Self-Force-Derived Mass of an Electron Bunch

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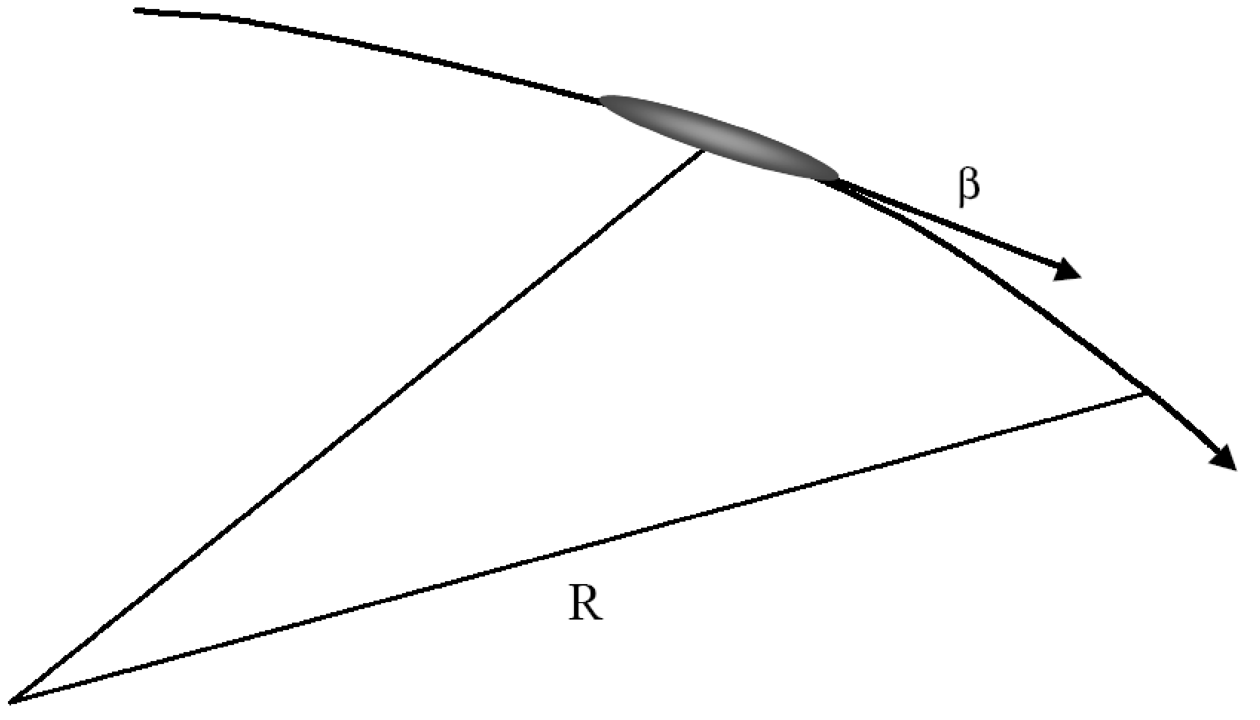
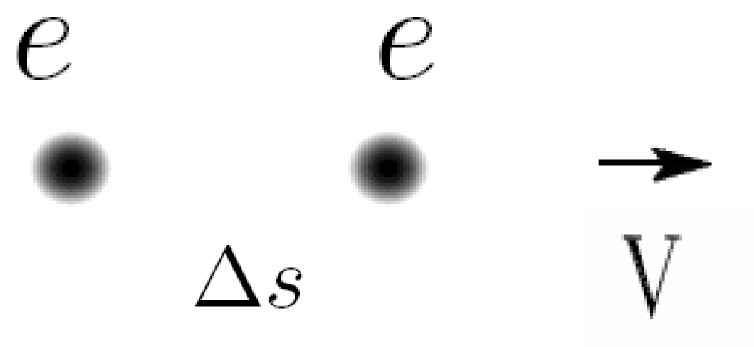
The properties of Lorentz transformations for energy and momentum in electromagnetic system are illustrated in a example involving a short electron bunch moving in a bending magnet



