DEVELOPMENT OF A COMPACT CHERENKOV FREE-ELECTRON LASER OPERATING IN TERAHERTZ WAVE RANGE

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Abstract

We designed a compact Cherenkov Free-Electron Laser (CFEL) oscillator driven by 50keV electron beam. The resonator, called double-slab resonator, consists of a rectangular waveguide partially filled with two lined parallel silicon slabs, whose dielectric constant ε equals to 11.6 in the terahertz frequency range. The operation frequency ranges from 40GHz to 1THz according to the thickness of the slabs. Numerical calculation predicted 100W-level output at 46GHz by electron beam with 50keV/400mA.

OPERATING FREQUENCY OF THE DOUBLE-SLAB RESONATOR

Our purpose is to develop a CFEL oscillator operating in the terahertz frequency range driven by a low energy electron beam [1]. We examine the operating frequency of the double-slab resonator fed by 50keV electron beam in this section.

Fig.1 shows the schematic view of the double-slab resonator. To simplify the problem, it is assumed that its transverse dimensions are $2(D+d)$ and infinite, where D and d are the vacuum half width and the thickness of the dielectric slab with dielectric constant of ε, respectively.

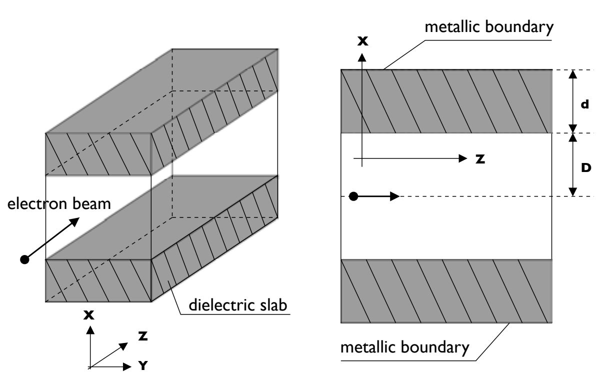

Figure 1: Schematic form of the waveguide partially filled with two lined parallel dielectric slabs with the thickness of d. Electron beam propagates along the Z-axis in the spacing of the slabs with the width of $2D$.

In CFEL oscillator, the radiation is gained by the interaction between the longitudinal electric field E_z and the straightly moving electron [2]. TM modes whose E_z distributes in a symmetrical pattern to the x=0 plane are, therefore, of interest. The electromagnetic field of the symmetric TM mode, in concrete terms TM_1, TM_3, TM_5.., is expressed as following. [3]
In vacuum region: $0 < |x| < D$

$$\begin{aligned} \mathbf{E}(x,z,t) &= E_{z0}\begin{pmatrix} \frac{k_z}{p}\sinh px \\ 0 \\ i\cosh px \end{pmatrix} e^{i\theta} \\ \mathbf{H}(x,z,t) &= E_{z0}\begin{pmatrix} 0 \\ \frac{k_0}{p}\sinh px \\ 0 \end{pmatrix} e^{i\theta} \end{aligned} \tag{1}$$

In dielectric region: $D < |x| < D+d$

$$\mathbf{E}(x,z,t) = E_{z0}\begin{pmatrix} \frac{k_z}{\varepsilon p}\left(F\cos\theta' + G\sin\theta'\right) \\ 0 \\ -i\frac{q}{\varepsilon p}(F\sin\theta' - G\cos\theta') \end{pmatrix} e^{i\theta}$$

$$\mathbf{H}(x,z,t) = E_{z0}\begin{pmatrix} 0 \\ \frac{k_0}{p}\left(F\cos\theta' + G\sin\theta'\right) \\ 0 \end{pmatrix} e^{i\theta} \tag{2}$$

where E_{z0} is the amplitude of E_z at x=0, k_z and ω are the wavenumber and angular frequency, respectively. The transverse wavenumbers in the vacuum and dielectric regions are defined as $p^2 = k_z^2 - \omega^2/c^2$ and $q^2 = \varepsilon\omega^2/c^2 - k_z^2$, respectively. Here the free-space wavenumber $k_0 = \omega/c$, the longitudinal phase of the radiation field $\theta = k_z z - \omega t$, the transverse phase $\theta' = (x-D)q$, the constants determined by the resonator configuration $F = \sinh pD$, $G = \varepsilon\frac{p}{q}\cosh pD$.

