Impact of Longitudinal Space-charge Wake from FEL Undulators on Current-enhanced SASE Schemes

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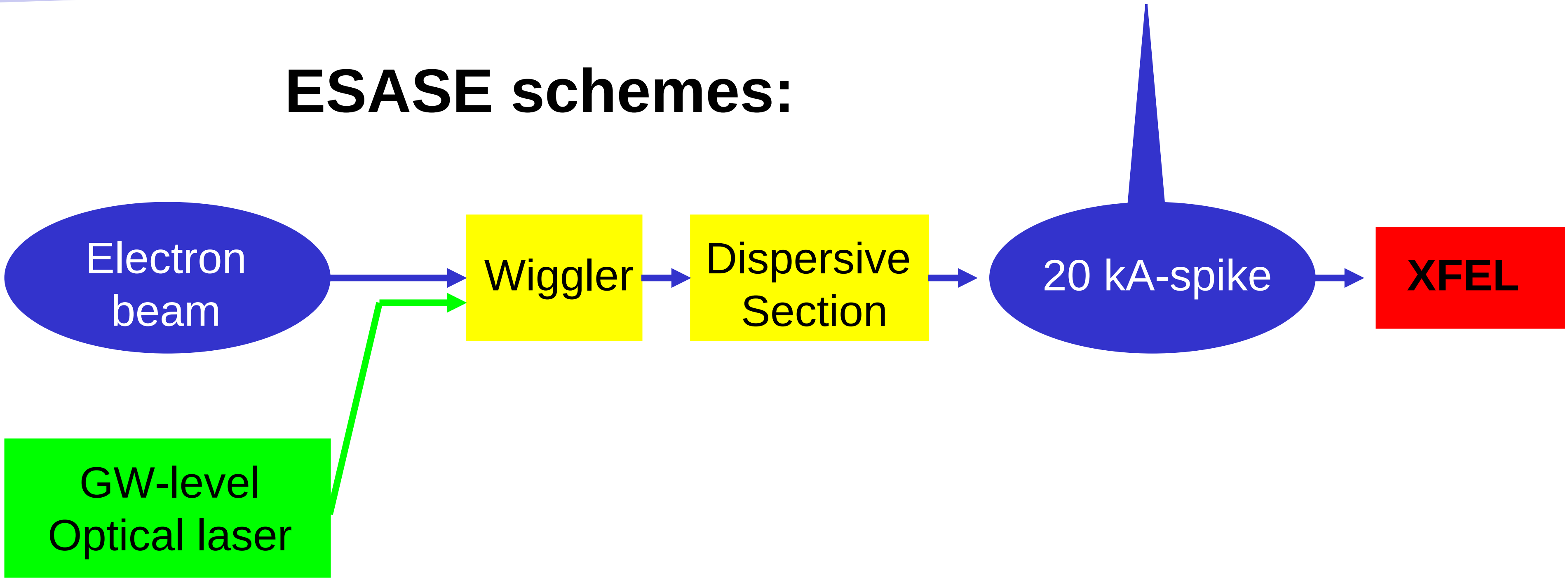
Motivation

ESASE schemes:



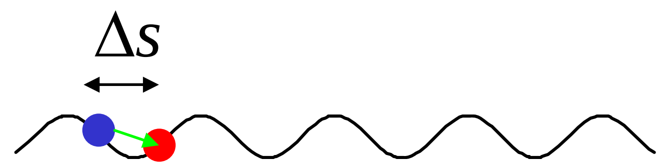
Comprehensive study of longitudinal wakes in XFEL (including transverse beam size) is needed

Impedance and wake

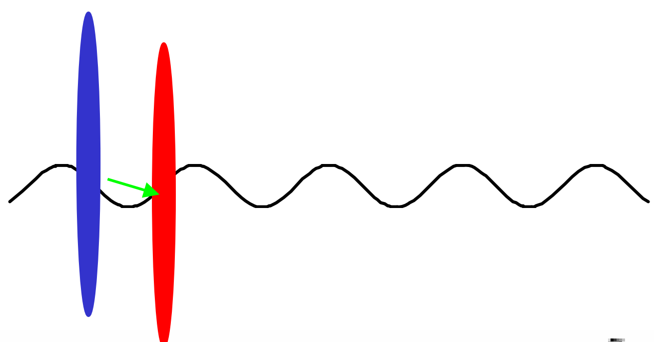
Usually wake function is defined from point-to-point:

$$Z_0 = \int_{-\infty}^{\infty} \frac{d(\Delta s)}{\beta c} G_0(\Delta s) \exp\left[i\omega \frac{\Delta s}{\beta c}\right],$$

$$G_0 = \frac{1}{(-e)} \int_{-\infty}^{\infty} d\vec{r}' \cdot \vec{E}^0(\Delta s, \vec{r}'(t), t)\big|_{t=z'/(\beta_z c)}$$