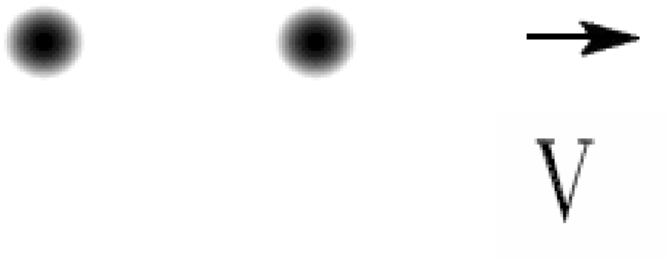
Short electron bunch moving in a bending magnet

The simplest possible charge distribution: a dumbbell, consisting of two point charges a short distance apart

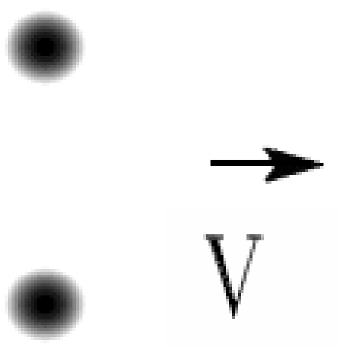


$$\Delta s \ll R/\gamma^3$$

Longitudinal motion of dumbbell



Vertical motion of dumbbell



Lorentz force

$$\boldsymbol{F}(\boldsymbol{r}_{\mathrm{T}},t)=e\boldsymbol{E}(\boldsymbol{r}_{\mathrm{T}},t)+ec\boldsymbol{\beta}_{\mathrm{T}}\times\boldsymbol{B}(\boldsymbol{r}_{\mathrm{T}},t),$$

Lienard-Wiechert fields

$$\boldsymbol{E}(\boldsymbol{r}_{\mathrm{T}},t)=\frac{e}{4\pi\varepsilon_0}\left\{\frac{1}{\gamma_{\mathrm{S}}^2}\frac{\hat{\mathbf{n}}-\boldsymbol{\beta}_{\mathrm{S}}}{R_{\mathrm{ST}}^2\left(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta}_{\mathrm{S}}\right)^3}+\frac{1}{c}\frac{\hat{\mathbf{n}}\times\left[(\hat{\mathbf{n}}-\boldsymbol{\beta}_{\mathrm{S}})\times\dot{\boldsymbol{\beta}}_{\mathrm{S}}\right]}{R_{\mathrm{ST}}\left(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta}_{\mathrm{S}}\right)^3}\right\}$$

$$\boldsymbol{B}(\boldsymbol{r}_{\mathrm{T}},t)=\frac{1}{c}\hat{\mathbf{n}}\times\boldsymbol{E}(\boldsymbol{r}_{\mathrm{T}},t)\ .$$

