# LIMIT FOR HARMONIC CONVERSION IN A SINGLE CASCADE OF COHERENT HARMONIC GENERATION

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*Abstract*

Harmonic generation is a reliable method for producing coherent high-brightness photon pulses from relativistic electron bunches. The standard process leading to Coherent Harmonic Generation is initiated by a powerful seed laser. As a consequence, reaching short wavelengths generally requires a high order frequency conversion. For that reason some of the projects which are presently under development for coherent VUV and soft-Xray emission are based on a series of two or more consecutive "cascades". In these setups, the radiation produced into one stage is used as a seed in a following cascade. The required number of cascades depends on the maximum harmonic conversion which can be obtained in single stages. In this paper we study the mechanism underlying CHG, i.e. the bunching creation into the modulator, and we investigate the physical limits of the single-stage CHG.. The identification of the limiting parameters may allow the implementation of new methods for enhancing the conversion efficiency. One of these methods, which rely on a simple modification of the standard CHG scheme, has been recently proposed [3] and shown to be able to significantly improve the system performance. Results are confirmed by 3D numerical simulations using the FERMI electron beam parameters as initial conditions.

## INTRODUCTION

It is well known that the electron bunching, which is needed for effective High Gain Harmonic Generation (HGHG), rapidly decreases with the harmonic number, unless a strong energy modulation is produced in the modulator. However, the exponential light amplification that follows the first quadratic growth is strongly sensitive to the total energy spread,i.e., the quadratic combination of initial incoherent energy spread and of the coherent modulation.

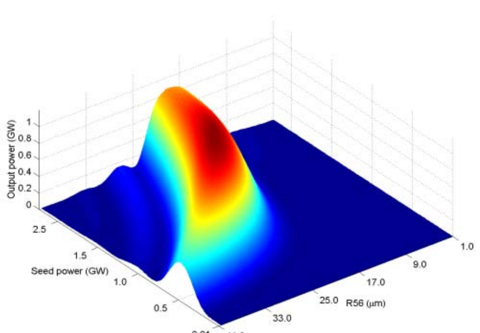


Figure 1: Example of Optimized output power for the HGHG FEL setup as a function of seed power and R56.

As a consequence, the optimization of a HGHG scheme relies on a compromise between the production of a sufficiently high bunching in the modulator (allowing a significant quadratic growth) and the need of keeping the total energy spread below the FEL parameter ρ. An example of the dependence of the produced FEL output power on the seed power and dispersive section strength (R56) is shown in Fig.1.

In this paper we study the possibility of optimizing the CHG setup once the electron-beam and the radiator parameters are given. For the presented work, electron beam parameters have been chosen to be those of the FERMI@Elettra project [1], see Table 1.

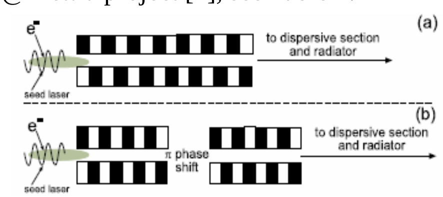


Figure 2: a) Schematic layout for the modulator in a standard HGHG setup. b) Proposed modified layout considering a phase shifter in between two modulator subsections.

The optimization is based on a modification of the standard modulator scheme, involving both the ondulator configuration and the power of the seed laser (Fig.2). The study has been done by perfomring numerical simulations with well established 3D FEL codes ginger and genesis.

Table 1: FERMI electron-beam parameters

| *Parameter* | *Value* | *Units* |
|---|---|---|
| Electron Beam Energy | 1.2 | GeV |
| Peak current | 1 | kA |
| Uncorrelated energy spread | 200 | keV |
| Norm. Transverse Emittance | 1.5 | mm-mrad |

## CASCADE LIMIT IN A STANDARD CHG SCHEME

According to theoretical predictions [2], the induced energy spread needed for efficient CHG increases with the harmonic number.

Figs.3 and .4 show that, while the optimum value of energy spread increases when reducing the emitted wavelength, the optimum bunching is approximately constant (order of 4% in our case). This result can be explained considering that in HGHG bunching is necessary to induce the quadratic emission occurring in the first part of the process, AND SIMPLY HAS TO BE LARGER THAN SHOT NOISE. However, the main contribution to the final output power comes from the successive exponential growth, which is rapidly depressed when the (total) energy spread increase.