To account for transverse dimensions: disk-to-disk



$$Z(\omega, z) = \frac{1}{|\bar{f}(\omega)|^2} \int_V \vec{j}^{\,*} \cdot \vec{E}\, dV = \frac{1}{|\bar{f}(\omega)|^2} \int_0^z dz' \int_A d\vec{r}'_\perp \vec{j}^{\,*} \cdot \vec{E}$$

To calculate Z: - Calculate sources - Calculate field

Parameter space

Parameter space

$$\lambda\!\!\!^{-} >> \lambda\!\!\!^{-}_r$$

Perturbation theory

LCLS case

$$\lambda\!\!\!^{-} = 50nm \qquad \lambda\!\!\!^{-}_r = \frac{0.15}{2\pi}nm$$

$\lambda\!\!\!^{-}$ red. wavelength of interest

$\lambda\!\!\!^{-}_r$ red. radiation wavelength

Parameter space

LCLS case

$$\lambdabar >> \lambdabar_r$$

Perturbation theory

$$\lambdabar = 50nm \qquad \lambdabar_r = \frac{0.15}{2\pi}nm$$

$$L_s >> 2\bar{\gamma}_z^2 \lambdabar$$

Steady state

$$L_s = 50m \qquad 2\bar{\gamma}_z^2 \lambdabar = 10m$$

$\lambdabar$ red. wavelength of interest L_s Sat. length

$\lambdabar_r$ red. radiation wavelength $\bar{\gamma}_z = \gamma / \sqrt{1+K^2/2}$

Parameter space

LCLS case

$\lambdabar \gg \lambdabar_r$

Perturbation theory

$\lambdabar = 50 nm \qquad \lambdabar_r = \frac{0.15}{2\pi} nm$

$L_s \gg 2\bar{\gamma}_z^2 \lambdabar$

Steady state

$L_s = 50 m \qquad 2\bar{\gamma}_z^2 \lambdabar = 10 m$

$a \gg \bar{\gamma}_z \lambdabar$

Free-space

$a = 2.5 mm \qquad \bar{\gamma}_z \lambdabar = 500 \mu m$

$\lambdabar$ red. wavelength of interest L_s Sat. length a Pipe tr. size

$\lambdabar_r$ red. radiation wavelength $\bar{\gamma}_z = \gamma / \sqrt{1+K^2/2}$

Parameter space

LCLS case

$$\bar\lambda \gg \bar\lambda_r$$

Perturbation theory

$$\bar\lambda = 50nm \qquad \bar\lambda_r = \frac{0.15}{2\pi} nm$$

$$L_s \gg 2\bar\gamma_z^2 \bar\lambda$$

Steady state

$$L_s = 50m \qquad 2\bar\gamma_z^2 \bar\lambda = 10m$$

$$a \gg \bar\gamma_z \bar\lambda$$

Free-space

$$a = 2.5mm \qquad \bar\gamma_z \bar\lambda = 500\mu m$$

$$(\ \sigma_\perp^2 \gg \bar\lambda \bar\lambda_w \qquad \sigma_\perp^2 = 8 \cdot 10^{-10} m^2 \qquad \bar\lambda \bar\lambda_w = 2 \cdot 10^{-10} m^2 \ )$$