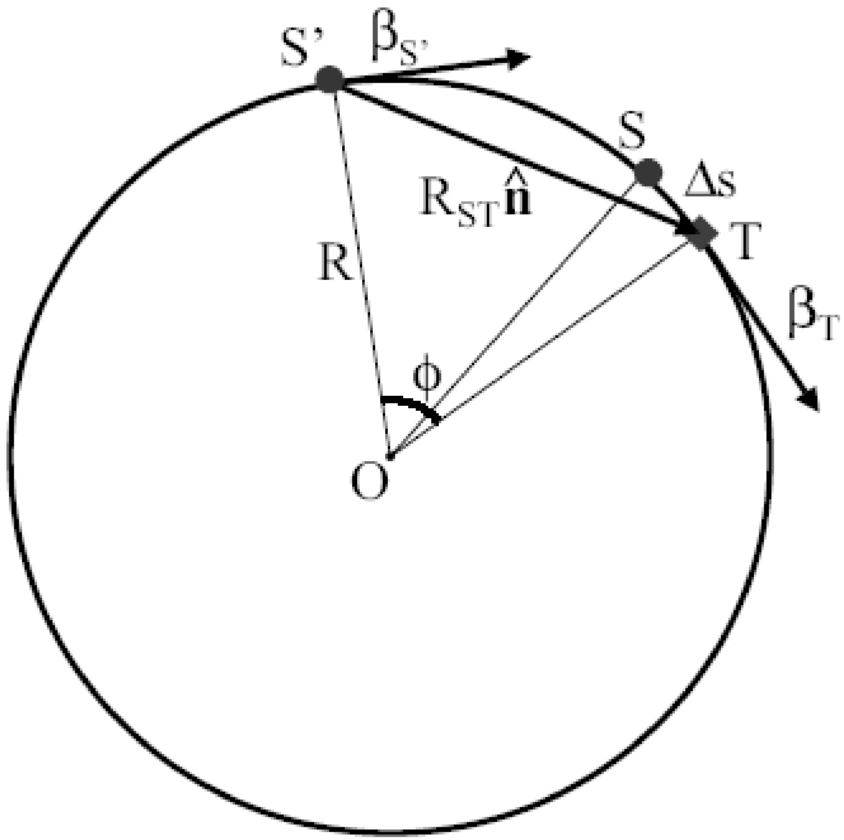


FIG. Geometry for the two-particle system in the steady state situation, with the test particle ahead of the source. Here T is the present position of the test particle, S is the present position of the source, while S' indicates the retarded position of the source.

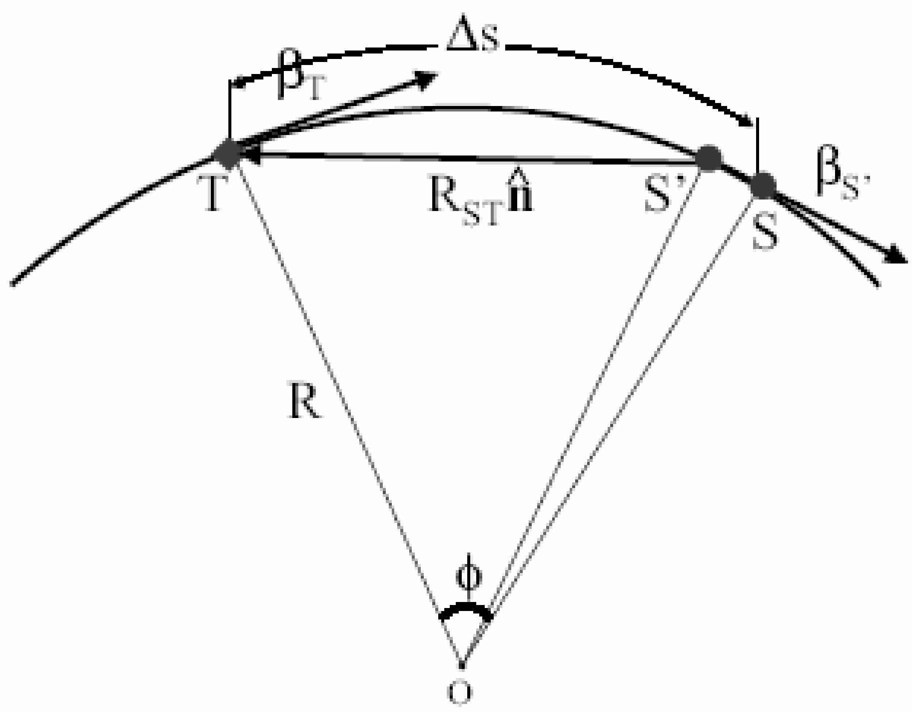
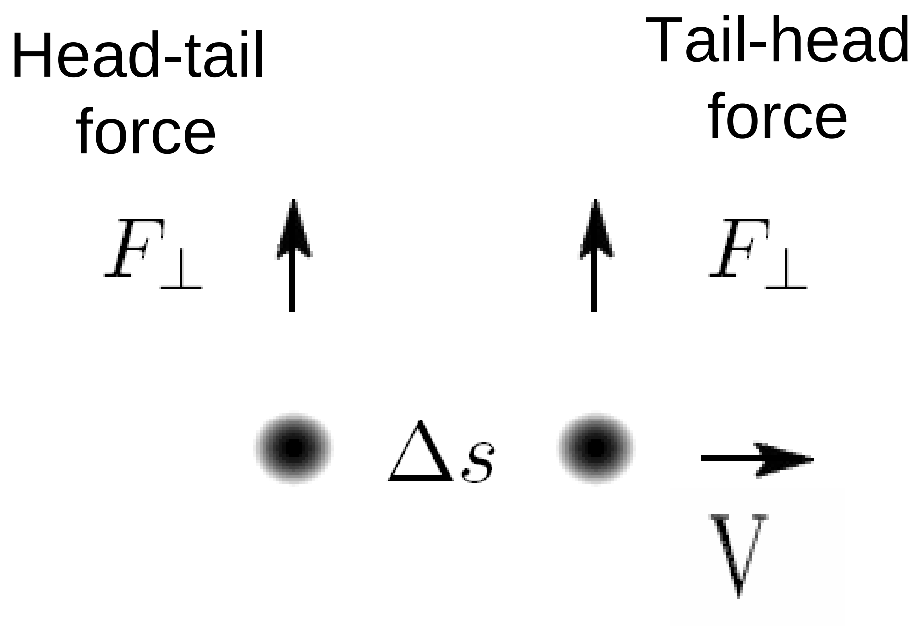


