COLLECTIVE BEAM-BEAM INSTABILITIES OF BUNCHES WITH TUNESPREADS

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Abstract

We discuss effects of Landau damping on the stability of coherent oscillations of short identical colliding bunches. Near the sum-type resonances $n/(2m)$, where n and m are integers, these oscillations are unstable. Comparing the stopbands calculated for monochromatic and non-monochromatic bunches, we have found that the beam-beam tunespreads increase the widths of the stopbands of coherent modes thus, resulting in Landau anti-damping of coherent beam-beam oscillations. The tunesperads due to octupole fields do not eliminate Landau anti-damping.

INTRODUCTION

Periodic perturbations of the particle oscillations at the interaction point (IP) of a collider by space charge forces of the counter-moving colliding bunches result in numerous resonant instabilities which can limit the operational performance of the collider. In particular, such beam-beam perturbations can result in the resonant instabilities of coherent oscillations of the colliding bunches. In the simplest case, if $\nu_{1,2}$ are the tunes of the betatron oscillations of particles in the colliding bunches 1 and 2, coherent oscillations of these bunches become unstable provided that the oscillation tunes approach the sum-type resonance region [1] $m_1\nu_1 + m_2\nu_2 = n$. Here, $m_{1,2}$ and n are positive integers. Nonlinear dependencies of the beam-beam forces on particle coordinates in the bunches produce the tuneshifts and the tunespreads of the particle oscillations. Therefore, some Landau damping of unstable modes could be expected. However, a qualitative study made in Ref.[2] has shown that the beam-beam tunespread is not sufficient to suppress the beam-beam coherent instability and that near the resonant tunes still can exist unstable coherent beam-beam modes. Using a different approach, this result was confirmed numerically for the beam-beam π–modes in Ref.[3].

Descriptions of such instabilities are usually complicated by a generic self-consistency of the beam-beam interactions. We simplify calculations of Landau damping effects of self-consistent coherent oscillations of short identical colliding bunches using the model described in Refs.[1, 2, 3], or in Ref.[4]. Using the technique developed in Refs.[5] and [6], we calculate the stopbands for coherent oscillations of colliding bunches taking into account and/or ignoring the beam-beam and others tunespreads. For the last case, we call colliding bunches as the monochromatic ones. Comparing results of such calculations, we find out that Landau damping due to the nonlinearity of the beam-beam forces, generally, increases the widths of the stopbands of coherent beam-beam modes. It also can change positions of the stopbands of unstable modes relative to the values of the resonant tunes. These features mean that together with the damping of unstable modes of the monochromatic colliding bunches the tunespreads result in the instabilities for the regions of tunes where the modes of the monochromatic bunches were stable – i.e. in the Landau anti-damping. Such an anti-damping is a specific feature for resonant instabilities of coherent oscillations near the sum-type coupling resonances [7].

THE MODEL

We consider collisions of two identical short counter-moving relativistic electron and positron bunches, which move in separate storage rings and interact head-on at a single interaction point (IP). In our calculations we assume a zero dispersion function at the IP. Incoherent e.g. the horizontal oscillations of particles in the bunches are described using $x = \sqrt{J\beta}\cos\psi$ and $p_x = px'/R_0 = -p\sqrt{J/\beta}\sin\psi$. Here, $I = pJ/2$ and ψ ($\psi(\theta+2\pi) = \psi(\theta) + 2\pi\nu_x$) are the action-phase variables of the unperturbed incoherent oscillations, $\Pi = 2\pi R_0$ is the perimeter of the closed orbit, $s = R_0\theta$ is the path along the closed orbit, primes mean $d/d\theta$, $p = \gamma Mc$ is the value of the momentum of the reference particle, β denotes the value of the β-functions of the horizontal oscillations at the center of IP.

Coherent oscillations of bunches are described using harmonics of their distribution functions f in phases ψ. We use a special model, where the bunches have very flat unperturbed distribution functions so that $f^{(1,2)}(I_y, x, \psi, \theta) = \delta(I_y) f^{(1,2)}(x, \psi, \theta)$,

$$f^{(1,2)}(x,\psi,\theta) = \frac{e^{-x}}{p\epsilon} + \sum_{m=-\infty}^{\infty} f_m^{(1,2)}(x,\theta) e^{im\psi}, \quad (1)$$

