ENHANCING FEL POWER WITH PHASE SHIFTERS*

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Abstract

Tapering the undulator parameter is a well-known method for maintaining the resonant condition past saturation, and increasing Free Electron Laser (FEL) efficiency. In this paper, we demonstrate that shifting the electron bunch phase relative to the radiation is equivalent to tapering the undulator parameter. Using discrete phase changes derived from optimized undulator tapers for the Linac Coherent Light Source (LCLS) x-ray FEL, we show that appropriate phase shifts between undulator sections can reproduce the power enhancement of undulator tapers. Phase shifters are relatively easy to implement and operate, and could be used to aid or replace undulator tapers in optimizing FEL performance.

INTRODUCTION

Despite the projected six orders of magnitude increase in peak power for Self-Amplified Spontaneous Emission (SASE) x-ray FELs, some applications, including single molecule imaging, may require still higher photon flux [1]. To increase the power in higher harmonics, it is possible to shift the FEL electron phase relative to the radiation, suppressing the power in the fundamental wavelength and prolonging growth of the harmonics [2]. We have studied the related use of phase shifts to correct slippage at saturation and boost the power output of the fundamental wavelength. Similar methods have been mentioned in the past in connection with harmonic radiation [2] and numerical simulations [3]. Here we undertake a detailed analytical and numerical study of enhancing FEL power with phase shifters. We explore the relation between phase shifts and undulator tapers to calculate optimal phase shifts for SASE FELs in the saturation regime. The phase shift method, while equivalent to tapering the undulator parameter, provide an independent knob to maximize the FEL performance.

ONE-DIMENSIONAL ANALYSIS

The resonant condition

$$\lambda_1 = \lambda_u \frac{1+K^2/2}{2\gamma_0^2} \tag{1}$$

sets the radiation wavelength, λ_1 for an FEL; after each undulator period, λ_u, the electron bunch slips behind the radiation by exactly λ_1. It follows that when the resonant condition holds, the phase, Ψ, between the electron bunch and radiation stays constant (up to 2π). Near saturation, the bunch loses significant energy to the radiation, and the FEL resonant condition begins to fail. As $\gamma_0 \to \gamma < \gamma_0$, the resonant wavelength increases, and the electrons slip more than one radiation wavelength during each undulator period. Introducing a phase shift (by means of a small chicane) can correct for the increase in slippage by shifting the bunch backwards into the previous bucket. (If, over many undulator periods, the bunch has accumulated extra slippage of $\Delta\theta$, then the chicane shifts the electrons an additional $2\pi - \Delta\theta$. There is no easy way to shift electrons forward.) After the shift, the electron bunch is once again in phase with the radiation, preserving the resonant condition farther into saturation.

To show that compensating for the additional slippage can optimize the radiation power in the undulator, we use a simplified 1-D FEL model and neglect the effect of detuning. The slowly varying radiation field $\tilde{a}$ is given by

$$\frac{d\tilde{a}}{d\bar{z}} = -\langle e^{-i\theta_j}\rangle\,. \tag{2}$$

The phases, $\theta \equiv (k+k_u)z - \omega t + \text{const}$, are the longitudinal positions of the electrons relative to the electron bunch given in units of $\lambda_1/2\pi = 1/k_u$. The variable $\bar{z} \equiv 2k_u\rho z$, is the scaled position along the undulator. Here ρ is the dimensionless FEL parameter [4]. Finally, the average in Eq. (2) is taken over all electrons. Taking $\tilde{a} \equiv Ae^{i\Psi}$, with Ψ the phase of the radiation relative to the electron bunch, we can separate out the magnitude and phase components of the radiation field:

$$\frac{d\tilde{a}}{d\bar{z}} = e^{i\Psi}\left[\frac{dA}{d\bar{z}} + iA\frac{d\Psi}{d\bar{z}}\right]. \tag{3}$$

Inserting Eq. (3) into Eq. (2) and separating real and imaginary parts gives

$$\frac{dA}{d\bar{z}} = -\langle\cos(\theta_j+\Psi)\rangle\,, \tag{4}$$

$$\frac{d\Psi}{d\bar{z}} = \frac{1}{A}\langle\sin(\theta_j+\Psi)\rangle\,. \tag{5}$$

Our strategy is to maximize A by introducing an arbitrary phase shift, ϕ between the radiation and electrons, and then maximize $\frac{dA}{d\bar{z}}$ with respect to ϕ at all points along the undulator. (A chicane delays all particles relative to the radiation, introducing an arbitrary change in relative phase of the bunch.) Our goal, then, is to choose ϕ to maximize the quantity $-\langle\cos(\theta_j+\Psi+\phi)\rangle$.

