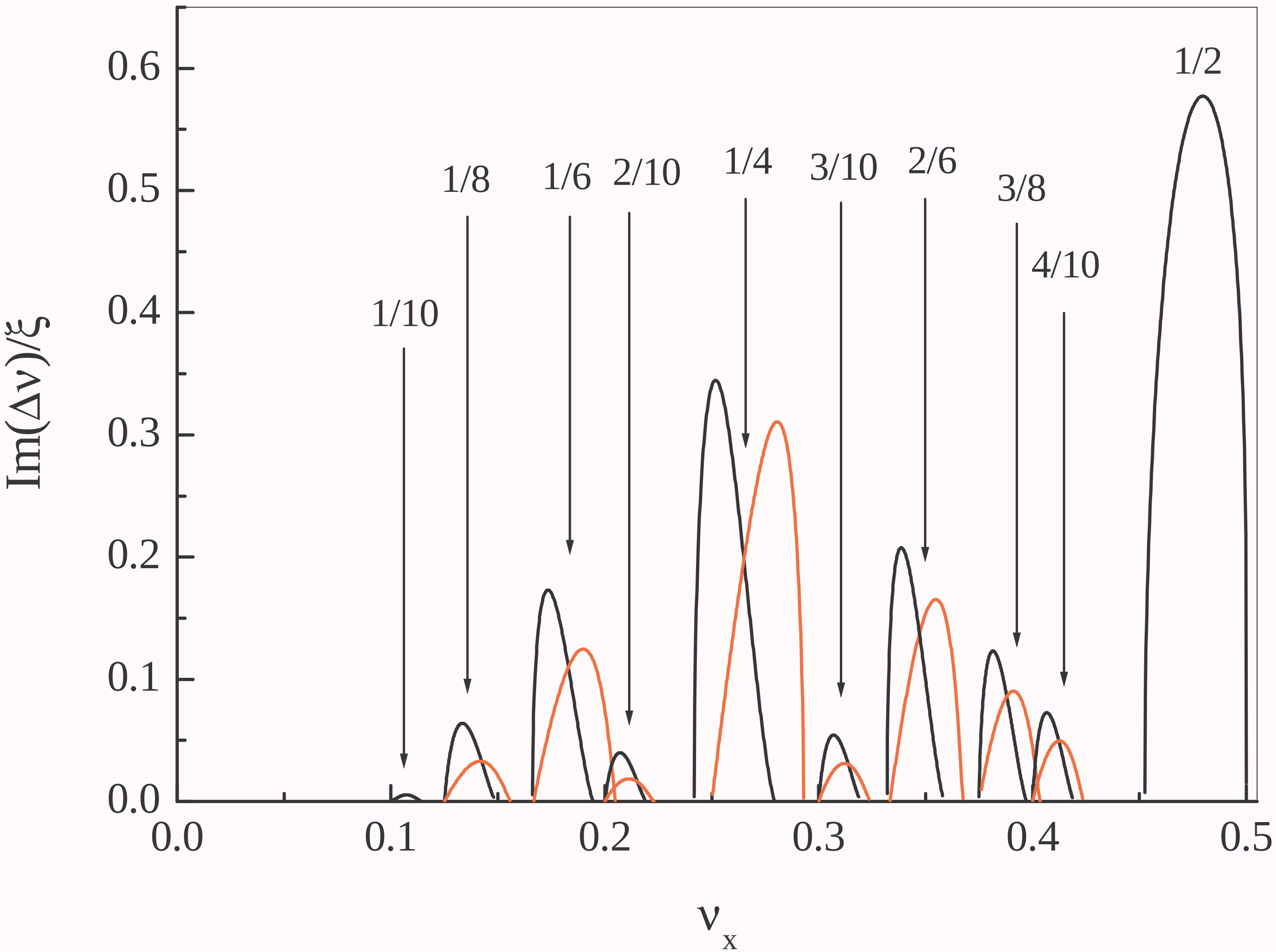


Line 1 – $\xi = 0.05$, line 2 – $\xi = 0.005$.