(y marks the values relating to the vertical oscillations, $I = xp\epsilon$, ϵ is the horizontal bunch emittance) and where the bunches execute coherent oscillations only in the horizontal plane. Assuming also that the bunches move in the rings with identical lattices, $p\epsilon|f_m^{(1,2)}| \ll 1$, and neglecting in the linearized Vlasov equations for $f_m^{(1,2)}$ the fast-oscillating terms, we find that the combinations

$$\begin{aligned} f_m^{(\pm)} &= f_m^{(1)} \pm f_m^{(2)} \\ &= e^{-x} p_m^{(\pm)}(x) \frac{x^{m/2}}{i^m} \sum_{n=-\infty}^{\infty} \frac{e^{-i(\nu+n)\theta}}{\nu + n - m\nu_x(x)} \end{aligned}$$

describe normal modes of identical colliding bunches [8]. Since the functions $f_m^{(1,2)}$ are linear combinations of the modes $f_m^{(\pm)}$, coherent oscillations of identical colliding bunches are stable only in the regions, where both modes $f_m^{(+)}$ and $f_m^{(-)}$ are stable. For small values of the beam-beam parameter $\xi = Ne^2/(2\pi pc\epsilon)$ (e.g. $B = 2\pi\xi < 1$) and near the resonances $\nu_x = n/(2m)$, we can neglect in equations for $f_m^{(\pm)}$ the contributions of non-diagonal in $|m|$ modes. Resulting equations for $p_m^{(\pm)}(z_1, x)$ read (more details in Ref.[6], $m > 0$)

$$\frac{d}{dx}\left(x^{m+1}\frac{dp_m^{(\pm)}(x)}{dx}\right) = \pm e^{-x}x^m V(x)\, p_m^{(\pm)}(x). \quad (2)$$

These equations should be solved with the boundary conditions $p_m^{(\pm)}(z_1, 0) = 1$, and $dp_m^{(\pm)}(z_1, 0)/dx = \pm V(0)/(m+1)$. The dispersion equations of the problem read

$$p_m^{(\pm)}(z_1, \infty) = 0. \quad (3)$$

Here, $V(x) = 2\delta(x)/(z_1^2 - \delta^2(x))$ ($\mathrm{Im} z_1 > 0$), and

$$z_1 = \frac{1}{m\xi}\left(\nu - \frac{n}{2}\right),\ \delta(x) = \frac{1}{\xi}\left(\nu_x(x) - \frac{n}{2m}\right), \quad (4)$$

n is the azimuthal number of the resonance harmonic.

RESULTS

Near the resonances $\nu_x \simeq n/(2m)$ Eqs.(2) and (3) always have solutions with the eigenvalues $\mathrm{Re}(z_1) = 0$ and $\mathrm{Im}(z_1) \neq 0$ [2]. Since $p_m^{(\pm)}(z_1, \infty)$ are functions of z_1^2, these solutions describe unstable modes. The values of the increments of unstable modes as well as the widths of relevant stopbands and their positions in ν_x are found solving Eqs.(2) and (3) numerically. In these calculations we also took into account self-consistent variations of the oscillation tunes and of β-functions by the beam-beam interactions: $\cos\mu_0 = \cos\mu_x + B\sin\mu_x$, $\beta\sin\mu_x = \beta_0\sin\mu_0$, where $\mu = 2\pi\nu$, the subscript 0 marks bare values. Below, we neglect possible flip-flop spitting of the betatron functions. Using simulations, we have found that in our model the tunes depend on x according to

$$\nu_x(x) = \nu_x - \Delta\nu_x(0)\left(1 - \frac{1 - e^{-x}}{x}\right), \quad (5)$$

where $\Delta\nu_x(0) = \nu_x - \nu_0$ is the linear beam-beam tuneshift (e.g. in Fig. 1) Calculating increments for the modes $(\pm)$ with $1 \leq m \leq 5$ and $1 \leq n \leq 4$, we find the stopband depicted in Fig. 2. Although only the segment $0 \leq \nu_x \leq 1/2$ is shown in Fig. 2, the stopbands for higher, or lower values of ν_x appear periodically with a period in ν_x of $1/2$. For dipole oscillations $m = 1$ we have found no roots of the dispersion equation (3) for $0 \leq \nu_x \leq 1/2$. This result means that only $(-)$ dipole mode has the stopband below the resonance $\nu_x = 1/2$. The stopband for the mode $(-, 2)$ starts slightly below the resonance $\nu_x = 1/4$. The lower ends of all other found stopbands are found to be close to the corresponding resonant tunes $\nu_x = n/(2m)$. Numerical values of the maximum increments and of the widths

