Space charge effect in an accelerated beam

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Introduction - 1

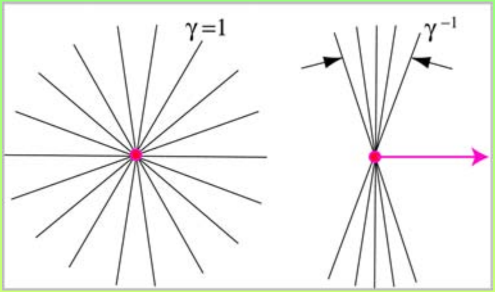
- An important characteristic of an FEL beam is its energy spread.
- One of the dominant contributions, for nanocoulomb bunches, comes from the space charge effect; it scales with the beam energy as γ^{-2}. Space charge effects here are understood for a beam moving with a constant velocity v.
- There is a contribution of the coherent radiation of the beam (CSR wake). It keeps balance between the electromagnetic energy that is carried away by the radiation and the kinetic energy of the beam particles. Effect of radiation due to a longitudinal acceleration has been recently studied by Geloni et al. (2007) and Bosch (2007)
- A converging (or diverging) beam has a component of the self field due to the radial motion [Bane & Chao, 2002].

Introduction - 2

- Note that many computer codes calculate the space charge effect by assuming that the beam is moving with a constant velocity v.

In this work we point out to a new component of the space charge field that arises during longitudinal acceleration of the beam. At large γ it scales as a/γ, where a is acceleration. We assume that the beam is moving as a rigid body and does not change its shape during the acceleration.

Physical mechanism of the effect



The electromagnetic field of a moving bunch is compressed in the transverse direction. The moving field carries more electromagnetic energy then the field at rest.

Physical mechanism of the effect

The electromagnetic energy is easily calculated for a Gaussian bunch moving with velocity v in the z direction

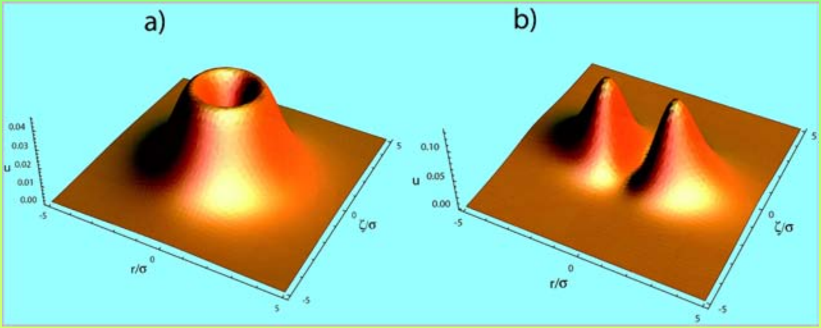
$$n(x, y, \zeta) = \frac{N}{(2\pi)^{3/2}\sigma_z\sigma_\perp^2} \exp\left(-\frac{r^2}{2\sigma_\perp^2} - \frac{\zeta^2}{2\sigma_z^2}\right) ,$$

where N is the number of particles in the bunch, $r = \sqrt{x^2 + y^2}$, $\zeta = z - vt$, $\sigma_x = \sigma_y = \sigma_\perp$ is the rms bunch size in the transverse direction, and σ_z is the rms bunch length in the longitudinal direction.

The electromagnetic field of such a bunch is calculated using the Lorentz transformation from the beam frame. The energy density

$$u(r, \zeta) = \frac{E^2 + H^2}{8\pi}$$

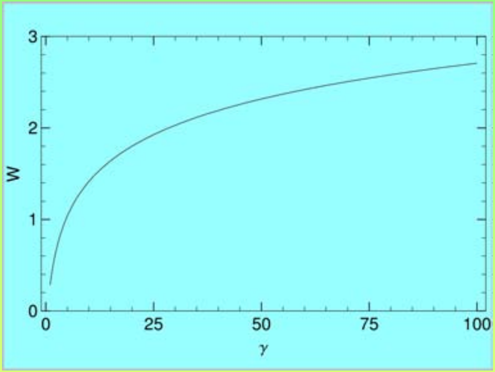
Physical mechanism of the effect



Electromagnetic energy density u in units $Q^2/8\pi\sigma^4$ for a spherical Gaussian bunch ($\sigma_z = \sigma_\perp$) with *a)* $\gamma = 1$ and *b)* $\gamma = 2$. Note the difference in the vertical scales.

Physical mechanism of the effect

The total electromagnetic energy $W = \int u dV$ grows with γ.



Plot of W in units Q^2/σ for a spherical Gaussian bunch ($\sigma_\perp = \sigma_z = \sigma$). The asymptotic expression for W is $W = \frac{Q^2}{\sqrt{\pi}\sigma_z} \log\gamma\,,\ \gamma \gg 1\,.$

Physical mechanism of the effect

The increasing energy of the EM field is taken from the kinetic energy of the particles via a longitudinal electric field that is generated inside the beam. This field is caused by acceleration, but it is NOT due to radiation!