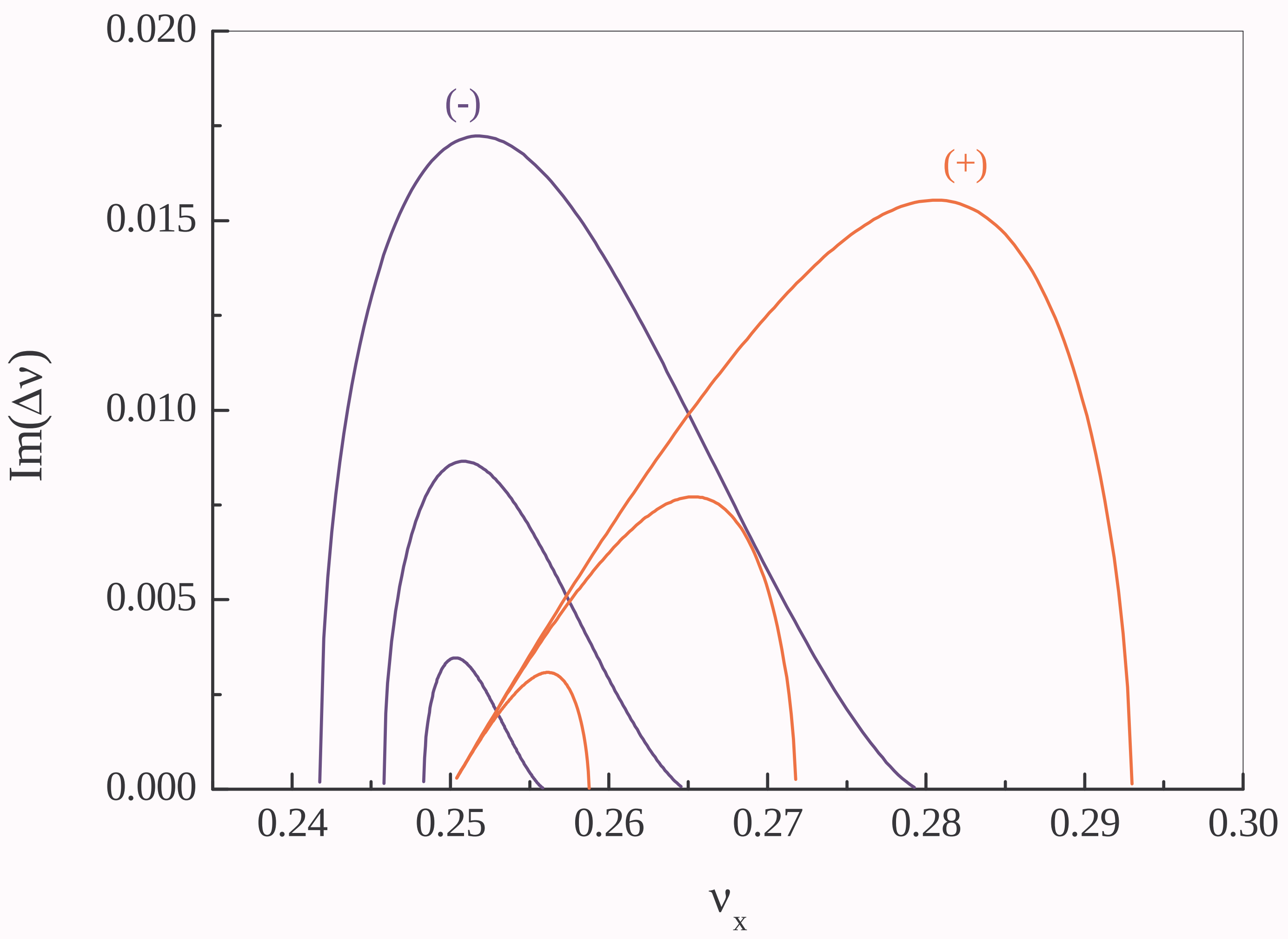
Example: dispersion equation: $p_m^{(-)}(z_1, \infty) = 0$



Rez_1=0, solid near $1/2$, dashed – $1/4$, $\xi = 0.05$.