FIG. Geometry for the two-particle system in the steady state situation, with the source particle ahead of the test one. Here T is the present position of the test particle, S is the present position of the source, while S' indicates the retarded position of the source.

Longitudinal motion of dumbbell.
Total centrifugal force is

$$2\,F_\perp$$



$$F_\perp \simeq \frac{e^2}{4\pi\varepsilon_0 R\Delta s}\ .$$

Longitudinal motion of dumbbell

$$\Delta s \ll R/\gamma^3$$

Transverse self-force calculations in a lab system based on the use Lienard-Wiechert fields

Tail-Head force = Head-Tail force and equal to

$$F_\perp \simeq \frac{e^2}{4\pi\varepsilon_0 R\Delta s} \ .$$

Total centrifugal force for whole system

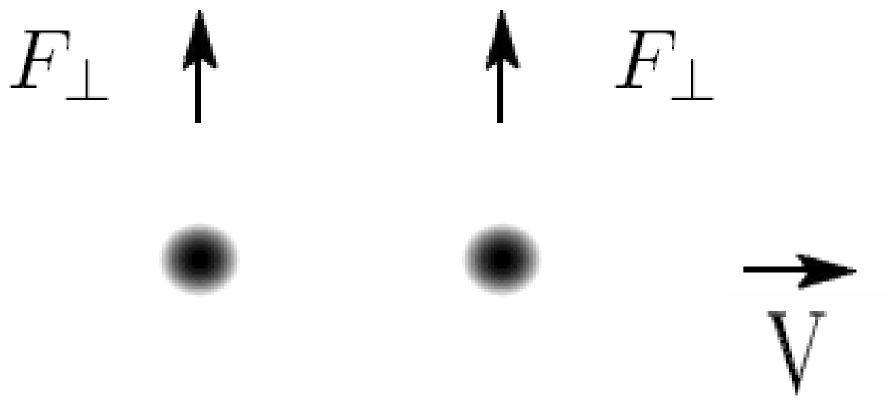
$$F_\perp \simeq 2\frac{e^2}{4\pi\varepsilon_0 R\Delta s} \ ,$$

Longitudinal motion of dumbbell. Total centrifugal force is

$$2F_{\perp}$$

Head-tail force

Tail-head force



$$F_{\perp} \simeq \frac{e^2}{4\pi\varepsilon_0 R\Delta s} \ .$$

Longitudinal motion of dumbbell

Naive relativistic prediction: Lorentz transformation for energy and momentum in a electromagnetic system behave as four-vector