Adding the arbitrary phase, ϕ, gives

$$\frac{dA}{d\bar{z}} = \langle\cos(\theta_j+\Psi)\rangle\cos(\phi) - \langle\sin(\theta_j+\Psi)\rangle\sin(\phi) \tag{6}$$

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To find the optimal phase shift, we differentiate with respect to ϕ to find

$$\phi_{max}(\bar{z}) = \arctan\left[-\frac{\langle\sin(\theta_j+\Psi)\rangle}{\langle\cos(\theta_j+\Psi)\rangle}\right]. \qquad (7)$$

Carrying out the maximization then requires the calculation of $\phi_{max}(\bar{z})$ at all points past saturation along the undulator. (ϕ is a function of $\bar{z}$, because θ_j and Ψ are also functions of $\bar{z}$.)

Substituting the result from Eq. (7) into Eq. (6) we find, for $\langle\cos(\theta_j+\Psi)\rangle > 0$,

$$\left(\frac{dA}{d\bar{z}}\right)_{max} = \sqrt{\langle\cos(\theta_j+\Psi)\rangle^2 + \langle\sin(\theta_j+\Psi)\rangle^2} \equiv |b|\,, \qquad (8)$$

where b is the bunching factor of the electrons.

We originally motivated the phase shift by the need to maintain a constant Ψ, the phase between the electron bunch and the radiation. We would like to check that our best phase, ϕ_{max}, optimized to increase the radiation amplitude, A, will simultaneously keep Ψ fixed. Returning to Eq. (5), now with an arbitrary phase inserted, we have

$$\frac{d\Psi}{d\bar{z}} \propto \langle\sin(\theta_j+\Psi)\rangle\cos\phi + \langle\cos(\theta_j+\Psi)\rangle\sin\phi\,. \qquad (9)$$

Plugging in ϕ_{max} from the optimization condition, Eq. (7), gives

$$\begin{aligned}\frac{d\Psi}{d\bar{z}} &\propto \langle\sin(\theta_j+\Psi)\rangle - \langle\cos(\theta_j+\Psi)\rangle\frac{\langle\sin(\theta_j+\Psi)\rangle}{\langle\cos(\theta_j+\Psi)\rangle} \\ &= 0\,. \end{aligned} \qquad (10)$$

We find that implementing the phase shifts, ϕ_{max}, does indeed keep the phase, Ψ, fixed, consistent with our initial goal of maintaining the correct slippage length.

EQUIVALENCY TO UNDULATOR TAPER

To find the optimal phases, Eq. (7), we can numerically evaluate $\langle\sin(\theta_j+\Psi)\rangle$ and $\langle\cos(\theta_j+\Psi)\rangle$ at each desired phase shifter location. This, of course, is not very practical for an actual SASE FEL. However, we motivated this approach by the need to maintain the resonant condition (Eq.1). Tapering the undulator, an established method for optimizing FELs, works through the same principle [5]. From the resonant condition, we see that shifting the phase is equivalent to decreasing the undulator parameter. To express a phase shift in terms of a taper, we start with the ponderomotive phase equation

$$\frac{d\theta}{d\bar{z}} = k_u - k_1\frac{[1+K^2/2]}{2\gamma^2}\,. \qquad (11)$$

relating the change in phase, $\phi = \Delta\theta$, to the undulator parameter, K. An undulator taper is a small shift $K_0 \to K(\bar{z})$. Plugging into Eq. (11), dropping the second order term, and using the resonant condition, $k_u/k_1 = (1+K_0^2/2)/(2\gamma_r^2)$ gives

$$\frac{d\theta}{d\bar{z}} = k_u - k_u\frac{\gamma_r^2}{\gamma^2} + k_u\frac{\gamma_r^2}{\gamma^2}\frac{K_0(K(\bar{z})-K_0)}{1+K_0^2/2}\,. \qquad (12)$$