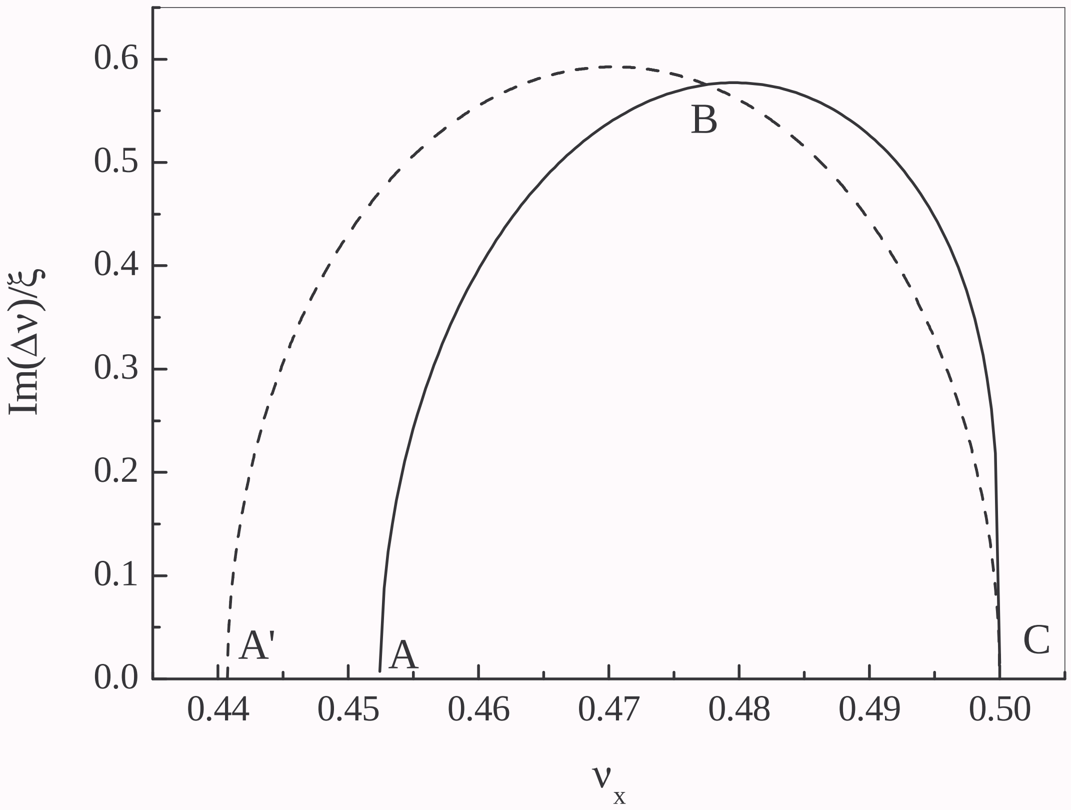
Non-monochromatic bunches, $\xi = 0.05$.



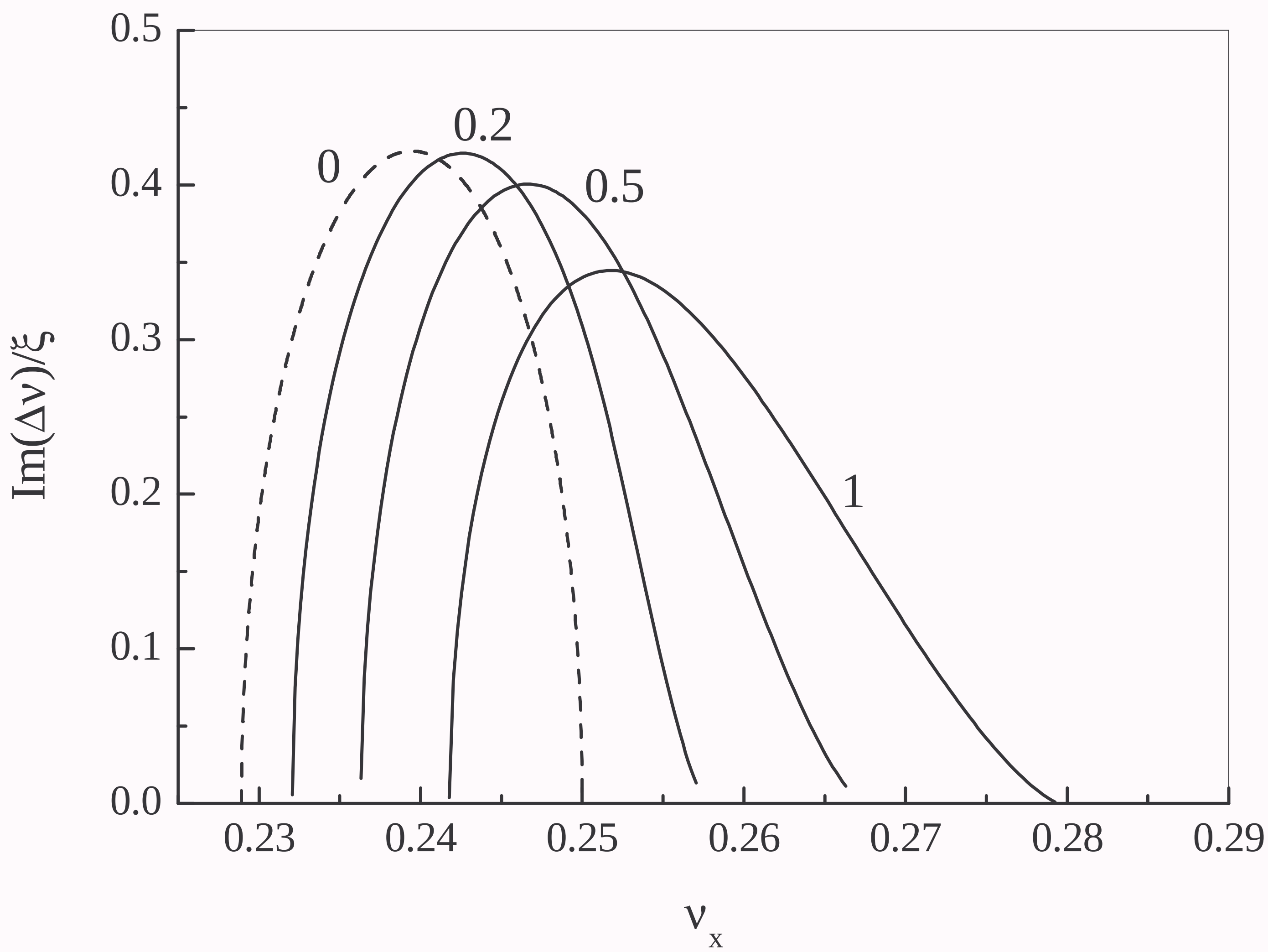
Non-monochromatic, $m = 2$, $\xi = 0.05,\ 0.025,\ 0.01$.