$\bar\lambda$ red. wavelength of interest

$\bar\lambda_r$ red. radiation wavelength

L_s Sat. length

$\bar\gamma_z = \gamma \Big/ \sqrt{1+K^2/2}$

a Pipe tr. size

$\bar\lambda_w$ red. undul. period

$\sigma_\perp$ Beam tr. size

Electromagnetic sources

We will work in the space-frequency domain

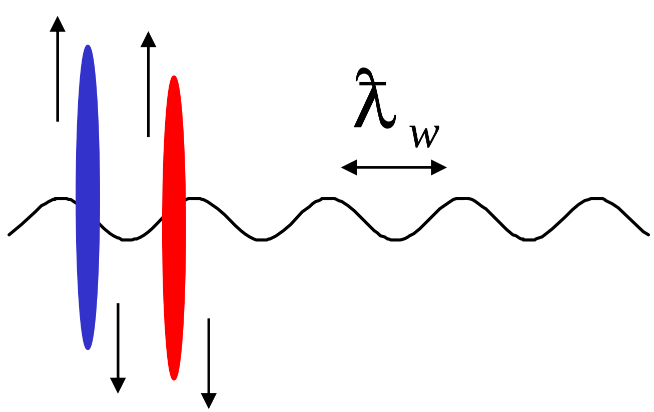
Sources can be written as

$$\bar{\rho}(\vec{r}_\perp, z, \omega) = \rho_o\left(\vec{r}_\perp - \vec{r'}_{o\perp}(z)\right)\bar{f}(\omega)\exp\left[i\omega s_o(z)/v_o\right] \qquad \vec{\bar{j}} = \bar{\rho}\vec{v}_o$$

where

$$r'_{ox}(z) = \frac{K}{\gamma k_w}\cos(k_w z) = r_w \cos(k_w z) \quad , \quad r'_{oy}(z) = 0$$

$\vec{v}_o$ is the correspondent velocity,
s_o the curvilinear abscissa,
f(t) the longitudinal distribution of electrons



Oscillating SOURCE disk
Oscillating TEST disk

Field calculations (1) : paraxial approximation

Paraxial approximation can be used to get the field envelope:

$$\left(\nabla_\perp{}^2 + \frac{2i\omega}{c}\frac{\partial}{\partial z}\right)\left[\vec{\tilde{E}}_\perp(z,\vec{r}_\perp,\omega)\right] = -4\pi \exp\left[-\frac{i\omega z}{c}\right]\left(\frac{i\omega}{c^2}\vec{\tilde{j}}_\perp - \vec{\nabla}_\perp\bar{\rho}\right)$$

$$\vec{\tilde{E}}_\perp = \vec{\bar{E}}_\perp \exp\left[-i\omega z/c\right]$$

Current term

Gradient term

Must calculate transverse and longitudinal field

$$\vec{\bar{E}}_\perp(z,\vec{r}_\perp) = -\frac{i\omega}{c}\int_0^z dz'\frac{1}{z-z'}\int d\vec{r'}_\perp\left(\frac{K\vec{e}_x}{\gamma}\sin(k_w z') + \frac{\vec{r}_\perp - \vec{r'}_\perp}{z-z'}\right)$$
$$\times\rho_o\left(\vec{r'}_\perp - r_w\cos(k_w z')\vec{e}_x\right)\bar{f}(\omega)\exp\left\{i\omega\left[\frac{|\vec{r}_\perp - \vec{r'}_\perp|^2}{2c(z-z')}\right] + \frac{i\omega z'}{2c\bar{\gamma}_z^2}\right\} .$$

Transverse field

$$\tilde{E}_z(z,\vec{r}_\perp) = -\frac{i\omega}{c}\int_0^z dz'\frac{1}{z-z'}\int d\vec{r'}_\perp\left[\frac{1}{\bar{\gamma}_z^2} + \frac{K}{\gamma}\sin(k_w z')\frac{x-x'}{z-z'}\right]$$
$$\times\rho_o\left(\vec{r'}_\perp - r_w\cos(k_w z')\vec{e}_x\right)\bar{f}(\omega)\exp\left\{i\omega\left[\frac{|\vec{r}_\perp - \vec{r'}_\perp|^2}{2c(z-z')}\right] + \frac{i\omega z'}{2c\bar{\gamma}_z^2}\right\}$$

Longitudinal field

Field calculations : results

Radiative current term (slow wave)

Radiative current term (fast wave)

Radiative gradient term (slow wave)

Radiative gradient term (fast wave)

Space-charge gradient term

Transverse field

Field calculations (3) : results

$$
\begin{aligned}
\vec{\tilde{E}}_\perp(z,\vec{r}_\perp) = -\frac{i\omega\bar{f}(\omega)}{c}\int d\vec{r'}_\perp \rho_o\left(\vec{r'}_\perp\right)\exp\left[\frac{i\omega z}{2c\bar{\gamma}_z^2}\right] \\
\times\Bigg\{\exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\left[+\frac{K\vec{e}_x}{2i\gamma}\exp[ik_w(z'-z)]\right] && \text{Rad. Curr.} \\
+\exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\left[-\frac{K\vec{e}_x}{2i\gamma}\exp[ik_w(z-z')]\right] && \text{Rad. Curr.} \\
+\exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right] \\
\times\left[-\frac{r_w\vec{e}_x}{2(z-z')}-\frac{i\omega r_w(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{2c(z-z')^2}\right]\exp[ik_w(z'-z)] && \text{Rad. Gr.} \\
+\exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right] \\
\times\left[-\frac{r_w\vec{e}_x}{2(z-z')}-\frac{i\omega r_w(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{2c(z-z')^2}\right]\exp[ik_w(z-z')] && \text{Rad. Gr.} \\
+\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\exp\left[\frac{i\omega(z'-z)}{2c\bar{\gamma}_z^2}\right]\left[\frac{\vec{r}_\perp-\vec{r'}_\perp}{z-z'}\right]\Bigg\} && \text{S.C. Gr.}
\end{aligned}
$$

