1D LINEAR INTENSITY SPIKING EVOLUTION IN A SINGLE SHOT OF A SASE FEL

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Abstract

It is shown that the simple system of equations used by Bonifacio et al. in 1994, leads to signals characterized by the usual spiky behaviour of the FEL radiation intensity, as well as to "coherent" signals in which the spiking is reduced to a small amplitude random fluctuation on a smooth and nearly constant average value. The two types of signals are obtained with different classes of initial conditions. In particular, coherent signals correspond to initial shot-noise configurations whose closely spaced spikes have widths much smaller than the cooperation length.

INTRODUCTION

The intensity spiking in the radiation of a high-gain free-electron laser starting from noise consists of a sequence of random spikes with wide top-to-bottom variations in the signal received at the end of the undulator [1-4]. The random nature of the spiking is a direct consequence of the random character of the small electromagnetic fields created on the electron beam as it enters the undulator field and the main aspects of the spiking are described accurately by the linear form of the one-dimensional FEL equations.

Starting from the Maxwell-Lorentz 1D equations, the disturbances in the radiation field $\delta A(z,t)$ and bunching factor $\delta b(z,t)$ are given by the following simple linear system in the limit of small radiation fields and when the radiation wave length λ is much smaller than the cooperation length $L_C = \lambda/4\pi\rho$ (ρ is the FEL parameter):

$$\left(\frac{\partial}{\partial t} + c(1-\beta_0)\frac{\partial}{\partial z}\right)\delta A(z,t) = \delta b(z,t),$$
$$\frac{\partial^2}{\partial t^2}\delta b(z,t) = \frac{i}{T_G^3}\delta A(z,t) \qquad (1)$$

These equations are written in the frame moving with the undisturbed beam velocity $c\beta_0$, $T_G = \lambda_w/4\pi c\rho$ is the gain time and λ_w the undulator period. The reason why we reconsider this very simple system of equations is that it may lead to final situations in which there is no spiking and the signal found at the end of 10-20 gain times T_G consists of one single and smooth bump, even when the length L_b of the beam is much longer than the cooperation length L_C.

ONE SINGLE SPIKE

We consider the particular solution of (1)

$$\delta A(\bar{z},\bar{t}) = \int_{-\infty}^{+\infty} dh\, \delta A_0(h) e^{i(h\bar{z}-\omega(h)\bar{t})}$$

where $\omega(h)$ is the root of the dispersion relation $\omega^3 - h\omega^2 - 1 = 0$ with positive imaginary part. The integral in the preceding equation is solved numerically for the case of a Gaussian spike at t=0 and different values of the parameter $\alpha = L_0/L_C$, where L_0 is the FWHM value of the initial Gaussian spike.

Fig.1 shows that the time evolution of the spike depends on the value of its width at t=0 as compared with L_C. In particular, when $L_0 \approx L_C$, the spike increases exponentially while its width increases only slightly. Instead, when $L_0 << L_C$, the width of the spike increases by several orders of magnitude starting from the very small value at t=0.

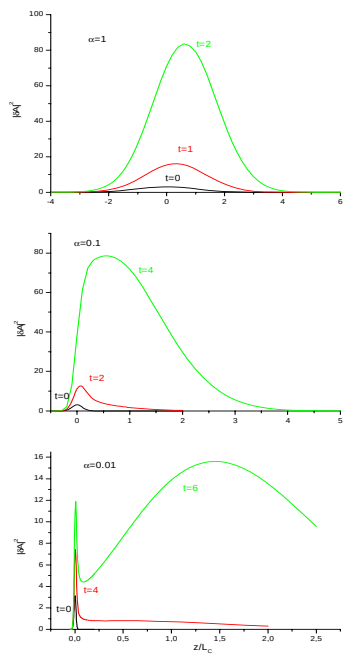


Fig.1(color) – Values of $|\delta A|^2$ *vs* z/L_C. Top figure $\alpha = 1$, curves (a, black),(b, red),(c, green) for $t/T_G = 0, 1, 2$, respectively; middle figure $\alpha = 0.1$, (a),(b),(c) for $t/T_G = 0, 2, 4$; bottom figure $\alpha = 0.01$, (a),(b),(c) for $t/T_G = 0, 4, 6$.

Fig.2 shows what happens for large values of t/T_G and for α = 1, 0.1, and 0.01, while Fig.3 gives the value of the radiation intensity at the peak of the spike vs t/T_G.