Non-monochromatic versus monochromatic

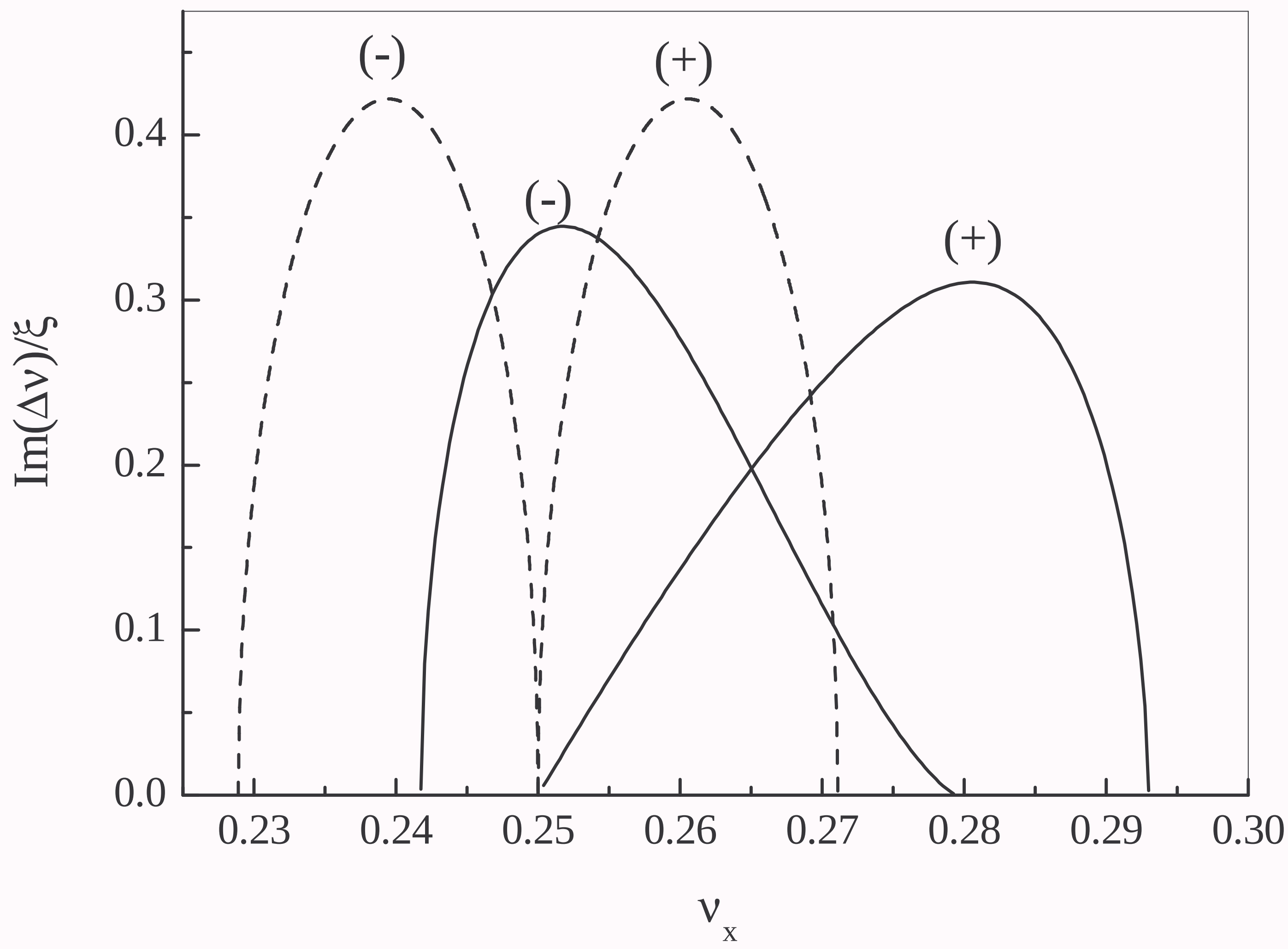
$\nu_x(x) = \nu_x$ (dashed line; for all: $\xi = 0.05$).