Transverse field

Field calculations (3) : results

$$\vec{\tilde{E}}_\perp(z,\vec{r}_\perp) = -\frac{i\omega \bar{f}(\omega)}{c}\int d\vec{r'}_\perp \rho_o\left(\vec{r'}_\perp\right)\exp\left[\frac{i\omega z}{2c\bar{\gamma}_z^2}\right]$$

$$\times\left\{\exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\left[+\frac{K\vec{e}_x}{2i\gamma}\exp[ik_w(z'-z)]\right]\right.$$

$$+\exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\left[-\frac{K\vec{e}_x}{2i\gamma}\exp[ik_w(z-z')]\right]$$

$$+\exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]$$

$$\times\left[-\frac{r_w\vec{e}_x}{2(z-z')}-\frac{i\omega r_w(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{2c(z-z')^2}\right]\exp[ik_w(z'-z)]$$

$$+\exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]$$

$$\times\left[-\frac{r_w\vec{e}_x}{2(z-z')}-\frac{i\omega r_w(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{2c(z-z')^2}\right]\exp[ik_w(z-z')]$$

$$\left.+\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\exp\left[\frac{i\omega(z'-z)}{2c\bar{\gamma}_z^2}\right]\left[\frac{\vec{r}_\perp-\vec{r'}_\perp}{z-z'}\right]\right\}$$

Rad. Curr.

Rad. Curr.

Rad. Gr.

Rad. Gr.

S.C. Gr.

Transverse field

Field calculations (3) : results

$$\vec{\tilde{E}}_\perp(z,\vec{r}_\perp) = -\frac{i\omega \bar{f}(\omega)}{c}\int d\vec{r'}_\perp \rho_o\left(\vec{r'}_\perp\right)\exp\left[\frac{i\omega z}{2c\bar{\gamma}_z^2}\right]$$

$$\times\Bigg\{\exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\left[+\frac{K\vec{e}_x}{2i\gamma}\exp[ik_w(z'-z)]\right]$$

$$+\exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\left[-\frac{K\vec{e}_x}{2i\gamma}\exp[ik_w(z-z')]\right]$$

$$+\exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]$$

$$\times\left[-\frac{r_w\vec{e}_x}{2(z-z')}-\frac{i\omega r_w(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{2c(z-z')^2}\right]\exp[ik_w(z'-z)]$$

$$+\exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]$$

$$\times\left[-\frac{r_w\vec{e}_x}{2(z-z')}-\frac{i\omega r_w(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{2c(z-z')^2}\right]\exp[ik_w(z-z')]$$

$$+\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\vec{r}_\perp-\vec{r'}_\perp|^2}{2c(z-z')}\right]\exp\left[\frac{i\omega(z'-z)}{2c\bar{\gamma}_z^2}\right]\left[\frac{\vec{r}_\perp-\vec{r'}_\perp}{z-z'}\right]\Bigg\}$$

Rad. Curr.

Rad. Curr.

Rad. Gr.

Rad. Gr.

S.C. Gr.

Transverse field

Field calculations (3) : results

$$\vec{\tilde{E}}_\perp(z,\vec{r}_\perp) = -\frac{i\omega \bar{f}(\omega)}{c}\int d\vec{r'}_\perp \rho_o\left(\vec{r'}_\perp\right)\exp\left[\frac{i\omega z}{2c\bar{\gamma}_z^2}\right]$$

$$\times\left\{\exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{\mid\vec{r}_\perp-\vec{r'}_\perp\mid^2}{2c(z-z')}\right]\left[+\frac{K\vec{e}_x}{2i\gamma}\exp[ik_w(z'-z)]\right]\right.$$

Rad. Curr.

$$+\exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{\mid\vec{r}_\perp-\vec{r'}_\perp\mid^2}{2c(z-z')}\right]\left[-\frac{K\vec{e}_x}{2i\gamma}\exp[ik_w(z-z')]\right]$$