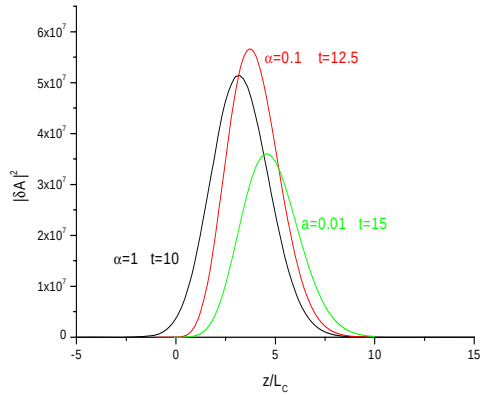


Fig.2 (color) – Values of $|\delta A|^2$ *vs* z/L_C. Curve (a, black) α = 1, t/T_G = 10; curve (b, red) α = 0.1, t/T_G = 12.5; curve (c, green) α = 0.01, t/T_G = 15.

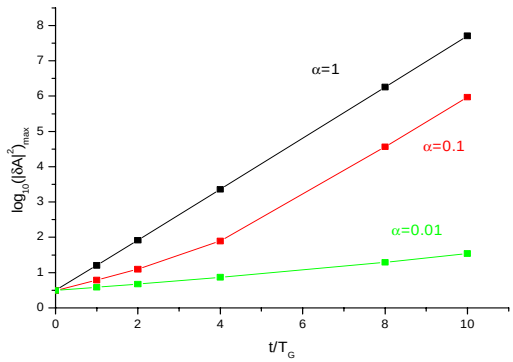


Fig.3 (color) – $|\delta A|^2$ at the maximum of the spike *vs* t/T_G for: (a, black) α = 1; (b, red) α = 0.1; (c, green) α = 0.01

RANDOM SEQUENCE OF SPIKES

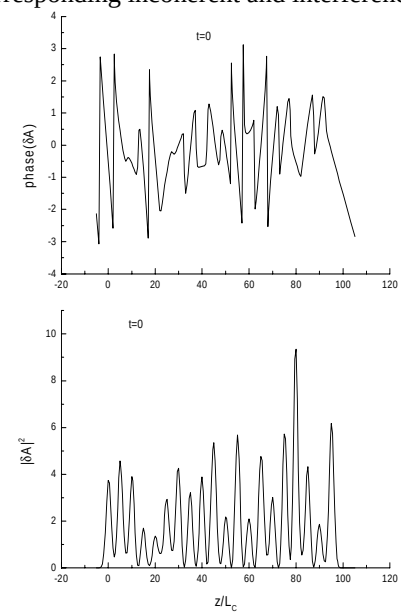
Infinitely long electron beam

In this case we assume $\delta A(z,0)$ as the following random sequence of Gaussian spikes

$$\delta A(\bar{z},0) = \sum_{j=1}^{N_{sp}} A_{0j} e^{-\mu_j(\bar{z}-\bar{z}_{0j})^2 + i\theta_j \bar{z}}$$

where N_{sp} is the number of spikes, the peak of each single spike being at $z=z_{0j}$ while the complex numbers A_{0j} and the real numbers μ_j, θ_j and z_{0j} are all random functions of the integer j defined in terms of their average values and r.m.s. deviations.

Fig.4 shows modulus and phase of the complex number δA at t=0 and $t=10T_G$ for an initial random sequence in which all spikes have widths comparable with L_C, the average distance D_{sp} between successive spikes being $5L_C$. At $t=10T_G$, $|\delta A|^2$ has the usual spiky form with wide top-to-bottom variations of the signal intensity. Fig.5 shows modulus and phase of δA at t=0 and 10 T_G, for a sequence of spikes whose widths are all much smaller than L_C. In this case the signal at t = 10 T_G has only a small amplitude fluctuation on top of a smooth and nearly constant average value. The number of "spikes" that characterise the shape of the signal intensity at this time follows approximately the usual rule, i.e., number of spikes ~ bunch length/ $2\pi L_C$ ~ $40/2\pi$ ~ 6-7 but the amplitude of the spikes as compared with the average value of the intensity is much reduced. The two parts (d) in figures 4 and 5 show the profile of the signal intensity at t=10 T_G for the two cases, together with the corresponding incoherent and interference terms.