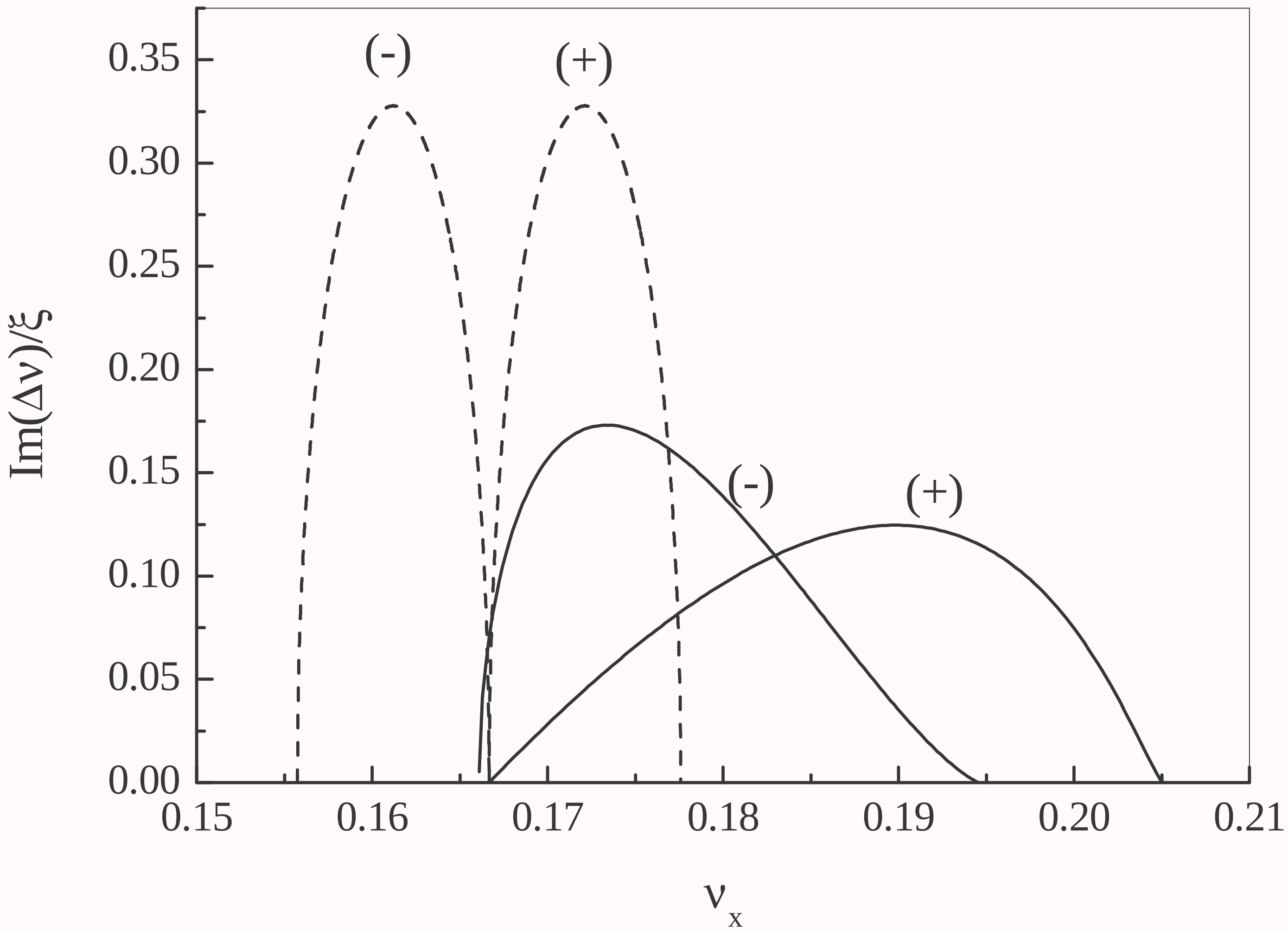
Mode (-,1), weak Landau anti-damping.



