# ANALYTICAL STUDIES OF TRANSVERSE COHERENCE PROPERTIES OF X-RAY FELS

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*Abstract*

The explicit solution of the initial value problem for a SASE FEL operating with a large ratio of electron beam emittance to the wavelength, $\hat{\epsilon} = 2\pi\epsilon/\lambda \gg 1$, is presented. The degree of transverse coherence is explicitly calculated, too. It is shown to be dependent on the ratio of the number of FEL gain lengths to the parameter $\hat{\epsilon}$. In particular, in the multi-mode limit the radiation from a SASE FEL has by the squared number of gain lengths higher degree of transverse coherence than a synchrotron radiation generated by a beam with the same emittance.

## INTRODUCTION

Free electron lasing at wavelengths shorter than ultraviolet can be achieved with a single-pass, high-gain FEL amplifier. Due to a lack of powerful, coherent seeding sources short-wavelength FEL amplifiers work in so called Self-Amplified Spontaneous Emission (SASE) mode when amplification process starts from shot noise in the electron beam [1, 2, 3]. The first VUV FEL user facility FLASH ("F"ree-Electron-"LAS"er in "H"amburg) [4, 5] operates in SASE mode and produces GW-level, laser-like radiation pulses with 10 to 50 fs duration in the wavelength range 13-45 nm. Present level of accelerator and FEL techniques holds potential for SASE FELs to generate wavelengths as short as 0.1 nm [6, 7, 8].

The condition $\hat{\epsilon} < 1/2$ is often formulated as necessary one for an optimal design of a SASE FEL. It is meant that under this condition the radiation from SASE FEL has a full transverse coherence. However, as it is shown in [9], the maximal degree of transverse coherence (and brilliance as well) is achieved at $\hat{\epsilon} \simeq 1$. For smaller emittances the degree of transverse coherence decreases due to the effect discovered in [10]. Moreover, the above mentioned condition is strongly violated in the project parameters of hard X-ray FELs [6, 7, 8]: there the parameter $\hat{\epsilon}$ is in the range 2-5. Even without discussing exotic proposals [11, 12], one can notice a general trend towards lower energies of the electron beam, i.e. cost-saving solutions. Since achievable normalized emittance $\gamma\epsilon$ is limited by beam physics and technology issues, this would lead to a further increase of $\hat{\epsilon}$. Thus, theoretical understanding of properties of a SASE FEL, operating in this regime, becomes practically important. In this paper we present the main results of the theoretical analysis performed in Ref. [13] dealing with the limit $\hat{\epsilon} \gg 1$.

## EIGENVALUE EQUATION

Let us have at the undulator entrance a continuous electron beam with the current $I_0$, with the Gaussian distribution in energy

$$F(\mathcal{E}-\mathcal{E}_0) = \left(2\pi\langle(\Delta\mathcal{E})^2\rangle\right)^{-1/2} \exp\left(-\frac{(\mathcal{E}-\mathcal{E}_0)^2}{2\langle(\Delta\mathcal{E})^2\rangle}\right) , \quad (1)$$

and in a transverse phase plane

$$f(x,x') = (2\pi\sigma^2 k_\beta)^{-1} \exp\left[-\frac{x^2+(x')^2/k_\beta^2}{2\sigma^2}\right] , \quad (2)$$

the same in $y$ phase plane. Here $k_\beta = 1/\beta$ is the wavenumber of betatron oscillations and $\sigma = \sqrt{\epsilon\beta}$.