The dispersion relation is given by

$$\varepsilon\frac{p}{q}\cos qd\cosh pD - \sin qd\sinh pD = 0. \tag{3}$$

Fig.2 shows the dispersion curves for TM_1 mode for slabs of silicon (ε=11.6) with thickness d=0.65mm and $30\mu m$. In CFEL, the radiation resonates with electron beam if the phase velocity w/k_z is equals to the electron velocity. Thus the intersections of the curves and the beam mode line $w = \beta_z k_z$ are the resonant points. The resonant frequencies are 46.0GHz and 983 GHz for d=0.65mm and $30\mu m$, respectively. The numerical calculations of the dispersion relations found that the resonant frequency was almost inversely proportional to the thickness d.

On the other hand, the resonant frequency suggests a little dependence on the width of vacuum region D as shown in Fig.3. It decreases only by 3% as the thickness D increases from 0.325mm to 1.3mm.

However, the vacuum width D has a substantial effect on the radiation filed strength of E_z in the vacuum region,

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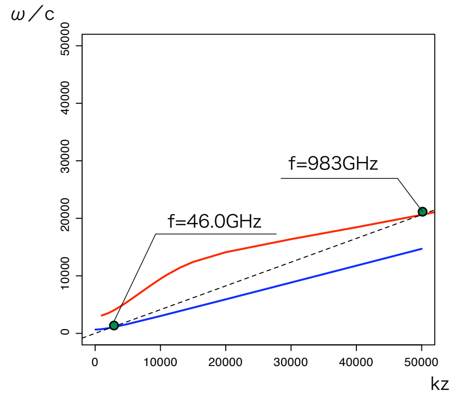

Figure 2: Dispersion curves of the double-slab resonator. Lower and upper curves show the dispersion for d=0.65mm and 30μm, respectively. Dashed line shows the velocity of 50keV.

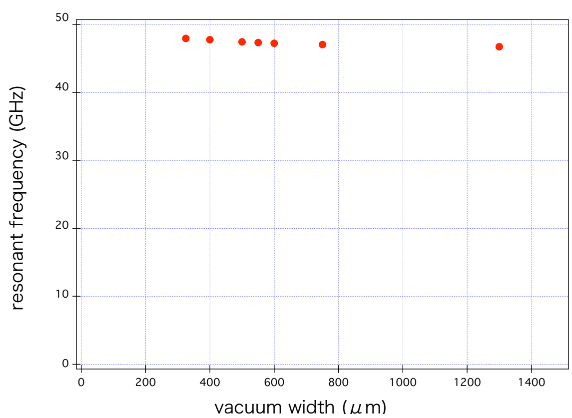

Figure 3: The resonant frequency versus the vacuum width D.

i.e. the interaction region. Fig.4(a) and Fig.4(b) show the E_z distributions at 46.0GHz for D=0.75mm and 0.5mm, respectively. As the width $2D$ decreases from 1.5 mm to 1.0 mm, the E_{z0} increases by a factor of 1.6. To obtain substantial interaction, one should reduce the width of vacuum region D less than $2d$. For higher operating frequency, thus, the beam focusing will be the issue.

NUMERICAL STUDY ON THE CFEL OSCILLATOR AT 46GHZ

In this section, we describe the design of a 46GHz CFEL oscillator for the preliminary experiment. The required resonator configuration, start-up current, the gain and the saturation power were investigated by using the one-dimensional numerical simulation code.