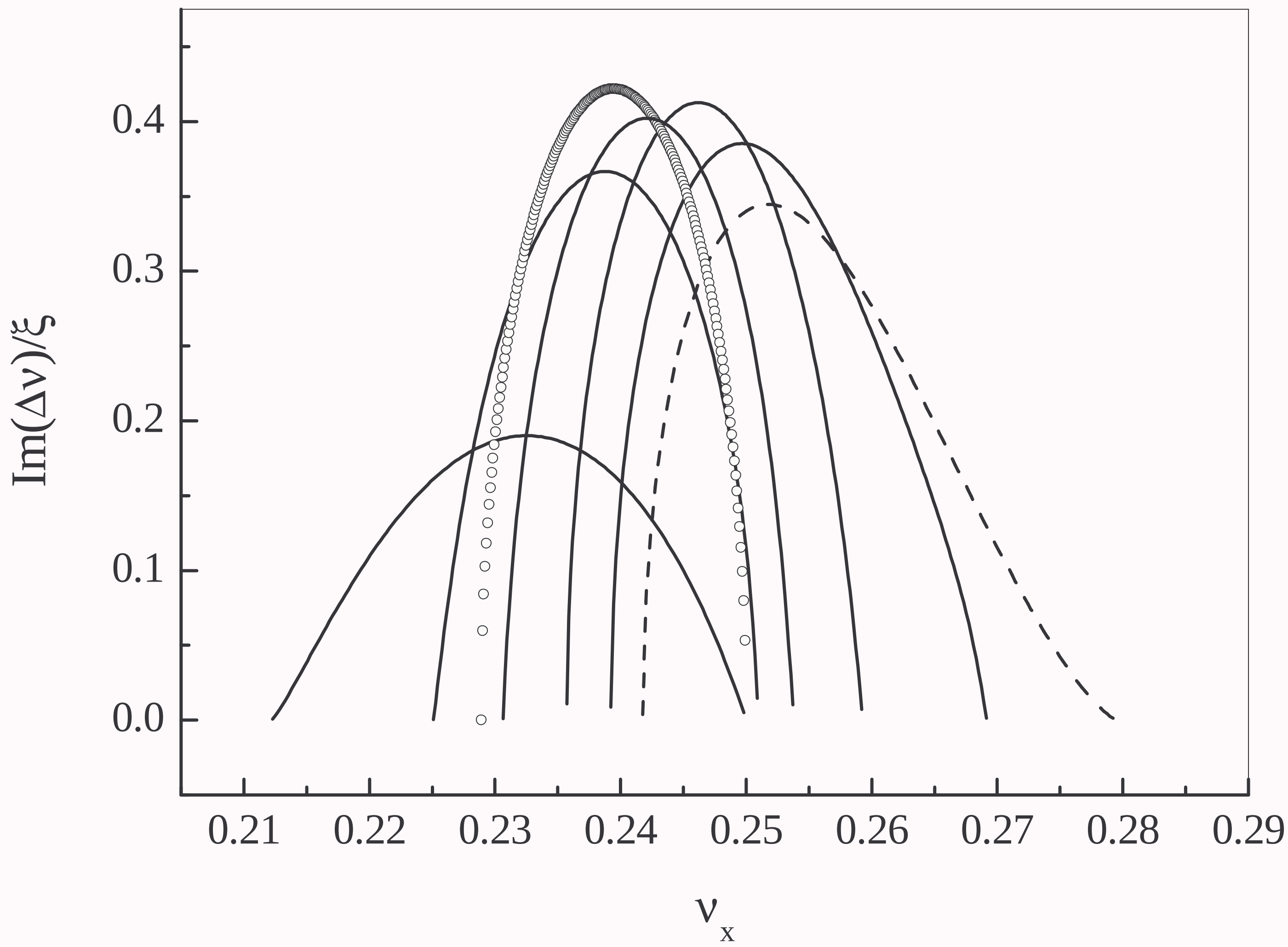
From monochromatic to non-monochromatic (-, 2).