Using cylindrical coordinates, in the high-gain limit we seek the solution for a slowly varying complex amplitude of the electric field of the electromagnetic wave in the form [14]:

$$\tilde{E}(z,r,\varphi) \propto \Phi_{nm}(r)\exp(\Lambda z)e^{\pm\,\mathrm{i}\,n\varphi}, \quad (3)$$

where $n$ is an integer, $n \ge 0$. For each $n > 0$ there are two orthogonal azimuthal modes and many radial modes that differ by eigenvalue $\Lambda$ and eigenfunction $\Phi_{nm}(r)$. The integro-differential equation for radiation field eigenmodes [15, 16, 17] can be written in the following normalized form:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\,\hat{r}^2} + \frac{1}{\hat{r}}\frac{\mathrm{d}}{\mathrm{d}\,\hat{r}} - \frac{n^2}{\hat{r}^2} + 2\,\mathrm{i}\,B\hat{\Lambda}\right]\Phi_{nm}(\hat{r}) =$$
$$-4\int\limits_0^\infty \mathrm{d}\,\hat{r}'\hat{r}'\Phi_{nm}(\hat{r}')$$
$$\times\int\limits_0^\infty \mathrm{d}\,\xi\frac{\xi}{\sin^2(\hat{k}_\beta\xi)}\exp\left[-\frac{\hat{\Lambda}_{\mathrm{T}}^2\xi^2}{2} - (\hat{\Lambda}+\mathrm{i}\,\hat{C})\xi\right]$$
$$\times\exp\left[-\frac{(1-\mathrm{i}\,B\hat{k}_\beta^2\xi/2)(\hat{r}^2+\hat{r}'^2)}{\sin^2(\hat{k}_\beta\xi)}\right]$$
$$\times I_n\left[\frac{2(1-\mathrm{i}\,B\hat{k}_\beta^2\xi/2)\hat{r}\hat{r}'\cos(\hat{k}_\beta\xi)}{\sin^2(\hat{k}_\beta\xi)}\right] , \quad (4)$$

where $I_n$ is the modified Bessel function of the first kind. The following notations are used here: $\hat{\Lambda} = \Lambda/\Gamma$, $\hat{r} = r/(\sigma\sqrt{2})$, $B = 2\sigma^2\Gamma\omega/c$ is the diffraction parameter, $\hat{k}_\beta = k_\beta/\Gamma$ is the betatron motion parameter,

$\hat{\Lambda}_{\mathrm{T}}^2 = \langle(\Delta\mathcal{E})^2\rangle/(\bar{\rho}^2\mathcal{E}^2)$ is the energy spread parameter, $\hat{C} = \left[k_{\mathrm{w}} - \omega/(2c\gamma_z^2)\right]/\Gamma$ is the detuning parameter, $\Gamma = \left[A_{\mathrm{JJ}}^2 I_0 \omega^2 \theta_{\mathrm{s}}^2 \left(I_{\mathrm{A}} c^2 \gamma_z^2 \gamma\right)^{-1}\right]^{1/2}$ is the gain factor, $\bar{\rho} = c\gamma_z^2\Gamma/\omega$ is the efficiency parameter, $\omega$ is the frequency of the electromagnetic wave, $\theta_{\mathrm{s}} = K_{\mathrm{rms}}/\gamma$, $K_{\mathrm{rms}}$ is the rms undulator parameter, $\gamma$ is relativistic factor, $\gamma_z^{-2} = \gamma^{-2} + \theta_{\mathrm{s}}^2$, $k_{\mathrm{w}}$ is the undulator wavenumber, $I_{\mathrm{A}} = 17$ kA is the Alfven current, $A_{\mathrm{JJ}} = 1$ for helical undulator and $A_{\mathrm{JJ}} = J_0(K_{\mathrm{rms}}^2/2(1+K_{\mathrm{rms}}^2)) - J_1(K_{\mathrm{rms}}^2/2(1+K_{\mathrm{rms}}^2))$ for planar undulator. Here $J_0$ and $J_1$ are the Bessel functions of the first kind. Note that the efficiency parameter $\bar{\rho}$ is related to the corresponding parameter $\rho$ [18] of the one-dimensional model as follows: $\bar{\rho} = \rho B^{1/3}$. The equation (4) can be reduced to the integral equation by means of the Hankel transformation [17] and then solved numerically.