Using the notation $\eta = (\gamma-\gamma_r)/\gamma_r$, $|\eta| \ll 1$ yields the usual FEL phase equation with a correction term of order $K_0 - K(\bar{z})$:

$$\frac{d\theta}{d\bar{z}} = 2k_u\left[\eta - \frac{K_0(K_0-K(\bar{z}))}{2+K_0^2}\right]. \qquad (13)$$

Thus we find a change, $K_0 \to K(\bar{z})$, results in a change, $\theta \to \theta + \Delta\theta$, with accumulated phase shift

$$\Delta\theta(\bar{z}) = -2k_u\int_{\bar{z}_0}^{\bar{z}}\frac{K_0(K_0-K(\bar{z}'))}{2+K_0^2}d\bar{z}' \qquad (14)$$

where $K(\bar{z})$ is some arbitrary function (i.e. an optimized taper) and the integral starts from the last phase shift at position $\bar{z}_0$.

The advantage of deriving phase shifts in this manner is that SASE FEL undulator tapers are well understood, and the optimal function $K(\bar{z})$ can be calculated from the FEL parameters, for instance by the GINGER self-taper algorithm [6]. With a predetermined taper function, we can plug into Eq. (14) and calculate the optimal phase shifts for an arbitrary FEL.

We have shown that shifting the phase continuously is identical to tapering the undulator parameter. However, the phase shift is most interesting if implemented discretely, with the taper from each undulator section replaced by a single shift. In this case, the exact equivalence breaks down. The larger the shift, the more time the FEL spends at a suboptimal phase, and for long undulators the accumulated difference $K(\bar{z}') - K_0$ might require large phase shifts, $\Delta\theta(L_u)$, near the end of the undulator (Eq. 14). To determine the practicality of discrete shifts, we use FEL parameters similar to the LCLS at 1.5 Å [7] for numerical examples (Table 1). Although LCLS does not currently have periodic phase shifters, designs for similar projects such as the European XFELs [8] have incorporated phase shifters between undulator sections. Using Eq. (14), we find the taper-equivalent shifts for LCLS have $\Delta\theta < 1$, suggesting that the method of phase shifts will match the performance of an undulator taper.

Table 1: SASE Simulation Parameters

Radiation wavelength (λ_1)	1.5 Å
Bunch current	3.4 kA
Undulator period (λ_u)	3 cm
Undulator parameter (K)	3.5
Electron energy	13.6 GeV
Relative RMS energy spread	$1x10^{-4}$
Normalized transverse emittance	1.2 μm
Beta function	25 m

NUMERICAL RESULTS

1-D Simulations

To simulate phase shifts, we started with a 1-D FEL code, including energy spread but not emittance effects. We used four different types of optimization:

1. Phase scan,
2. Numerically optimized phase shifts,
3. Linear energy taper, and
4. Phase shifts derived from a linear taper.

First we used a brute force phase scan. To determine the n^{th} phase shift at position z_n, we optimized the power at position z_{n+1} by trying 5 (or more) different phase shifts. Then, picking the best phase for the n^{th} position, we optimized position z_{n+1}.

Second, we used the results from Eq. (7) to calculate optimal phase shifts numerically. We calculate the phase for 2 cases: shifts every 1m and every 3m. The 3m shifts are less effective than the phase scan because the shifts are infrequent, and $\Delta\theta > 1$. The results for the first two methods are given in Fig. 1.

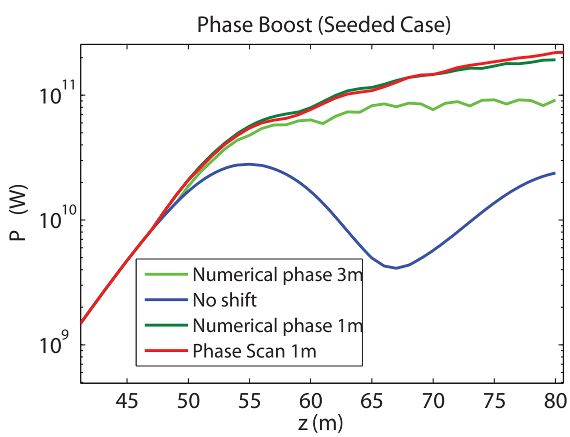

Figure 1: Fundamental radiation power from phase shifts. Phases done 3 ways: Numerical optimization every 1m (dark green), Numerical optimization every 3m (light green), Phase scan every 1m (red). The fourth plot (blue) has no phase shifts.