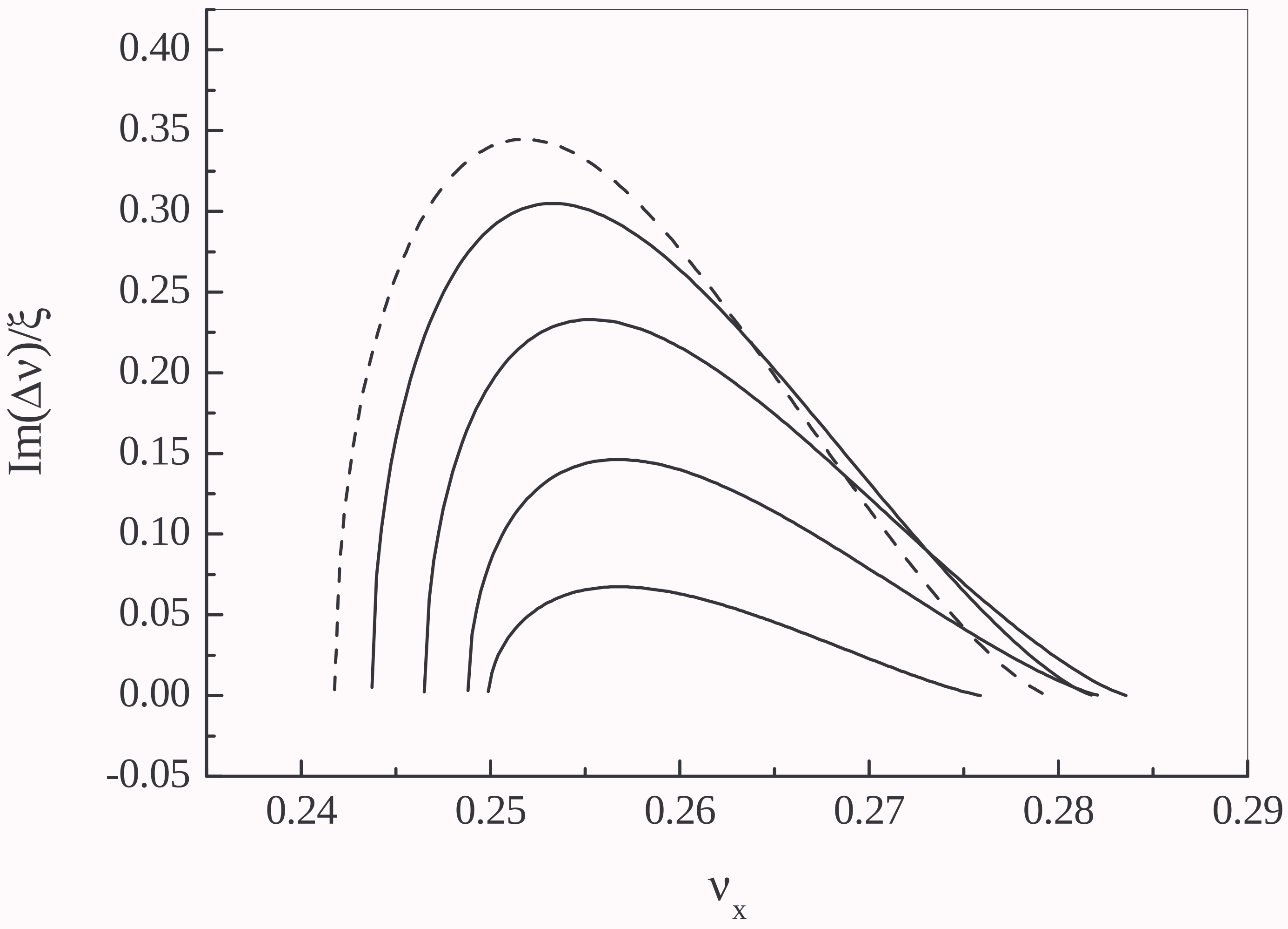
Near 1/4; Dashed – monochromatic.