Effects of emittance play the dominant role in X-ray FELs. In this paper we will mainly concentrate on the case when beta-function is optimized for the highest FEL gain as it happens in practice. Since diffraction parameter depends on beta-function, it is more convenient to go over to the normalized parameters other then those introduced above. Indeed, diffraction parameter can be rewritten as $B = 2\hat{\epsilon}/\hat{k}_\beta$, where $\hat{\epsilon} = 2\pi\epsilon/\lambda$. Then we can go from parameters $(B, \hat{k}_\beta)$ to $(\hat{\epsilon}, \hat{k}_\beta)$. In the following we will consider the case when $\hat{\epsilon} \gg 1$, the energy spread effect can be neglected, and the beta-function is optimized for the maximum growth rate. We apply the variational method [17, 19] to the Eq. (4) with the trial functions [13]

$$\Phi_{nm}(\hat{r}) = \hat{r}^n \exp(-a\hat{r}^2) L_m^n(2a\hat{r}^2) \,, \qquad (5)$$

where $L_m^n$ are assotiated Laguerre polynomials. Solving the obtained equations [13], we find the zeroth-order eigenvalue

$$\hat{\Lambda}_0 \simeq \frac{0.3695 + 0.2735\,\mathrm{i}}{\hat{\epsilon}} \qquad (6)$$

for the optimal betatron wavenumber $\hat{k}_\beta \simeq 0.503/\hat{\epsilon}^2$ and at the optimal detuning $\hat{C}_0 \simeq 0.391/\hat{\epsilon}$. The diffraction parameter for this operating point is $B \simeq 3.98\,\hat{\epsilon}^3$. Then we find [13] the next order correction (in $\hat{\epsilon}^{-1}$) to the eigenvalue

$$\hat{\Lambda}_{nm} \simeq \hat{\Lambda}_0 - \frac{(1+n+2m)(0.3080 + 0.0988\,\mathrm{i})}{\hat{\epsilon}^2} \,, \qquad (7)$$

and the mode parameter

$$a \simeq (0.4355 - 0.5123\,\mathrm{i})\hat{\epsilon} \,. \qquad (8)$$

Eqs. (5), (7) and (8) are the solutions for field distributions and growth rates of eigenmodes of a high-gain FEL with optimized beta-function in the limit $\hat{\epsilon} \gg 1$. We compared these asymptotical solutions with the exact solution [17] at the optimal detuning for different modes and found a good agreement for $\hat{\epsilon} \gg 1$.

Concluding this section, we should analyze some scaling relations. The FEL gain length in the case under consideration scales as $L_{\mathrm{g}} = (\mathrm{Re}\,\hat{\Lambda}_0\Gamma)^{-1} \propto \hat{\epsilon}/\Gamma$, while the beta-function as $\beta = k_\beta^{-1} \propto \hat{\epsilon}^2/\Gamma$. In other words, the betatron phase advance per gain length, $k_\beta L_{\mathrm{g}}$, is of the order of $\hat{\epsilon}^{-1}$. The size of a radiation mode scales as $\hat{\epsilon}^{-1/2}$, and a typical change of transverse coordinates of particles during the passage of one gain length as $\hat{\epsilon}^{-1}$ in units of the electron beam size. The rms size of the intensity distribution of a fundamental mode (with $n = m = 0$) can be expressed in dimensional units as $\sigma_{rad} \simeq 1.07\sqrt{\beta\lambda/(2\pi)} \simeq 0.92\sqrt{\epsilon L_{\mathrm{g}}}$ for the optimal beta-function.

## INITIAL VALUE PROBLEM

The solution of initial value problem for a SASE FEL with a parallel electron beam, accounting for diffraction effects, was obtained in [15, 20]. Initial value problem, that also includes emittance effect, was solved in [21, 22, 23] in a general form and then approximated in single-mode limit. Here we present an explicit solution [13] for a large emittance, $\hat{\epsilon} \gg 1$, and the optimal (for the growth rate) focusing, $\hat{k}_\beta \simeq 0.503/\hat{\epsilon}^2$. The normalized output power (normalized FEL efficiency) in high-gain linear regime can be expressed as [13]:

$$\hat{\eta} = \frac{\eta}{\bar{\rho}} \simeq \frac{0.0755\exp(2N_g)}{N_c\hat{\epsilon}^2\sqrt{N_g}\,[f\,(N_g/\hat{\epsilon}) - 1]} \qquad (9)$$

where $\eta$ is the ratio of the radiation power to the electron beam power, $N_g = \mathrm{Re}\,\hat{\Lambda}_0\hat{z} = 0.3695\hat{z}/\hat{\epsilon}$ is the number of field gain lengths within a given undulator length, $\hat{z} = \Gamma z$, $N_c = I/(e\omega\bar{\rho})$, and the function $f$ is given by

$$f\left(\frac{N_g}{\hat{\epsilon}}\right) = 0.419\cosh\left(1.667\frac{N_g}{\hat{\epsilon}}\right) + 0.581\cos\left(0.535\frac{N_g}{\hat{\epsilon}}\right) \,. \qquad (10)$$

Note that parameter $\bar{\rho}$ for the given operating point is related to the corresponding one-dimensional parameter [18] as $\bar{\rho} = B^{1/3}\rho \simeq 1.58\hat{\epsilon}\rho$.

The expression (9) is valid when $\hat{\epsilon} \gg 1$ and $N_g \gg 1$, but the ratio $N_g/\hat{\epsilon}$ may take any value[1]. In particular, for a sufficiently long undulator, $N_g \gg \hat{\epsilon} \gg 1$, the fundamental $\mathrm{TEM}_{00}$ gives the dominating contribution to the total power. In this case Eq. (9) reads

$$\hat{\eta} \simeq \frac{0.360}{N_c\hat{\epsilon}^2\sqrt{N_g}}\exp\left[2N_g(1 - 0.834/\hat{\epsilon})\right] \simeq \frac{0.36}{N_c\hat{\epsilon}^2\sqrt{N_g^{00}}}\exp(2N_g^{00}) \,, \qquad (11)$$

[1] In practice the maximal value of $N_g$ is limited by saturation effects that are not considered in the linear theory presented here. For any reasonable set of parameters $N_g < 10$ in linear regime of SASE FEL operation.

where $N_g^{00} \simeq N_g$ is the number of field gain lengths for the $TEM_{00}$ mode. This solution is identical to that given in [23], taken in the limit $\hat{\epsilon} \gg 1$ with the optimal beta-function, and integrated over FEL frequency band. Now let us consider the multi-mode limit, $\hat{\epsilon} \gg N_g \gg 1$. In this case the radiation power (9) is expressed as

$$\hat{\eta} \simeq \frac{0.151}{N_c N_g^{5/2}} \exp(2N_g) \ . \qquad (12)$$

Relative partial contributions to the total power of modes with an azimuthal index $n$ can be calculated as follows [13]:

$$p_0 = \sqrt{\frac{f-1}{f+1}} \qquad for \quad n = 0 \qquad (13)$$

$$p_n = \frac{2\sqrt{f-1}}{\sqrt{f+1}(f+\sqrt{f^2-1})^n} \qquad for \quad n > 0 \qquad (14)$$

For $n > 0$ we consider the sum of the contributions of the two independently excited azimuthal modes with the angular dependence $\exp(\pm \mathrm{i} n\varphi)$. The contribution of the azimuthal-symmetric mode goes pretty linearly for $N_g < \hat{\epsilon}$:

$$p_0 \simeq 0.5 \frac{N_g}{\hat{\epsilon}}$$

and asymptotically approaches unity when $N_g \gg \hat{\epsilon}$. Other modes have maxima of which locations can be found from the equation

$$f\left(\frac{N_g}{\hat{\epsilon}}\right) = \frac{\sqrt{n^2+1}}{n} \ .$$

Maximal partial contribution of $n-$th azimuthal mode in linear regime of SASE FEL operation is given by

$$\max(p_n) = \frac{2n^n(\sqrt{n^2+1}-n)}{(\sqrt{n^2+1}+1)^n} \qquad for \quad n > 0 \qquad (15)$$

For instance, a possible contribution is limited by 34.3% for $n = 1$, by 18.0% for $n = 2$, etc. For large $n$ a maximum is located at $N_g/\hat{\epsilon} \simeq n^{-1}$ and takes the value $\max(p_n) \simeq (ne)^{-1}$, $e$ being the base of natural logarithm.