In the rest frame

$$E'_e = U' = e^2/(4\pi\epsilon_0\gamma\Delta s)$$

$$\boldsymbol{P}'_e = 0.$$

In the lab frame

$$\boldsymbol{P}_{e3} = \gamma\boldsymbol{v}U'/c^2,$$

Centrifugal force

$$F_\perp \simeq \frac{e^2}{4\pi\varepsilon_0 R\Delta s}.$$

For the longitudinal motion it disagree with lab frame calculations.
They differ by a factor 2

Vertical motion of dumbbell

$$\Delta s \ll R/\gamma^3$$

Transverse self-force calculations in a lab system based on the use Lienard-Wiechert fields

Tail-Head force = Head-Tail force

Total centrifugal force for whole system

$$F_\perp \simeq \frac{e^2}{4\pi\varepsilon_0 R\Delta s} \; .$$

For the vertical motion it agree with four-vector energy -momentum derived force

Electromagnetic energy and momentum constitute a four-tensor

$$P^i = \frac{1}{c} \int T'^{\mu\nu} \Lambda^i{}_\mu \Lambda^0{}_\nu \frac{dV'}{\gamma}$$

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$P_z = \gamma \left(2m + U'/c^2 - \frac{1}{c^2} \int T'_{33}\, dV' \right) \beta c$$

Space-space components of energy-momentum tensor

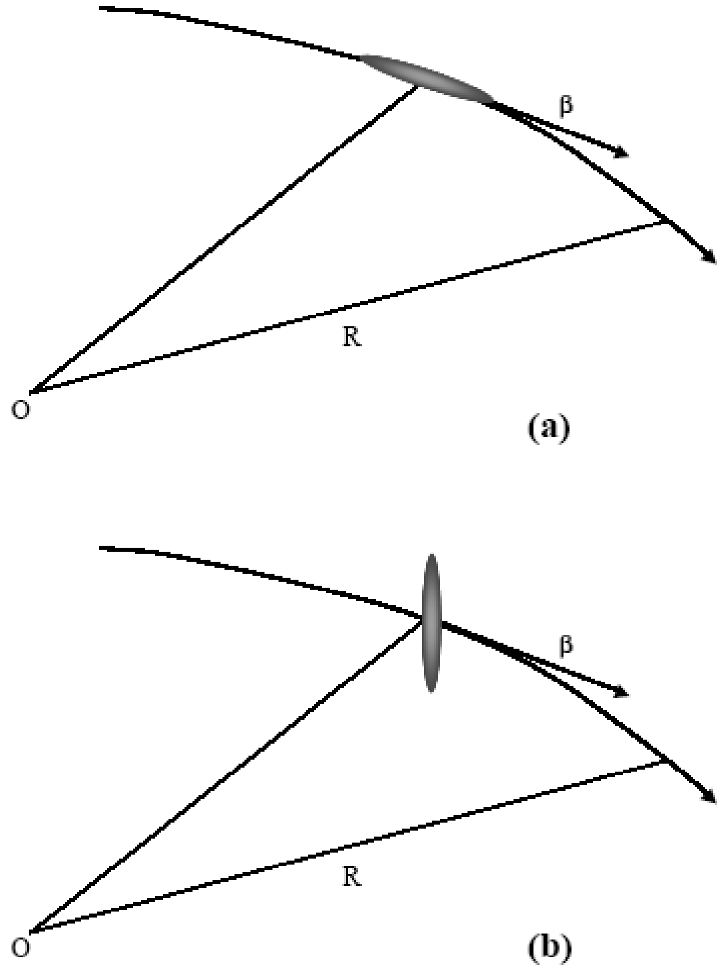
$$T'_{ij} = \varepsilon_0 (E'_i E'_j - \delta_{ij} E'^2 / 2) \ ,$$

where $i, \ j \ = \ 1 \ldots \ 3.$

$$T'_{33} = +E'^2_z - E'^2 / 2 \ .$$

$$\int T'_{33} \ dV' = -U'.$$

$$P_z = \gamma \left(2m + 2U' / c^2 \right) \beta c$$



Electromagnetic mass of a line distribution depend on the direction of the velocity