0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Im(Δν)/ξ
0.15
0.16
0.17
0.18
0.19
0.20
0.21
ν_x
(-)
(+)
(-)
(+)

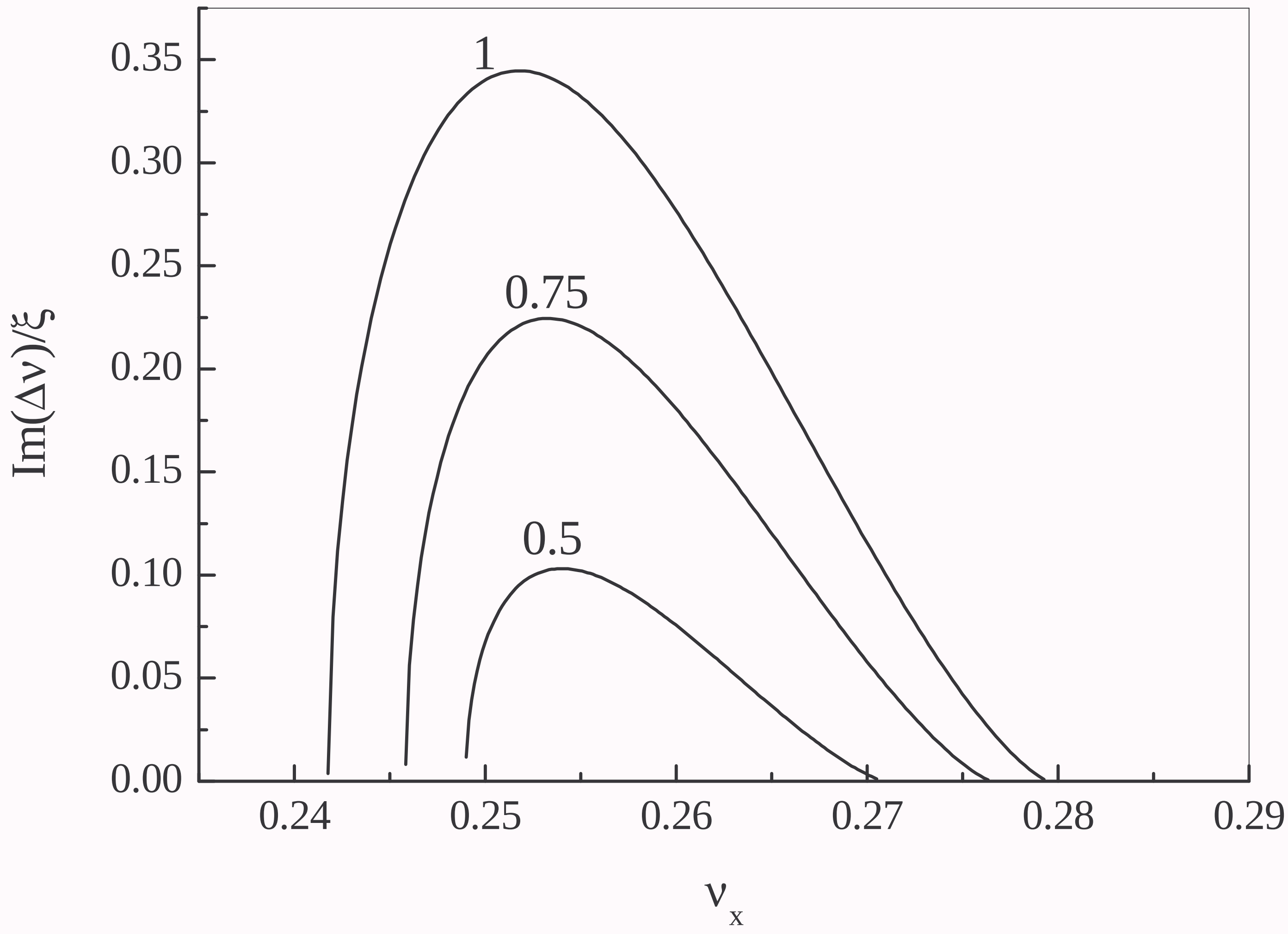
Near 1/6.



Positive octupole tuneshift $d\Delta\nu_x/dI > 0$; mode (-,2).



Negative octupole tuneshift; mode (-,2).



Simulation of the hour-glass reduction; mode (-,2).

Conclusions

The beam-beam tunespreads result in the instabilities of coherent beam-beam oscillations in the regions of betatron tunes ν_x where coherent oscillations of monochromatic bunches would be stable – i.e. in **Landau anti-damping**.

This Landau anti-damping is a generic feature for sum resonance coherent instabilities.

Octupole tunespreads and hour-glass reductions do not cancel Landau anti-damping.

However, coherent and/or incoherent oscillations interference.