Especially simple relation for the partial contributions of azimuthal modes can be deduced for the point where $p_0 = p_1$. There we have:

$$p_0 = \frac{1}{3} \ , \qquad p_n = \frac{1}{2^{n-1}3} \ . \qquad (16)$$

Note that the results (15) and (16) are universal since they do not depend on a specific choice of the function $f$.

## DEGREE OF TRANSVERSE COHERENCE

The definition of a degree of transverse coherence was introduced in [9]:

$$\zeta = \frac{\int\int |\gamma_1(\vec{r}_\perp, \vec{r\prime}_\perp)|^2 \langle I(\vec{r}_\perp)\rangle \langle I(\vec{r\prime}_\perp)\rangle \, \mathrm{d}\,\vec{r}_\perp \, \mathrm{d}\,\vec{r\prime}_\perp}{[\int \langle I(\vec{r}_\perp)\rangle \, \mathrm{d}\,\vec{r}_\perp]^2} \ . \qquad (17)$$

where $I(\vec{r}_\perp) = |\tilde{E}(\vec{r}_\perp)|^2$ is the radiation intensity,

$$\gamma_1(\vec{r}_\perp, \vec{r\prime}_\perp) = \frac{\langle \tilde{E}(\vec{r}_\perp)\tilde{E}^*(\vec{r\prime}_\perp)\rangle}{\left[\langle|\tilde{E}(\vec{r}_\perp)|^2\rangle\langle|\tilde{E}(\vec{r\prime}_\perp)|^2\rangle\right]^{1/2}} \qquad (18)$$

is the transverse correlation function, $\tilde{E}$ is the slowly varying amplitude of the electromagnetic wave, and $< \dots >$ means ensemble average. The definition (17) is valid for any stationary (or quasi-stationary [14, 24]) random process, in particular for the radiation from a SASE FEL with a long electron bunch [14] operating in linear and nonlinear regimes.

The radiation of a SASE FEL, operating in the linear regime, holds properties of a completely chaotic polarized light [14, 24]. In this case, as it was shown in [9], the definition (17) is equivalent to that given by the variance of the instantaneous power [10, 14]:

$$\zeta = \sigma_P^2 = \frac{< (P - < P >)^2 >}{< P >^2} \ , \qquad (19)$$

where $P = \int I(\vec{r}_\perp) d\vec{r}_\perp$. The degree of transverse coherence $\zeta$ can be thought of as an inverse number of transverse modes [9, 10, 14]:

$$\zeta = \frac{1}{M_\mathrm{T}} \ . \qquad (20)$$

However, the definitions (17) and (19) cannot be directly used for the purpose of this paper, namely for analytical calculations of the degree of transverse coherence of a SASE FEL. Nevertheless, analyzing the results of numerical simulations in Ref. [9], we found out that the degree of transverse coherence according to the definition (17) is well approximated by the squared partial contribution of azimuthal-symmetric mode:

$$\zeta \simeq p_0^2 \ . \qquad (21)$$

Accuracy of this approximation is connected with the residual effect [10] on transverse coherence originated from the finite frequency bandwidth of a SASE FEL. Indeed, for a sufficiently long undulator the only fundamental $TEM_{00}$ mode survives (other modes are exponentially suppressed), i.e. perfect transverse coherence would be achieved for a monochromatic wave. However, the amplitude and phase distributions of this mode change within the FEL bandwidth. As a result, the degree of transverse coherence approaches unity as [10]: $1 - \zeta \simeq \delta_r/N_g$ but not exponentially as in (21). A numerical factor $\delta_r$ is of the order of