тов de'. Только таким образом можно провести интегрирование по всему электрону. Переход от t' к t осуществляется посредством разложения функций $t' = t - r/c$ в ряд по степеням r/c. При этом следует помнить, что согласно пункту 1

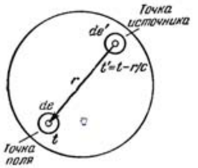


Рис. 20.2. К вычислению взаимодействия между частями одного и того же электрона.

$$\dot{u}(t') = \dot{u}(t) - \frac{r}{c}\ddot{u}(t) + \ \ldots, \qquad (20.21)$$

$$u(t') = -\frac{\dot{u}(t)\, r}{c} + \frac{\ddot{u}(t)\, r^2}{2c^2} - \ \ldots, \qquad (20.22)$$

$$s^{-3} = r^{-3}\left[1 - \frac{3\dot{u}(t)\cdot r}{c^2} + \frac{3r}{2}\left(\frac{\ddot{u}(t)\cdot r}{c^3}\right)\ldots\right]. \qquad (20.23)$$

В дальнейшем будем оставлять лишь члены порядка $(r/c)^3$. Тогда величиной $[u(t')/c]^2$ можно пренебречь и выражение (20.20) существенно упрощается

$$4\pi\varepsilon_0\, dE = de'\left[\frac{r(r\cdot\dot{u})}{r^3c^2} - \frac{\dot{u}}{rc^2} - \frac{r(r\cdot\ddot{u})}{r^2c^3} + \frac{\ddot{u}}{c^3} + \frac{r}{r^3} - \right.$$
$$\left. - \frac{3r(\dot{u}\cdot r)}{r^3c^2} + \frac{3}{2}\frac{r}{r^2c^3}(\ddot{u}\cdot r) + \frac{\dot{u}}{rc^2} - \frac{\ddot{u}}{2c^3}\right] =$$
$$= de'\left[-\frac{2r(\dot{u}\cdot r)}{r^3c^2} + \frac{1}{2}\frac{r(\ddot{u}\cdot r)}{r^2c^3} + \frac{r}{r^3} + \frac{\ddot{u}}{2c^3}\right]. \qquad (20.24)$$

The effect is known from the "electromagnetic mass of electron". Page 348 from Panofsky and Philips, "Classical Electricity and Magnetism").

The model

A beam is moving along the z axis with velocity $v(t)$ that is the same for all particles of the beam. Charge and current densities are

$$\rho = en(x, y, z - z_0(t)),\; j_z = ev(t)n(x, y, z - z_0(t))$$

with $v = dz_0/dt$.
The scalar and vector potentials

$$\phi(\mathbf{r}, t) = \int \frac{\rho(\mathbf{r'}, t - \tau)}{|\mathbf{r} - \mathbf{r}'|} d^3r' \,,$$
$$\mathbf{A}(\mathbf{r}', t) = \frac{1}{c} \int \frac{\mathbf{j}(\mathbf{r}', t - \tau)}{|\mathbf{r} - \mathbf{r}'|} d^3r' \,, \qquad (1)$$

where $c\tau = |\mathbf{r} - \mathbf{r}'|$. We assume that the acceleration $a(t) = dv/dt$ is small and expand the potentials in Taylor series keeping only linear terms in acceleration.

There are two such conditions

$$a \ll \frac{c^2}{l}\,, \qquad \left|\frac{\dot{a}}{a}\right| \ll \frac{c}{l}\,, \tag{2}$$

where l is the characteristic size of the bunch.
The electric field of the beam is a sum of the space charge field (which depends on $\mathbf{v}$ and does not depend on a) and a component that vanishes in the limit when $a = 0$:

$$\mathbf{E} \approx \mathbf{E}_{\mathrm{sc}} + \tilde{\mathbf{E}}\,, \tag{3}$$

We call $\tilde{\mathbf{E}}$ the *acceleration field*.

The acceleration field

This gives the following expression for $\tilde{E}_z$,

$$\begin{aligned}\tilde{E}_z = & -\frac{e}{c^2} a \int \frac{n\left(x', y', z' + v\tau\right)}{|\mathbf{r} - \mathbf{r}'|} d^3 r' \\ & - \frac{e}{2c^2} a\beta \int \left(\beta|\mathbf{r} - \mathbf{r}'| - (z - z')\right) \partial_{zz} n\left(x', y', z' + v\tau\right) d^3 r' \\ & + \frac{e}{2c^2} a \int \left(\frac{z - z'}{|\mathbf{r} - \mathbf{r}'|} - 4\beta\right) \partial_z n\left(x', y', z' + v\tau\right) d^3 r' \, .\end{aligned}$$

In the ultrarelativistic limit $\gamma \gg 1$,

$$\tilde{E}_z = -\sqrt{\frac{2}{\pi}} \frac{r_e N}{\sigma_z \gamma} E_{\mathrm{ext}} e^{-z^2/2\sigma_z^2} \, , \qquad \gamma \gg 1 \, ,$$

with $r_e = e^2/mc^2$. We see that the acceleration field is directed against the external field E_{ext} and scales as E_{ext}/γ. This contrasts to the usual scaling $\propto \gamma^{-2}$ of the longitudinal space charge forces.