Electromagnetic mass of a spherical shell
Abraham 1904

$$U' = \epsilon_0/2 \int \boldsymbol{E'}^2 dV',$$

Charges uniformly distributed on the surface of a sphera

$$\boldsymbol{P}_e = 4/3\gamma \boldsymbol{v} U'/c^2,$$

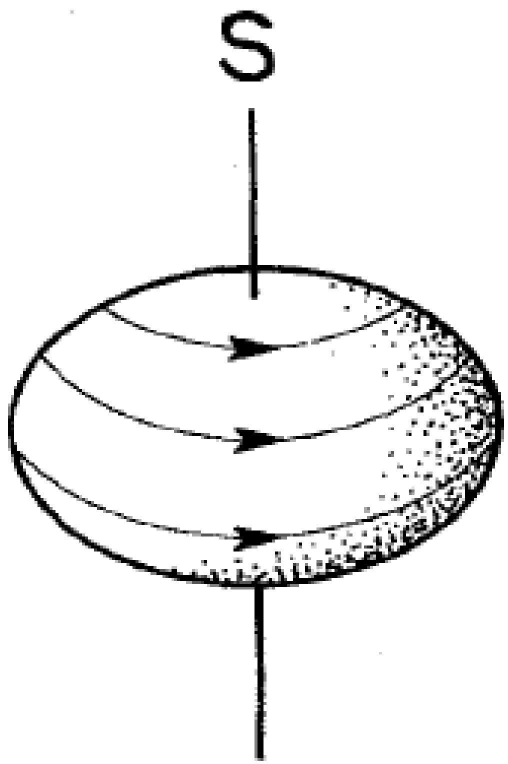
Electromagnetic mass of a spherical shell
is 4/3 its energy-derived mass

For longitudinal motion electromagnetic mass of a line
distribution is 2 its energy-derived mass

For transverse motion it agrees with energy-derived mass

Stable system with an extended charge distribution

Heavy nuclear



For heavy nuclear one has a concrete realization of the Poincare stresses through the nuclear field

The usual expressions for the electromagnetic energy and momentum

$$P_e^0 = \frac{1}{2c}\int\left(\epsilon_0 \boldsymbol{E}^2 + \frac{\boldsymbol{B}^2}{\mu_0}\right)dV$$

$$\boldsymbol{P}_e = \frac{1}{\mu_0 c^2}\int(\boldsymbol{E}\times\boldsymbol{B})\,dV,$$

The redefinition of the four-momentum that Rohrlich used to deal with electron problem

$$P_e^0 = \frac{1}{c}\int\left[\frac{1}{2}\left(\epsilon_0 \boldsymbol{E}^2 + \frac{\boldsymbol{B}^2}{\mu_0}\right) - \boldsymbol{v}\cdot\frac{\boldsymbol{E}\times\boldsymbol{B}}{\mu_0 c^2}\right]dV$$

$$\boldsymbol{P}_e = \frac{\gamma}{c}\int\left[\frac{\boldsymbol{E}\times\boldsymbol{B}}{\mu_0 c} - \frac{\boldsymbol{v}}{2}\cdot\left(\epsilon_0 \boldsymbol{E}^2 + \frac{\boldsymbol{B}^2}{\mu_0}\right) + \right.$$
$$\left. + \epsilon_0(\boldsymbol{v}\cdot\boldsymbol{E})\boldsymbol{E} + (\boldsymbol{v}\cdot\boldsymbol{B})\frac{\boldsymbol{B}}{\mu_0}\right]dV$$

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On energy and momentum of an ultrarelativistic unstable system

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arXiv:physics/0211093v1[physics.acc-ph]

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