Rad. Curr.

$$+\exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{\mid\vec{r}_\perp-\vec{r'}_\perp\mid^2}{2c(z-z')}\right]$$

$$\times\left[-\frac{r_w\vec{e}_x}{2(z-z')}-\frac{i\omega r_w(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{2c(z-z')^2}\right]\exp[ik_w(z'-z)]$$

Rad. Gr.

$$+\exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{\mid\vec{r}_\perp-\vec{r'}_\perp\mid^2}{2c(z-z')}\right]$$

$$\times\left[-\frac{r_w\vec{e}_x}{2(z-z')}-\frac{i\omega r_w(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{2c(z-z')^2}\right]\exp[ik_w(z-z')]$$

Rad. Gr.

$$\left.+\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{\mid\vec{r}_\perp-\vec{r'}_\perp\mid^2}{2c(z-z')}\right]\exp\left[\frac{i\omega(z'-z)}{2c\bar{\gamma}_z^2}\right]\left[\frac{\vec{r}_\perp-\vec{r'}_\perp}{z-z'}\right]\right\}$$

S.C. Gr.

Transverse field

Field calculations (3) : results

Radiative gradient term (slow wave)

Radiative gradient term (fast wave)

Space-charge gradient+current

Longitudinal field

Field calculations (3) : results

$$
\begin{aligned}
\widetilde{E}_z(z,\vec{r}_\perp) = & -\frac{i\omega \bar{f}(\omega)}{c}\int d\vec{r'}_\perp \rho_o\left(\vec{r'}_\perp\right)\exp\left[\frac{i\omega z}{2c\bar{\gamma}_z^2}\right] \\
& \times\Bigg\{ \exp[+ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\,\vec{r}_\perp-\vec{r'}_\perp\,|^2}{2c(z-z')}\right]\left[+\frac{K}{2i\gamma}\frac{x-x'}{z-z'}\exp[ik_w(z'-z)]\right] \\
& + \exp[-ik_w z]\int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\,\vec{r}_\perp-\vec{r'}_\perp\,|^2}{2c(z-z')}\right]\left[-\frac{K}{2i\gamma}\frac{x-x'}{z-z'}\exp[-ik_w(z'-z)]\right] \\
& + \int_0^z \frac{dz'}{z-z'}\exp\left[i\omega\frac{|\,\vec{r}_\perp-\vec{r'}_\perp\,|^2}{2c(z-z')}\right]\exp\left[\frac{i\omega(z'-z)}{2c\bar{\gamma}_z^2}\right]\frac{1}{\bar{\gamma}_z^2}\Bigg\}
\end{aligned}
$$

Rad. Gr.

Rad. Gr.

S.C.

Longitudinal field

Cross-check with Gauss law

Field expression has been cross-checked

In particular, it obeys Gauss law.

Interesting remark:

$$\vec{\nabla} \cdot \overline{E}_{sc} = 4\pi\overline{\rho}$$

$$\vec{\nabla} \cdot \overline{E}_{rad} = 0$$

Separately.

Longitudinal impedance: general result

Using expressions for field and Bessel functions we obtain:

$$Z_R = -\frac{K^2 \pi \omega z}{4\gamma^2} \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp) \rho_o\left(\vec{r''}_\perp\right) J_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)$$

$$Z_I = -\frac{K^2 \omega z}{2\gamma^2} \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp) \rho_o\left(\vec{r''}_\perp\right) \left\{ \frac{\pi}{2} Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) - K_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) + \frac{4+2K^2}{K^2} K_0\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\bar{\gamma}_z \bar{\lambda}}\right) \right\} ,$$

Longitudinal impedance: general result

Using expressions for field and Bessel functions we obtain:

$$Z_R = -\frac{K^2\pi\omega z}{4\gamma^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) J_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)$$

$$Z_I = -\frac{K^2\omega z}{2\gamma^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right)\left\{\frac{\pi}{2}Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) - K_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) + \frac{4+2K^2}{K^2}K_0\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\bar{\gamma}_z\bar{\lambda}}\right)\right\},$$

S.C. long.

$$L_f = \bar{\gamma}_z^2\bar{\lambda}; \quad \theta_d = \frac{1}{\bar{\gamma}_z^2};$$

Longitudinal impedance: general result

Using expressions for field and Bessel functions we obtain:

$$Z_R = -\frac{K^2\pi\omega z}{4\gamma^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) J_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)$$

$$Z_I = -\frac{K^2\omega z}{2\gamma^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right)\left\{\frac{\pi}{2}Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) - K_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) + \frac{4+2K^2}{K^2}K_0\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\bar{\gamma}_z\bar{\lambda}}\right)\right\},$$

R. tran. Slow

$L_f = \bar{\lambda}_w;\ \theta_d = \sqrt{\frac{\bar{\lambda}}{\bar{\lambda}_w}};$

S.C. long.