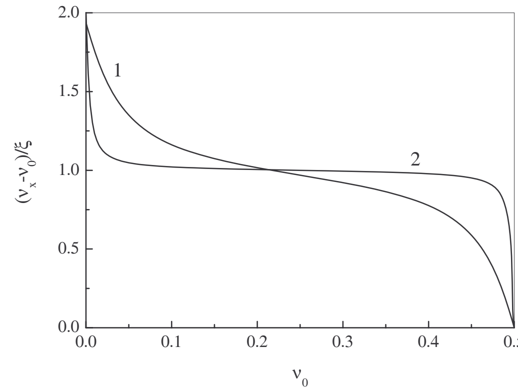

Figure 1: Dependence of the linear beam-beam tune shift on the bare tune (ν_0); line 1 – $\xi = 0.05$, line 2 – $\xi = 0.005$.

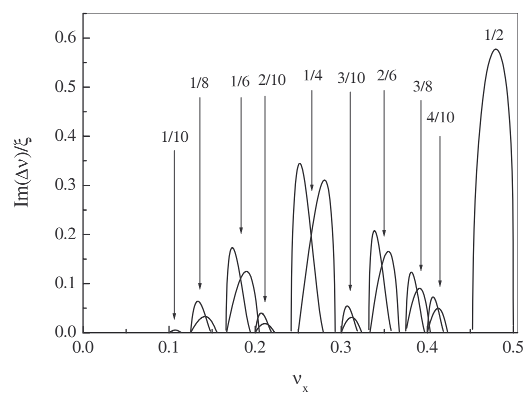

Figure 2: Dependencies of the increments of the coherent beam-beam modes on the tune of the horizontal incoherent betatron oscillations (ν_x). Modes $1 \leq m \leq 5$, arrows show positions of stopbands near particular resonances, wider curves to the right from the resonances $\nu_x = n/(2m)$ depict the increments of the modes $(+)$, $\xi = 0.05$.

of the stopbands for modes $(-)$ and $1 \leq m \leq 3$ in Fig. 2 are in general agreement with similar results reported in Ref.[3] and obtained using a different approach. We note that the values of the maximum increments for the modes $m \geq 3$ are a bit smaller in the region $\nu_x < 1/4$ than that above $\nu_x = 1/4$. The maximum increments of the modes near the resonances $\nu_x = 1/(2m)$ tend to zero, when ν_x approaches the border value

$$(\nu_x)_{\min} = \frac{1}{2\pi}\arccos\frac{1 - B^2}{1 + B^2}. \quad (6)$$

This fact is in a general agreement with experimental observations of the beam-beam instabilities in the electron-positron colliders. Although it is not shown here, a decrease in ξ results in decreases in the values of the mode increments ($\mathrm{Im}\nu$) and in the narrowing of the widths of the stopbands in ν_x. However, the ratios $\mathrm{Im}\nu/\xi$ and of these widths to ξ remain the same.

Outside the spectrum of incoherent oscillations (e.g. $z_1^2 > \delta^2(0)$) Eqs.(2) and (3) may have solutions with

$\mathrm{Re}(z_1) \neq 0$ and $\mathrm{Im}(z_1) = 0$. To avoid the mode interference, coherent tuneshifts of such stable solutions should not enter the stopbands of unstable modes.

To figure out effects of Landau damping on the stability of coherent beam-beam oscillations we compared the stopbands calculated for monochromatic and non-monochromatic bunches. For monochromatic bunches the roots of the dispersion equations corresponding to the largest increments are calculated using ([6]) $z_1^2 = \delta^2 \pm 2\Lambda_m \delta$, where the sign $+$ is taken for the modes $(-)$ and

$$\Lambda_m = \frac{4}{m\,(m+2)} \frac{1}{\left(1 + \frac{1}{m+1}\right)^{m+1}}. \tag{7}$$