The calculation of CFEL interaction was based on the following equations.

$$\frac{d^2 z_i(t)}{dt^2} = -\frac{e}{\gamma^3 m} E_z(z,t) \tag{4}$$

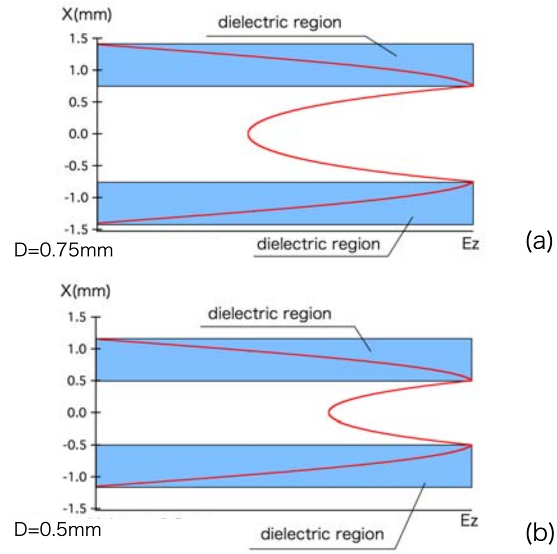

Figure 4: Field distributions for the vacuum width

$$\frac{d\gamma_i(t)}{dt} = -\frac{e}{mc^2} v_{z_i}(t) E_z(z_i,t) \tag{5}$$

where $E_z(z,t) = E_{z0}\cos(\omega t - k_z z_i)$, e is the electron charge, m is the electron rest mass, c is light velocity in the vacuum, and γ is the relativistic factor. The subscript i refers to the quantities of the i-th particle. The power increment for the radiation is related to the decrease in the average electron energy:

$$< \frac{d\gamma}{dt} >= \frac{1}{n} \sum \frac{d\gamma_i}{dt} \tag{6}$$

$$\frac{dP}{dt} = - < \frac{d\gamma}{dt} > \frac{I}{e} mc^2 \tag{7}$$

where n is the number of electron, and I is beam current.

The radiation power inside the resonator is calculated by

$$\begin{aligned} P &= \int_s \mathbf{S} \cdot \mathbf{e}_z dx dy \\ &= A(\omega, k_z) E_{z0}^2 \end{aligned} \tag{8}$$

where,

$$\begin{aligned} A(\omega, k_z) &= \frac{k_z k_0}{4\varepsilon p^2} \Bigg[\varepsilon \left(\frac{e^{2pD} - e^{-2pD}}{2p} - 2D \right) \\ &+ 4 \Bigg[F^2 \left(\frac{\sin 2qd}{4q} + \frac{d}{2} \right) + FG \left(\frac{1-\cos 2qd}{2q} \right) \\ &+ G^2 \left(\frac{d}{2} - \frac{\sin 2qd}{4q} \right) \Bigg] \Bigg] \end{aligned} \tag{9}$$

From these equations, E_{z0} of the gained radiation was evaluated, and then it is used to calculate the radiation field at the next time step.

To gain the radiation at 46.0GHz, the electron energy should be detuned from the resonant energy of 50keV. The amount of the detuning was represented with a factor α which means $\gamma = \alpha\gamma_0 : \gamma_0 = 50/511 + 1$. Fig.5 shows the dependence of the small-signal gain on the detuning factor α for the resonator length L=11cm with 400mA electron beam. The meaningful gain is obtained in the range $1.0005 < \alpha < 1.005$, and the gain reaches the maximum at $\alpha = 1.0022$. Because the optimal α is related to the slippage between the radiation and electron, the optimal value changes with the resonator length L.

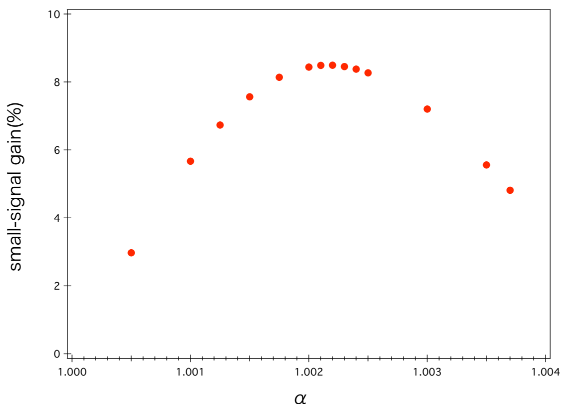

Figure 5: The dependence of the small-signal gain on the detuning factor α for the resonator length L=11cm.