$L_f = \bar{\gamma}_z^2\bar{\lambda};\ \ \theta_d = \frac{1}{\bar{\gamma}_z^2};$

Longitudinal impedance: general result

Using expressions for field and Bessel functions we obtain:

$$Z_R = -\frac{K^2 \pi \omega z}{4\gamma^2} \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) J_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\lambda \lambda_w}}\right)$$

$$Z_I = -\frac{K^2 \omega z}{2\gamma^2} \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) \left\{ \frac{\pi}{2} Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\lambda \lambda_w}}\right) - K_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\lambda \lambda_w}}\right) + \frac{4 + 2K^2}{K^2} K_0\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\bar{\gamma}_z \lambda}\right) \right\},$$

R. tran. Slow

$L_f = \lambda_w$; $\theta_d = \sqrt{\frac{\lambda}{\lambda_w}}$;

R. tran. Fast

$L_f = \lambda_w$; $\theta_d = \sqrt{\frac{\lambda}{\lambda_w}}$;

S.C. long.

$L_f = \bar{\gamma}_z^2 \lambda$; $\theta_d = \frac{1}{\bar{\gamma}_z^2}$;

Asymptote I : $\sigma_\perp^{\,2} << \bar{\lambda}\bar{\lambda}_w$

$$x << 1:\ \ J_0(x)\approx 1;\ \ K_0(x)\approx -\gamma_E - \ln(x/2)\,;\ \ Y_0(x)\approx (2/\pi)[\gamma_E + \ln(x/2)]\,;$$

$$Z_R = -\frac{K^2\pi\omega z}{4\gamma^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) J_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)$$

$$Z_I = -\frac{K^2\omega z}{2\gamma^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right)\left\{\frac{\pi}{2}Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) - K_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) + \frac{4+2K^2}{K^2}K_0\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\bar{\gamma}_z\bar{\lambda}}\right)\right\},$$

Asymptote I : $\sigma_\perp^2 << \lambda\!\!\!^{-}\lambda\!\!\!^{-}_w$

$$x << 1: \; J_0(x) \approx 1; \;\; K_0(x) \approx -\gamma_E - \ln(x/2)\,; \;\; Y_0(x) \approx (2/\pi)[\gamma_E + \ln(x/2)]\,;$$

$$Z_R = -\frac{K^2 \omega \pi z}{4\gamma^2} \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) = \boxed{-\frac{K^2 \pi z}{4c\lambda\!\!\!^{-}\gamma^2}}$$

$$Z_I = -\frac{K^2 \omega z}{2\gamma^2} \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) \left\{ \frac{\pi}{2} Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\lambda\!\!\!^{-}\lambda\!\!\!^{-}_w}}\right) - K_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\lambda\!\!\!^{-}\lambda\!\!\!^{-}_w}}\right) + \frac{4+2K^2}{K^2} K_0\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\bar{\gamma}_z \lambda\!\!\!^{-}}\right) \right\},$$

Asymptote I : $\sigma_\perp^{\ 2} << \lambdabar \lambdabar_w$

$$x << 1: \ J_0(x) \approx 1; \ \ K_0(x) \approx -\gamma_E - \ln(x/2) ; \ \ Y_0(x) \approx (2/\pi)[\gamma_E + \ln(x/2)] ;$$

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$$\begin{aligned} Z_I &= -\frac{K^2 \omega z}{\gamma^2} \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp) \rho_o \left(\vec{r''}_\perp\right) \\ &\times \left\{ \ln\left(\sqrt{\frac{\lambdabar}{\lambdabar_r}}\right) - \frac{2}{K^2} \ln\left(\sqrt{1+\frac{K^2}{2}}\right) - \frac{2\gamma_E}{K^2} - \frac{2}{K^2} \ln\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{2\lambdabar\gamma}\right) \right\} \\ &= -\frac{K^2 z}{c\lambdabar\gamma^2} \ln\left(\sqrt{\frac{\lambdabar}{\lambdabar_r}}\right) + \frac{2z}{c\lambdabar\gamma^2} \ln\left(\sqrt{1+\frac{K^2}{2}}\right) + Z_{I\ \mathrm{free}} , \end{aligned}$$

$$Z_{I\ \mathrm{free}} = \frac{2z\gamma_E}{c\lambdabar\gamma^2} + \frac{2\omega z}{\gamma^2} \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp) \rho_o \left(\vec{r''}_\perp\right) \ln\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{2\lambdabar\gamma}\right) = \frac{2z\gamma_E}{c\lambdabar\gamma^2} + \frac{2z}{c\lambdabar\gamma^2} \ln\left(\frac{\sigma_\perp}{\gamma\lambdabar}\right)$$