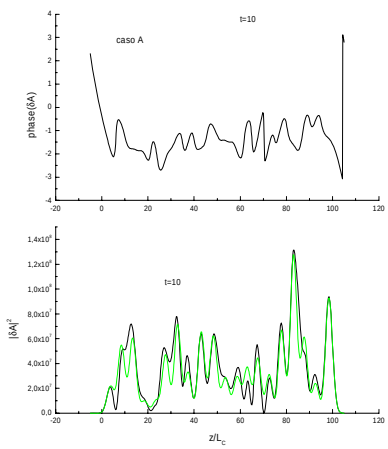


Fig.4 (color) – Phase and modulus squared of δA *vs* z/L_C at t=0 (first column, parts (a) and (b)) and $t=10T_G$ (second column, (c) and (d)). (d) shows the signal intensity (curve 1, black) and the incoherent part (curve 2, green). N_{sp}=20, $D_{sp}=5L_C$, α=1.

While $|\delta A|^2$ in Fig.4 is practically equal to its incoherent part indicating that the interference between different signals is always negligible, in Fig.5 the final signal intensity is the result of a strong process of constructive interference. If it seems appropriate to classify as "incoherent" the signal produced in the case of Fig.4, it seems likewise appropriate to consider as "coherent" the signal produced in the case of Fig.5.

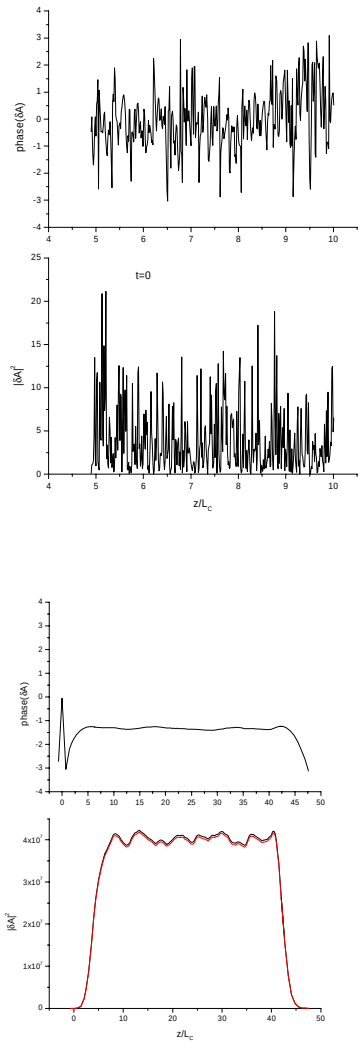


Fig.5 (color) – Same as Fig.6 with N_{sp}=1000, D_{sp}=0.04 L_C, α=0.007. Only part of the signal has been plotted at t=0. Part (d) shows the signal intensity (curve 1, black) and the interference part (curve 2, red).

As a third example, we consider the case in which the signal at t = 0 contains two characteristic lengths, the first of the order of the cooperation length L_C and the second much shorter than L_C. Fig.6 shows what happens in a case in which the initial signal is made up by 225 narrow spikes whose widths are uniformly of the order of 10^{-1} L_C (the average distance between narrow spikes is 0.2 L_C), together with 15 large spikes whose widths are uniformly of the order of L_C (average distance between large spikes 3 L_C). The whole signal at t = 0 covers about 45 cooperation distances. As one can see, spiking develops also in this case after a considerable "cleaning" of the signal, because at t = 10 T_G it is composed by 8 large spikes (number of spikes ~ $45/2\pi$ ~ 7-8) with the usual wide top-to-bottom oscillations. This is due to the long spikes at t = 0 increasing immediately with the largest growth rates.