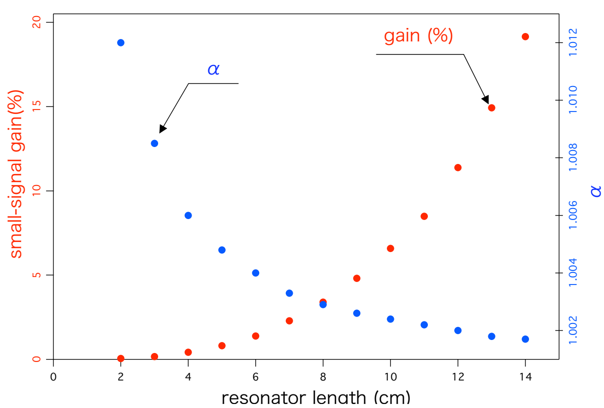

Figure 6: The dependence of the small-signal gain and optimal α on the resonator length.

Red circles in Fig.6 show the dependence of the small-signal gain on the resonator length. Blue circles show the optimal α for each resonator length. The small-signal gain varies as $L^{3.03}$. The optimal α decreases with L: $\alpha - 1$ is inversely proportional to L. We chose L=11cm for practical design because the gain exceeds enough the loss in the resonator, as well as the slabs of this length can be cut from the standard wafer for the semiconductor applications.

Fig.7 shows evolution of the CFEL output power and net gain. The resonator length is 11cm. We assumed the total loss of 5% each round-trip including the dielectric loss of 4% and the extraction efficiency of 1%.

The radiation grows with a net gain of 3.57% until 600 round-trips, then saturates around the 800 round-trips at the saturation power of 97W. In the practical situation, the duty cycle should be limited to 1/1000 to avoid thermal damage. The average power of the practical device will be 100mW level.

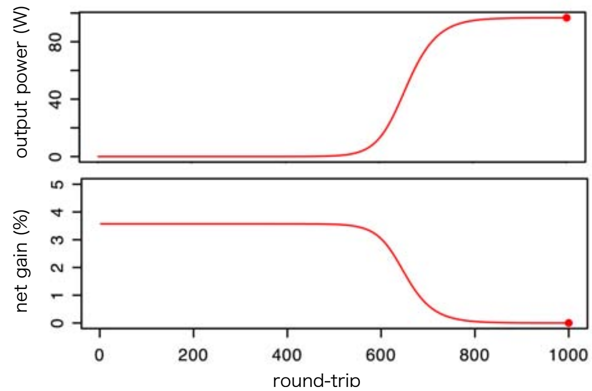

Figure 7: Upper trace shows the output power growth of the electromagnetic wave verses the round-trip. Bottom trace shows the net gain verses the round-trip. The lasing frequency is 46GHz, the resonator length is 11cm. The electron energy is 50keV, and the beam current is 400mA.

CFEL TEST BENCH

To demonstrate 46GHz lasing experiment, we constructed the CFEL device consisting of Spindt-Type Cathode, the silicon double-slab resonator, and superconducting coil. Spindt-Type Cathode can generate 100mA electron beam from emission area of $0.5^2 \times \pi\ mm^2$. Fig.8 shows picture of the silicon double-slab resonator. The slab has the thickness of 0.65mm and the width of 5mm. Superconducting coil produces magnetic flux density up to 5T. This strong field will focus the electron beam.

Figure 8: The double-slab resonator

REFERENCES

[1] M.R.Asakawa, K.Nakao, N.Miyabe, M.Kusaba, Y.Tsunawaki, Proceedings of FEL 2006, BESSY,Berlin,Germany, p364.

[2] J.Walsh, B.Johnson, Phys.Rev.Let.Vol.53 (1984) 779.

[3] Wladyslaw Zakowicz, J.Appl.phys.,Vol.55, (1984) 3421.