(Gaussian model)

Asymptote II : $\sigma_\perp^2 \gg \lambda\!\!\!^{-}\lambda\!\!\!^{-}_w$

$$Z_R = -\frac{K^2\pi\omega z}{4\gamma^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) J_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\lambda\!\!\!^{-}\lambda\!\!\!^{-}_w}}\right)$$

$$Z_I = -\frac{K^2\omega z}{2\gamma^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right)\left\{\frac{\pi}{2}Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\lambda\!\!\!^{-}\lambda\!\!\!^{-}_w}}\right)\right.$$
$$\left. -K_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\sqrt{\lambda\!\!\!^{-}\lambda\!\!\!^{-}_w}}\right) + \frac{4+2K^2}{K^2}K_0\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\bar{\gamma}_z\lambda\!\!\!^{-}}\right)\right\},$$

Large transverse size suppresses radiative contributions

Asymptote II : $\sigma_{\perp}^{2} >> \bar{\lambda}\bar{\lambda}_{w}$

$$Z_R = -\frac{K^2 \pi \omega z}{4\gamma^2} \int d\vec{r'}_{\perp} \int d\vec{r''}_{\perp} \rho_o^*(\vec{r'}_{\perp}) \rho_o\left(\vec{r''}_{\perp}\right) J_0\left(\frac{\sqrt{2}\left|\vec{r'}_{\perp} - \vec{r''}_{\perp}\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)$$

$$Z_I = -\frac{K^2 \omega z}{2\gamma^2} \int d\vec{r'}_{\perp} \int d\vec{r''}_{\perp} \rho_o^*(\vec{r'}_{\perp}) \rho_o\left(\vec{r''}_{\perp}\right) \left\{ \frac{\pi}{2} Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_{\perp} - \vec{r''}_{\perp}\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) - K_0\left(\frac{\sqrt{2}\left|\vec{r'}_{\perp} - \vec{r''}_{\perp}\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) + \frac{4+2K^2}{K^2} K_0\left(\frac{\left|\vec{r'}_{\perp} - \vec{r''}_{\perp}\right|}{\bar{\gamma}_z \bar{\lambda}}\right) \right\},$$

Large transverse size suppresses radiative contributions

Asymptote II : $\sigma_\perp^{\,2} >> \bar{\lambda}\bar{\lambda}_w$

$$Z = -i\frac{2\omega z}{\bar{\gamma}_z^2}\int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) K_0\left(\frac{\left|\vec{r'}_\perp - \vec{r''}_\perp\right|}{\bar{\lambda}\bar{\gamma}_z}\right)$$

Only space-charge term. Like free space, but $\gamma \rightarrow \bar{\gamma}_z$

Wake fields

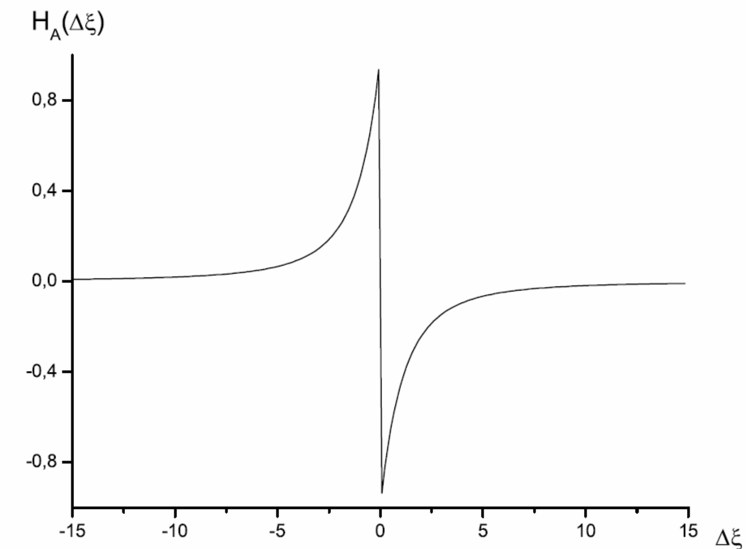
Gaussian longitudinal profile σ_z
Gaussian transverse profile $\sigma_\perp$