For the dipole mode $(-)$ Landau damping results only in minor changes of the stopband of the monochromatic bunches (Fig. 3). The maximum values of the increments almost coincide, Landau damping suppress the instability within a narrow band A′A and decreases the oscillation increments within the segment AB. Within the segment BC the beam-beam tunespread slightly increases the increments of unstable modes hence, resulting in some Landau anti-damping. Stronger Landau anti-damping indicate the stopbands of the modes with higher betatron multipole numbers ($m \geq 2$, e.g. in Fig. 4 and 5). For such modes, the beam-beam tunespread although decreases the values of the maximum increments, moves the lower border of the stopbands of the modes $(-)$ towards the resonant tune $n/(2m)$ and substantially increases the widths of the stopbands. Except for the case $m = 2$, the stopbands of the multipole modes $(\pm)$ are placed above the resonant tunes almost entirely. Hence, the beam-beam tunespread suppressing the modes of the monochromatic colliding bunches opens new wide regions of the tunes ν_x where the oscillations become unstable. The described Landau anti-damping of the coherent oscillations of the colliding bunches is a generic phenomenon for the coherent beam-beam interactions. These instabilities occur due to the coupling of the modes m and $-m$ near the sum-type resonance $m(\nu_x^{(1)} + \nu_x^{(2)}) = 2m\nu_x = n$. According to general properties of such instabilities [7] any damping can stabilize coupled coherent modes only in the case, when both coupled modes are damped sufficiently strongly. Otherwise, the oscillations become unstable.

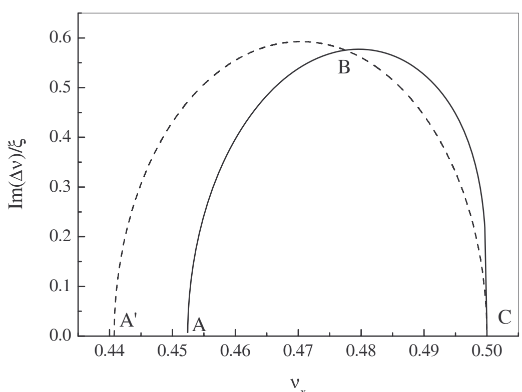

Figure 3: Dependence of the increment of the dipole mode $(-)$ on the tune of the horizontal incoherent oscillations ν_x. Solid line – Landau damped coherent oscillations, dashed line – monochromatic bunches, $\xi = 0.05$.

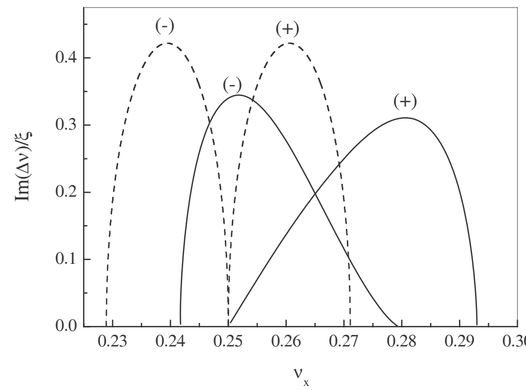

Figure 4: The stopbands of coherent oscillations near the resonance $1/4$. Solid line – Landau damped coherent oscillations, dashed line – monochromatic bunches, the symbols $(\pm)$ mark the curves for modes $(\pm)$, $\xi = 0.05$.

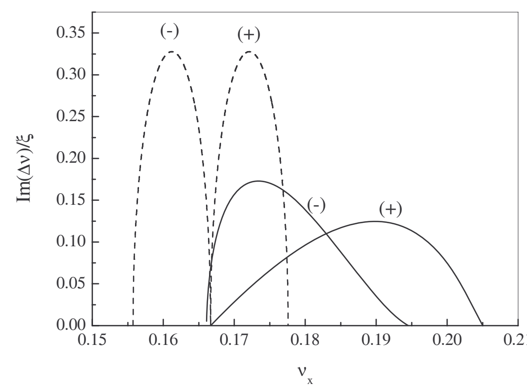

Figure 5: Same as in Fig. 4, but near the resonance $1/6$.

Replacing ν_x by $\nu_x + ax$, we can also inspect some effects of the octupole lattice non-linearity on the stability of coherent beam-beam oscillations. According to data depicted in Figs. 6 and 7, the octupole fields do not cancel the described Landau anti-damping. However, it can decrease the strength of the instability provided that the sign of the non-linearity is correct.