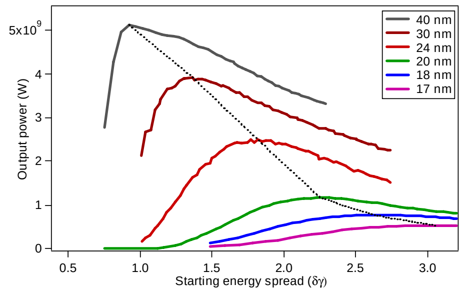


Figure 3: For each wavelength the maximum output power, obtained after optimization, is plotted as a function of the total energy spread (δγ) at the entrance of the radiator, which is correlated to the seed power .

Indeed, in first approximation, the required value for the bunching is independent from the harmonic we are interested in.

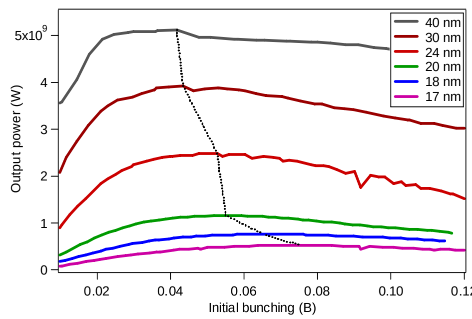


Figure 4: For each wavelength the maximum (optimized) output power is plotted as a function of the bunching produced after the dispersive section.

The fact that shorter wavelengths require larger energy spread implies that the final output power monotonically decreases (see inset of fig. 5) when going to shorter wavelengths.

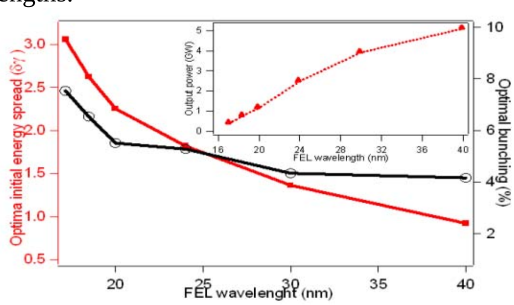


Figure 5: Form the data of figs. 3 and 4 we can see a strong dependence of the δγ necessary to optimize the output power as a function of the desired wavelength (red curve). On the contrary, the bunching (black curve) shows only a weak dependence. The inset shows the decrease of output power as a function of λ.

A limit in wavelength for HGHG to occur is provided by the harmonic at which it is no more possible to produce the required amount of bunching. In the considered setup (see Table 1) HGHG become inefficient for λ<20nm (N>12) and cannot be obtained for λ<17nm (fig.5).

We focus now on the 20nm case, that is the 12th harmonic of a frequency multiplied Ti:Sa tuned at 240nm. As shown in previous figures, a power of about 1 GW for the considered system can be obtained. Fig.6 shows the evolution of the radiation power (red) and bunching (black) along the radiator starting from a properly optimized initial condition. The considered radiator parameters are those used for the first FEL of the Fermi@Elettra project [1], with the only difference that the radiator period has been reduced to 55mm from the original 65mm in order to extend the tunability to shorter wavelengths.

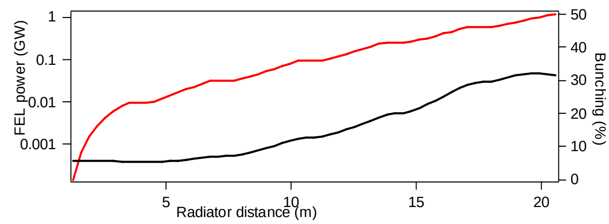


Figure 6: Results showing the quadratic (z<5) ad the exponential growth of the FEL power in the optimized standard setup for 20nm.

The mechanism underlying the FEL process during HGHG can be understood looking at the electron phase space at the entrance of the radiator, as shown in fig.7a. Figure 7b shows the localized density distribution of electrons, which is at the origin of initial bunching. Bunched electrons (blue in Fig.7a) are responsible for the coherent quadratic emission (z<5m in Fig.6). Remaining electrons interact with the produced radiation in the rest of radiator (z>5m) and are eventually responsible for the exponential growth.