Third, we used a linear taper to boost the fundamental power. Fourth, we calculated the taper-equivalent phase shifts (Eq. (14)), implemented every 1m. The final two methods are computationally identical if the phase is shifted on each iteration. However, the results still match for less frequent phase shifts, so long as the shifts are small ($\Delta\theta < 1$). Results for both seeded and SASE cases are given in Figs. 2 and 3. Performance is worse for the third harmonic (green), because the shifts are relatively larger than in the case of the fundamental.

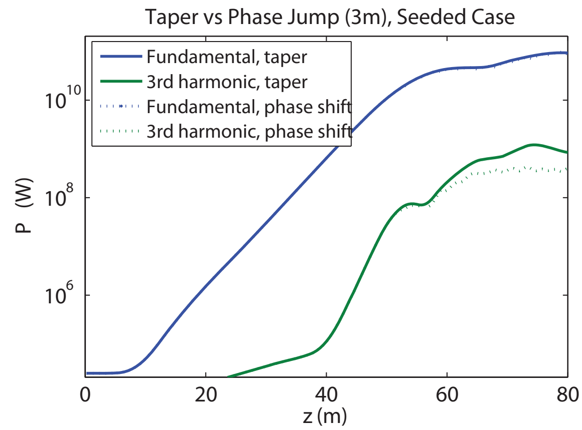

Figure 2: Comparison of taper and equivalent phase shifts every 3m for Seeded FEL, 1-D simulation.

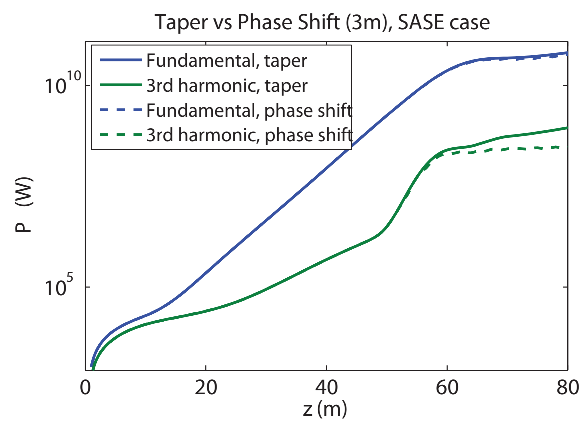

Figure 3: Comparison of taper and equivalent phase shifts every 3m for SASE FEL, 1-D simulation.

Finally, we compare phase shifts derived from the optimal linear taper to numerically optimized phase shifts (Eq. (7)), and the results from the phase scan. The phase shifts are all well matched (Fig. 4), confirming the equivalence of the four optimization methods.

3-D Simulations

We repeated the same study using the FEL code GINGER. To find an optimal undulator taper, $\Delta K(z)$, we used GINGER's self-taper algorithm [6]. With η defined above

$$\frac{d\eta}{dz} = -\frac{1}{\gamma_0}\frac{d\gamma_r}{dz} - \frac{eK[JJ]}{2\gamma_0^2 mc_2}E\sin(\theta+\Psi)$$

When the energy change is small (in the exponential regime), we can ignore the $\frac{d\gamma_r}{dz}$ term. However, at saturation, the energy loss is significant (of order ρ), and the resonant energy γ_r changes. Following [5], we define a synchronous phase Ψ_r from

$$-\frac{1}{\gamma_0}\frac{d\gamma_r}{dz} \equiv \frac{eK[JJ]}{2\gamma_0^2 mc_2}E\sin(\Psi_r).$$