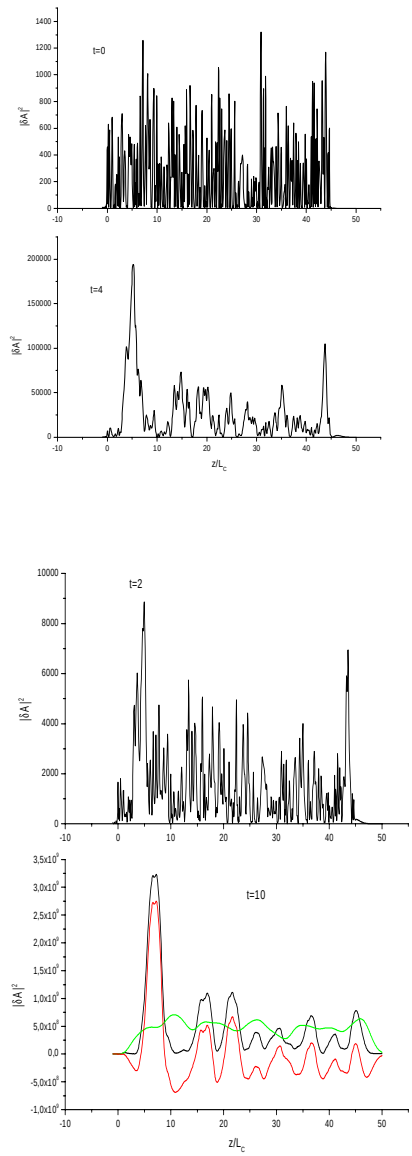


Fig.6 (color) - $|\delta A|^2$ *vs* z/L_C at t/T_G=0,2,4,10 (part (a),(b),(c),(d) respectively). (d) shows also the incoherent (curve 2, green) and interference (curve 3, red) parts of the total signal (curve 1, black) at t=10T_G.

Finite-length electron beam

System (1) has been integrated numerically also in the case of a beam of finite length. Fig.7 shows the behaviour of the signal intensity as obtained by solving the system with appropriate initial and boundary conditions. The initial condition for this case consists of 8000 Gaussian spikes with the average distance between successive spikes $D_{sp} = 0.01L_C$. The bunch length is $L_b = 80\ L_C$.

CONCLUSION

The main result of this paper is that according to system (1), the light emitted by an FEL is not always a chaotic light even if the initial signal is noisy [5]. In fact, this system leads to signals dominated by the spiking as well as to signals in which the spiking is reduced to a small amplitude fluctuation superimposed on a smooth pulse. Spiking in the usual sense is associated with initial conditions characterized by a scale-length L_0 of the order of the cooperation length L_C, while absence of spiking requires that the initial shot-noise condition consists of a random sequence of uniformly narrow spikes with widths of the order of $10^{-2}\ L_C$ or smaller, i.e., that $L_0 << L_C$. Since system (1) is based on the slowly-varying envelope approximation (SVEA), the initial conditions are consistent with its nature only if the characteristic length L_0 is longer than the wave-length λ of the emitted radiation. Absence of spiking as deduced on the basis of (1), therefore, requires that L_0 satisfies the inequality $\lambda < L_0 << L_C$. This means small values of the FEL parameter ρ, of the order of 10^{-4} or smaller which are typical of FELs based on static or optical undulators and operating in the regime of very high frequencies or even X-ray FELs.

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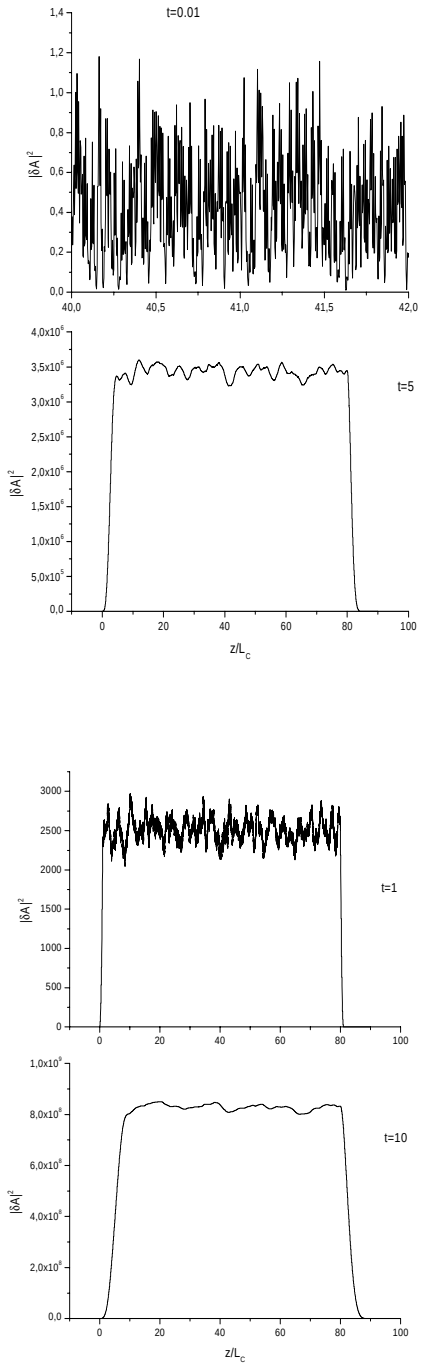

Fig.7 – Signal intensity *vs* z/L_C at times $t/T_G = 0.01$, 1, 5, 10 (parts (a), (b), (c) and (d)) in the case of a finite-length electron beam with $L_b = 80\ L_C$. $N_{sp} = 8000$, $D_{sp} = 0.01L_C$. Only part of the signal is given at t=0.01.