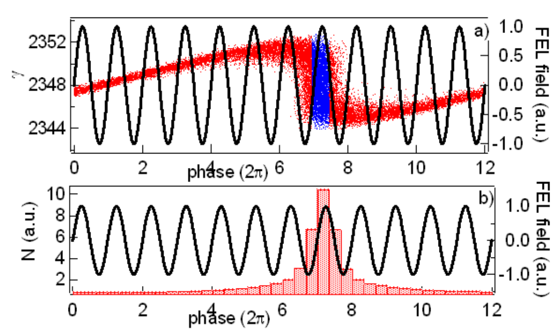


Figure 7: a) Phase-space representation of electrons at the entrance of the radiator in the optimized standard configuration for the 20nm case. b) Histogram of the electron density distribution. In both figures, the field evolution at 20 nm is reported as a reference.

Due to the energy modulation necessary for HGHG, "blue" electrons are characterized by a large energy spread and will only participate to the initial coherent emission. The rest of electrons (red) are characterized by a small "slice" energy spread, but their mean energy present quite large variations along the original phase (0-

24π referring to one period of the $12^{th}$ harmonic of the seed wavelength, i.e. 20nm). Such a detuning limits the exponential growth.

## NEW OPTIMIZED CONFIGUATIONS

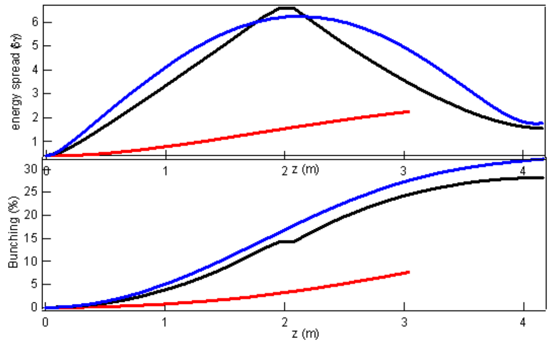


Figure 8: Evolution of the energy spread (a) and bunching (b) inside the modulator for the three considered setups: standard (red curve), double modulator (black curve) and detuned modulator (blue curve).

The modulator of a HGHG system can be optimized if one is able to produce the needed amount of bunching by inducing a smaller amount of coherent energy spread. For this purpose, in [3] we have proposed a simple scheme relying on a two-segment modulator separated by a de-phasing unit (Fig.1). In the first segment, electrons interact with the coherent radiation and coherent energy spread is produced. After the de-phasing unit, electrons and radiation are de-phased by about π, so that the interaction in the second modulator section is used to partially absorb the coherent energy spread.

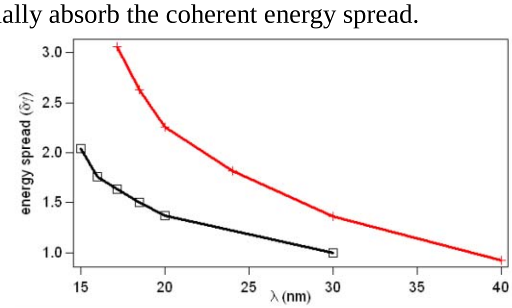


Figure 9: Comparison between the standard setup and the one based on the double modulator in terms of the energy spread necessary for creating a certain amount of bunching (4%) at the desired λ. Red curve refers to the standard configuration while the black one refers to the double modulator scheme.

We propose here a new setup which is able to produce the same effect on the electron beam using a single modulator section. The method exploits the natural de-phasing that occurs to electrons moving in a detuned undulator. As demonstrated by the data reported in Fig.8, by properly choosing the detuning value it is possible to obtain the same effect of the two modulator sections with π de-phasing in-between. The benefits of using one of the proposed methods with respect to the standard one are shown in Fig.8, where the minimum energy spread necessary to induce a bunching of 4% is reported as a function of the wavelength.