Acceleration field for a Gaussian Bunch

The acceleration is related to the rate of change of the gamma factor, $a = (c/\gamma^3\beta)d\gamma/dt$. The energy change of a particle in the beam due to the acceleration field is

$$\Delta\mathcal{E}(r,z) = \int ve\tilde{E}_z dt = -\frac{I_0}{I_A}mc^2\int_{\gamma_i}^{\gamma_f}\frac{d\gamma}{\gamma^3}G\left(\frac{r}{\sigma_\perp},\frac{z}{\sigma_z}\right), \quad (4)$$

where $I_0 = Nec/\sqrt{2\pi}\sigma_z$ is the peak current in the bunch, $I_A = mc^3/e$ is the Alfvén current, γ_i and γ_f are the initial and final values of the gamma factor.

Effect of conducting pipe

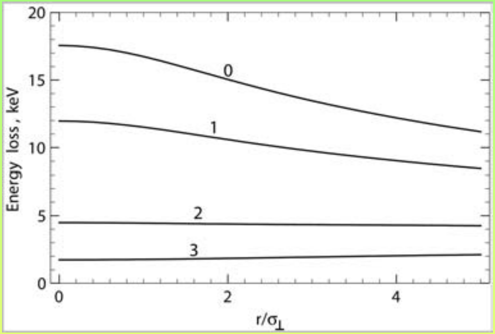
In reality, the beam is being accelerated inside a vacuum volume that has a characteristic transverse size b. There is a field generated by image charges and currents in the wall. If $b/\sigma \gg 1$, these charges are located relatively far from the beam, and variation of their field at the location of the beam over the distance $\sim \sigma$ is small. Hence, the calculation of the *energy spread* in the beam can be carried out using only the free space field. Also the electromagnetic energy of a relativistic beam propagating in a pipe of radius b does not change with γ when $\gamma \gtrsim b/\sigma_z$. We assume

$$\gamma_f = \frac{b}{\sigma_z}\,.$$

Note that typically $b/\sigma_z \gg 1$, and because our result has only a logarithmic dependence on γ, it is rather insensitive to the exact value of γ_f.

LCLS example

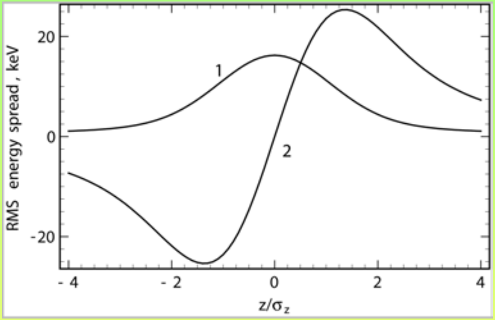
LCLS rf-gun parameters: $\sigma_z = 0.86$ mm, $\sigma_x = 0.6$ mm, and $Q = 0.72$ nC (corresponding to the peak current of $I_0 = 100$ A). We choose $\gamma_i = 1$ and $\gamma_f = 20$, corresponding to the beam pipe radius of about 1.2 cm.



Energy loss induced by the acceleration field for four different slices in the bunch ($z/\sigma_z = 0$, 1, 2, and 3; this number is indicated near the curves) as a function of electron radial position.

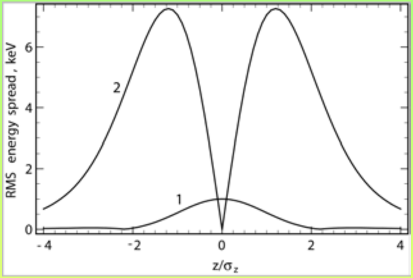
LCLS example - slice energy loss

The energy loss averaged over the transverse coordinate as a function of the position z, curve *1* (for comparison, we also show the energy loss introduced by the space charge effect, curve *2*)



LCLS example - slice energy spread

The rms energy spread in slices as a function of z, curve *1* (we also show the energy loss and the rms energy spread introduced by the space charge forces, curve *2*).



Energy balance

As was mentioned above, the acceleration field keeps balance between the electromagnetic energy of the beam and the kinetic energy of the particles:

$$\frac{dW}{dt} = -e \int \tilde{E}_z v n d^3 r \, .$$

We calculated the electromagnetic energy difference for the beam at the final state with $\gamma_f = 20$ and the initial state with $\gamma_i = 1$, which gave us $\Delta W = 22.5$ μJ. When we integrated the right side of this equation over time from the initial to the final state, we found that the work of the acceleration numerically is equal to ΔW, in perfect agreement with the energy balance equation.

Conclusions

- We point out to a new component of the self field of an accelerated field, which is not due to coherent radiation. It contributes to the beam energy chirp and the slice energy spread in RF guns.
- The physical origin of the field is due to the changing energy of the Coulomb field. It keeps the energy balance between the kinetic energy of the beam and the energy of the field.
- We illustrated the effect for a typical RF gun parameters.
- Many simulation computer codes neglect this field, and our result allows to evaluate if this is important for a particular problem.