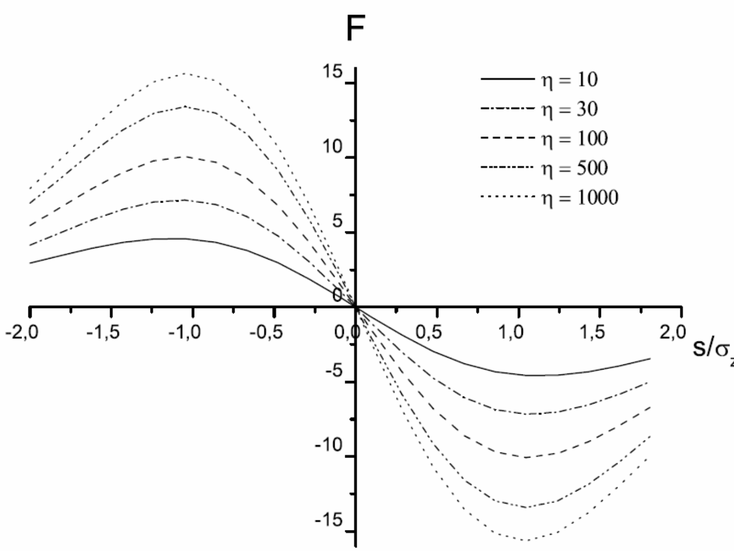
$$\frac{\Delta\mathcal{E}_A}{\mathcal{E}_0}\left(\frac{s}{\sigma_z};\eta\right)=\frac{I_{\max}\hat{z}}{\gamma I_A}F\left(\frac{s}{\sigma_z};\eta\right)$$

$$F\left(\frac{s}{\sigma_z};\eta\right)=\int_{-\infty}^{\infty}d(\Delta\xi)\,\eta\,H_A\left(\eta\frac{s}{\sigma_z}-\Delta\xi\right)\exp\left[-\frac{(\Delta\xi)^2}{2\eta^2}\right]$$

$$H_A(\Delta\xi)=-\frac{1}{2\sqrt{\pi}}(\Delta\xi)\left\{2\frac{\sqrt{\pi}}{|\Delta\xi|}-\pi\exp\left[\frac{(\Delta\xi)^2}{4}\right]\mathrm{erfc}\left[\frac{|\Delta\xi|}{2}\right]\right\}$$

$$\eta=\frac{\bar{\gamma}_z\sigma_z}{\sigma_\perp}\qquad \hat{z}=\frac{z}{2\bar{\gamma}_z^2\sigma_z}$$





Application: ESASE schemes

LCLS case

$a >> \bar{\gamma}_z \lambda\!\!\!^{-}$ **Free-space**	$a = 2.5mm$	$\bar{\gamma}_z \lambda\!\!\!^{-} = 500\,\mu m$
$L_s >> 2\bar{\gamma}_z^2 \lambda\!\!\!^{-}$ **Steady state**	$L_s = 50m$	$2\bar{\gamma}_z^2 \lambda\!\!\!^{-} = 10m$
$\lambda >> \lambda_r$ **Perturbation theory**	$\lambda = 50nm$	$\lambda_r = 0.15nm$
$\sigma_\perp^2 >> \lambda\!\!\!^{-} \lambda\!\!\!^{-}_w$ **Asymptote**	$\sigma_\perp^2 = 8 \cdot 10^{-10} m^2$	$\lambda\!\!\!^{-} \lambda\!\!\!^{-}_w = 2 \cdot 10^{-10} m^2$

$$\Delta\mathcal{E}_{A,\mathrm{peak}} = 2m_e c^2 \frac{I_{\max}}{I_A} \hat{z} F_{\max} \simeq 30\ \mathrm{MeV}$$

+20 MeV from 200 m –straight section

→ **TOTAL** $\Delta E_{A,peak} = 50MeV$

$$\hat{\alpha} = -\frac{1}{\gamma \omega \rho_{1D}} \frac{d\gamma}{dt}; \quad \rho_{1D} \sim 10^{-3}\ at\ I_{peak} = 18\,kA$$

→ $\hat{\alpha} \sim 1$

Conclusions

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DESY 07-87 at http://arxiv.org/abs/0706.2280