Additional suppression of the strength of the coherent beam-beam instability can occur in collisions of long bunches due to hour-glass effect [9]. In the simplest case and provided that the disruption parameter of the bunches $4\pi\xi\sigma_s/\beta$ is small, the stopbands of the betatron modes of the bunches with the lengths σ_s comparable to β can be calculated using Eqs.(2) and (3) after a reduction in Eq.(2) of the function $V(x)$ by the suppressing factor times. According to data depicted in Fig. 8, the hour-glass suppres-

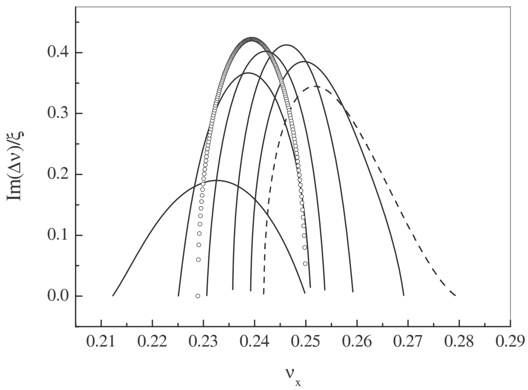

Figure 6: Modifications of the stopband of the $(-)$ mode near $\nu_x = 1/4$ due to octupoles. Solid lines right to left $a/\xi = 0.1, 0.2, 0.3, 0.4, 0.8$; dashed line $a = 0$, open circles – the stopband for monochromatic bunches, $\xi = 0.05$.

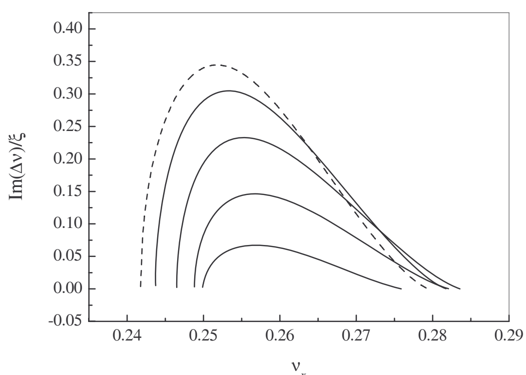

Figure 7: Same as in Fig. 6, but top to bottom $a/\xi = -0.1, -0.3, -0.6, -1$; dashed line: $a = 0$.

sion decreases the increments of unstable modes and the width of the stopband, but does not eliminate Landau anti-damping of the modes.

CONCLUSIONS

Using the simplifying model, we have studied the influence of tunespreads on the stability of coherent oscillations of short, identical colliding e^+e^- bunches. Comparing the spectra of coherent oscillations which are calculated taking into account and/or ignoring the tunespreads we have found out the Landau anti-damping of coherent oscillations of colliding bunches. Namely, together with the damping of unstable modes of the monochromatic colliding bunches the tunespreads result in the instability of coherent oscillations in the regions of betatron tunes ν_x where coherent oscillations of monochromatic bunches were stable. Effects of this anti-damping increase with an increase in the value of the betatron multipole number m. It is almost negligible for the dipole modes, but for the modes with $m \geq 2$ the calculations ignoring the beam-beam tunespread result in strong underestimation of the widths of the stopbands

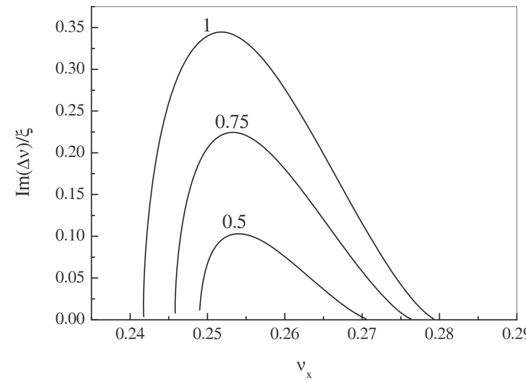

Figure 8: Modifications of the stopband of the $(-)$ mode near $\nu_x = 1/4$ due to hour-glass like suppression. The figures above the curves give the values of the suppression factors, $\xi = 0.05$.

of coherent oscillations of the colliding bunches as well as in wrong positions of these stopbands relative the resonant values of the tunes ν_x. Generally, effects of the tunespreads decrease the maximum values of the oscillation increments as compared to those calculated for monochromatic bunches. Octupole fields do not cancel Landau anti-damping, but can decrease increments of unstable modes. Initial estimations also show, that the hour-glass reductions do not eliminate Landau anti-damping of the coherent beam-beam modes.

We simplified our half-analytic calculations ignoring possible effects of incoherent beam-beam resonances on the stability of collective beam-beam modes assuming that only small amount of the bunch particles are captured in the resonance buckets. If the incoherent resonances are strong and/or are wide enough in ν_x, the described calculations may predict the results which are not reliable (see, e.g. in Ref.[10]). In such cases, the stability of collective beam-beam modes should be studied using numerical simulations.

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