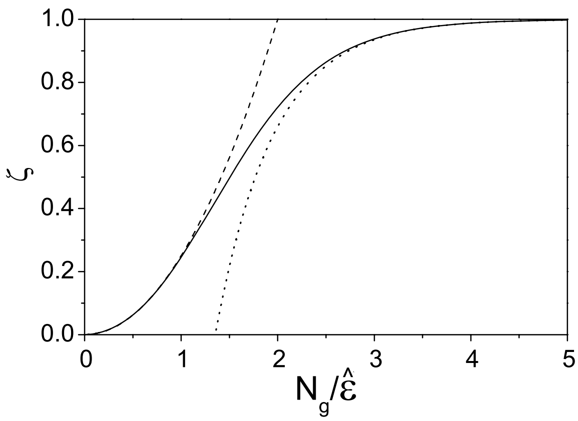


Figure 1: Degree of transverse coherence versus parameter $N_g/\hat{\epsilon}$. The asymptotes (24) and (25) are shown as dot and dash lines, respectively.

one for $\hat{\epsilon} \simeq 1$ [9] and strongly increases in the limit of small electron beam size. However, for considered here case $\hat{\epsilon} \gg 1$ we have found analytically [13] that $\delta_r \simeq 0.026$ for the optimal beta-function:

$$\zeta \simeq 1 - \frac{0.026}{N_g} \ , \qquad (22)$$

i.e. this effect is negligible for large $N_g$. We have seen from numerical simulations [9] that the relative difference between (17) and (21) is indeed very small for large $\hat{\epsilon}$ and $N_g$ in both linear and nonlinear regimes. Thus, despite the definition (21) was introduced heuristically, it is sufficiently accurate and adequate for our purposes. We rewrite it, using (13), in a more explicit form

$$\zeta \simeq \frac{f-1}{f+1} \ . \qquad (23)$$

The plot of the degree of transverse coherence versus $N_g/\hat{\epsilon}$ for the function $f$ from (10) is presented in Fig 1. For $N_g \gg \hat{\epsilon} \gg 1$ the degree of transverse coherence is close to one:

$$\zeta \simeq 1 - \frac{2}{f} \simeq 1 - 9.54 \exp\left(-1.667\frac{N_g}{\hat{\epsilon}}\right) \ , \qquad (24)$$

but one should keep in mind the residual effect (22). In the limit $\hat{\epsilon} \gg N_g \gg 1$ we get

$$\zeta = \frac{1}{M_{\mathrm{T}}} \simeq 0.25 \left(\frac{N_g}{\hat{\epsilon}}\right)^2 \ , \qquad (25)$$

or, in dimensional units

$$\zeta = \frac{1}{M_{\mathrm{T}}} \simeq \left(\frac{\lambda N_g}{4\pi\epsilon}\right)^2 \ . \qquad (26)$$

Amazingly, a numerical factor in front of the last expression is equal to one (with the accuracy of the order of $10^{-3}$, accepted in this paper) for the case of the optimal beta-function. Note that in the case of synchrotron radiation for an axisymmetric beam one would have in the limit $\hat{\epsilon} \gg 1$ for optimal focusing: $\zeta = 1/M_{\mathrm{T}} \simeq (\lambda/4\pi\epsilon)^2$. Thus, the radiation from a SASE FEL has $N_g^2$ higher degree of transverse coherence in this limit.

The asymptotes (24) and (25) are plotted in Fig. One can see that Eq. (25) is pretty accurate for $N_g/\hat{\epsilon} < 1$, and Eq. (24) - for $N_g/\hat{\epsilon} > 3$.

Finally, we present an estimate of the degree of transverse coherence at saturation for the case $\hat{\epsilon} \gg N_g \gg 1$. Number of gain lengths at the end of linear regime (where the formation of transverse coherence stops) can be estimated [13] as $N_g \simeq \frac{1}{2}\ln(N_c/\hat{\epsilon})$. Thus, using (25) we get:

$$\zeta^{sat} \simeq \left(\frac{\ln(N_c/\hat{\epsilon})}{4\hat{\epsilon}}\right)^2 \ . \qquad (27)$$

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