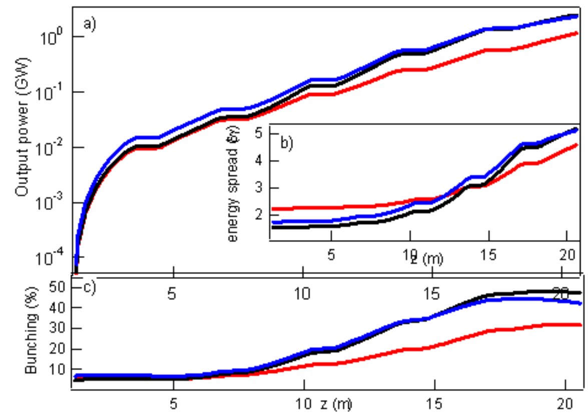


Figure 10: Example of the improved performance at 20nm obtained by using the proposed modified modulators. FEL power evolution along the radiator for the setup using the standard modulator (red curve), the double modulator (blue curve) and the detuned modulator (black curve).

Data show that for the same wavelength the new proposed methods allow to obtain the required bunching with about half of the energy spread with respect to the standard method (Fig.9). This implies that a significantly larger output power can be reached. This is shown in Fig.10 where the evolution along the radiator of the output power and bunching for the 20nm case is shown for the standard and two new proposed methods. A factor two can be gained with the new modulator schemes.

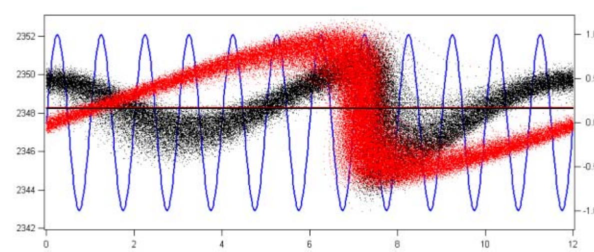

Figure 11: Phase space representation of the bunched electrons at the entrance of the radiator in the case of the standard modulator scheme (red dots) and the one using the 2 modulator sections with a π de-phasing unit (black dots).

Looking at Fig.10, one can appreciate the difference between the standard and new methods. While the bunched electrons (ϕ~7), responsible for coherent emission, are very similar for the two considered configurations, the unbunched electrons look different. Indeed, electrons that have been modulated by using the double modulator setup (black dots) have in general an energy which is closer to the nominal one.

## EXTENDING THE LIMIT

According to data shown in fig.9, using the proposed optimized modulator setups is possible to extend the limit for HGHG to shorter wavelengths. Indeed, data show that with the same induced energy spread using the new proposed schemes it is possible to obtain a bunching of 4% at a wavelength which is about half of the one that can be reached with the standard method.

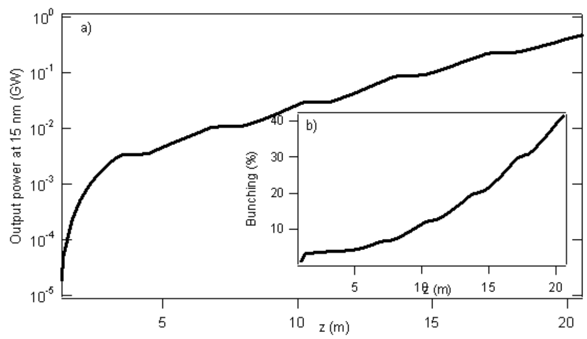


Figure 12: Power growth of the FEL into the radiator at the 16$^{th}$ harmonic (15nm) The use of the new proposed schemes allow to reach more than half GW, In the inset the evolution of the bunching is shown.

This is confirmed by data reported in Fig.12 showing the power evolution along the radiator using an electron bunch which has been passed through the two- modulator scheme for creating the bunching. Data shows that with the proposed modified methods it is possible to go close to saturation at 15 nm. It is important to note that better results in terms of extracted power can be obtained at shorter wavelengths by optimizing the radiator period..

## CONCLUSIONS

The limit for the harmonic conversion which can be obtained with the standard HGHG scheme in a realistic machine has been analyzed. Moreover two methods have been presented allowing to extend the limit of HGHG toward shortest wavelengths. These methods only require a modification of the undulator used in the modulator section and a more powerful seed laser.

## REFERENCES

[1] FERMI design technical report, in press.
[2] L. H. Yu, Phys. Rev. A **44**, 5178 (1991).
[3] E. Allaria, G. De Ninno, PRL **99,** 014801 (2007).