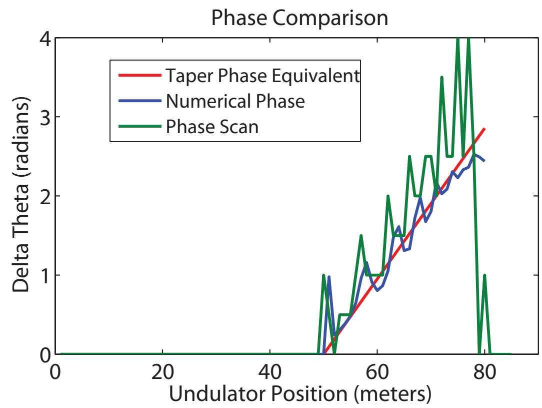

Figure 4: Phases determined by phase scan, numerical optimization and taper equivalent (Seeded case).

with Ψ_r determined by both $\frac{d\gamma_r}{dz}$ and K. Any particle with the synchronous phase will then define the center of the bucket, with particles nearby in phase space performing a synchrotron oscillation around the synchronous phase. (By definition, the energy η is constant.) An optimal phase Ψ_r is determined by optimizing bucket size and $d\gamma_r/dz$, which together set the energy transferred from electrons to radiation. We can then find a function $K(z)$ to maintain the phase Ψ_r as a constant throughout the saturation regime. Alternatively, we can manipulate the phase directly through periodic phase jumps, as described in Eq. (14) above. If the phase jumps are frequent enough so that $\Delta\theta < 1$ (or equivalently, $\Delta\Psi$), the system will effectively maintain the resonant condition without resorting to altering K.

Using GINGER's self-design taper function we created an optimal taper and then found the equivalent phase shifts (Fig. 5). We confirm the 1-D results, showing an equivalence between a taper and phase shift for an LCLS-like SASE FEL (Fig. 6, Table 1). The equivalence starts to fail at the very end of the undulator, when $\Delta\theta > 1$.

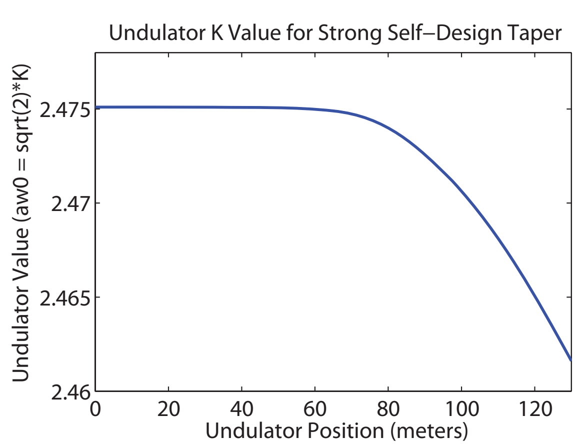

Figure 5: Taper for Seeded and SASE cases. Simulation using GINGER's 3D code.

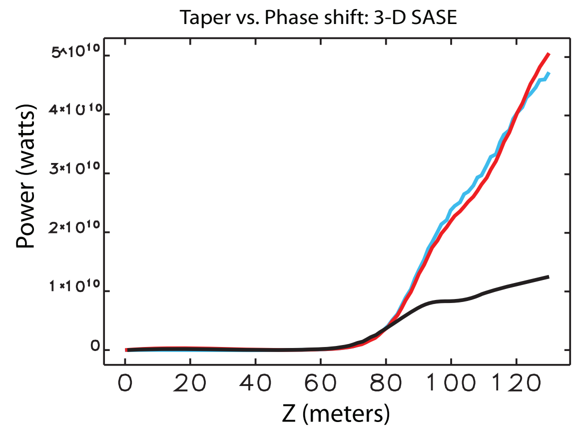

Figure 6: Power for normal (black), tapered (red) and phase-shifted (blue) LCLS-like SASE FEL, 3-D simulation. 12 Phase shifts between undulator sections (3.8 m).

CONCLUSION

We demonstrate the equivalence between phase shifts and undulator tapers. If the required phase shifts are small ($\Delta\theta < 1$), then the shifts can be implemented discretely and still mimic the effect of an undulator taper. In particular, we simulate phase shifts placed between undulator sections for an x-ray FEL and find the results are equivalent to those of an optimized taper. The phase shift method could be useful as a replacement or enhancement of undulator tapers when the use of tapers is constrained by technical issues. Even when the undulator parameter of each undulator section can be individually adjusted, placing phase shifters between the undulator sections provides independent control and fine tuning capability over the FEL power.

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