CONSERVATION LAWS IN QUASILINEAR THEORY OF RAMAN FREE-ELECTRON LASER

B. Maraghechi, Department of Physics, Amirkabir University of Technology, Tehran, Iran
A. Chakhmachi, Plasma Physics and Nuclear Fusion Research School,
Nuclear Science and Technology Research Institute, AEOI, Tehran, Iran
and Department of Physics, Amirkabir University of Technology, Tehran, Iran

Abstract

A quasilinear theory of the free-electron laser, in Raman regime, is presented to establish that conservation laws on number, energy, and momentum are upheld. A high density electron beam is assumed so that the space-charge potential is no longer negligible. A sufficiently broad band spectrum of waves is assumed so that saturation will be due to the quasilinear spread of the beam electrons. Otherwise, for the single mode excitation, saturation will be due to the electron trapping in the space-charge potential. It is shown that the quasilinear slow variation of the background distribution function is in the form of the diffusion equation in momentum space. An expression for the time evolution of the spectral energy density is derived. Conservation laws to the quasilinear order (second order) are derived and are proved to be satisfied. Results of the present investigation may be used to study the quasilinear saturation of a free-electron laser in the presence of the space-charge wave.

INTRODUCTION

Saturation and nonlinear evolution of free-electron lasers (FEL) are of considerable importance both experimentally and theoretically. The main reason is that saturation determines the efficiency of the device.

For sufficiently broad spectrum of unstable waves, saturation is due to quasilinear energy spread of the beam electrons. This problem has been studied only in the Compton regime, where the electrostatic potential of the space-charge wave is negligible due to low density of the electron beam [1-5]. On the other hand, when there is only a single exited mode, saturation of the amplification is considered to be caused by electrons trapped in the electrostatic field of the space-charge wave in the Raman regime or the pondermotive wave in the Compton regime [6-8].

The purpose of the present investigation is to derive conservation laws in the quasilinear analysis of an FEL in the presence of the space-charge field of electrons. The method of analysis and notations are similar to Ref. 1. The quasilinear slow variation of the background distribution function is in the form of the diffusion equation in momentum space. An expression for the time evolution of the spectral energy density is derived. Conservation laws, to the quasilinear order (second order), for particle, energy and momentum are derived and are proved to be satisfied.

PHYSICAL MODEL

We consider a collisionless, relativistic, electron beam with uniform cross section propagating in the z direction. The electron beam propagates through a constant amplitude helical wiggler magnetic field specified by

$$\mathbf{B}_0 = -B_w cos(k_0 z)\widehat{\mathbf{e}_x} - B_w sin(k_0 z)\widehat{\mathbf{e}_y}, \qquad (1)$$

The transverse electromagnetic and longitudinal electrostatic perturbed fields $\delta\mathbf{E}$ and $\delta\mathbf{B}$ are defined as

$$\delta\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{A} - \nabla\delta\varphi, \qquad \delta\mathbf{B} = \nabla\times\delta\mathbf{A}. \qquad (2)$$

The relativistic, nonlinear Vlasov equation for the electron beam distribution function $f_b(z,\mathbf{p},t)$ is given by

$$[\frac{\partial}{\partial t} + v_z\frac{\partial}{\partial z} - e(\delta\,\mathbf{E} + \frac{\mathbf{v}\times(\mathbf{B}_0+\delta\mathbf{B})}{c}).\frac{\partial}{\partial\mathbf{p}}]f_b(z,\mathbf{p},t) = 0. \qquad (3)$$

In the present analysis, we investigate the class of exact solution to Eq. (3) of the form

$$f_b(z,\mathbf{p},t) = n_0\delta(P_x)\delta(P_y)G(z,p_z,t), \qquad (4)$$

where $n_0 = const$, and P_x and P_y are the canonical momenta transverse to the beam propagation direction. Substituting the distribution function (4) into Eq. (3) and integrating the resting equation over p_x and p_y, gives

$$[\frac{\partial}{\partial t} + v_z\frac{\partial}{\partial z} - \frac{\partial}{\partial z}H(z,p_z,t)\frac{\partial}{\partial p_z}]G(z,p_z,t) = 0. \qquad (5)$$

In this equation, $H(z,p_z,t)$ is defined by

$$H(z,p_z,t) = \gamma_T mc^2 - e\delta\varphi(z,t)$$

$$= [m^2c^4 + c^2p_z^2 + e^2(A_x^0+\delta A_x)^2 + e^2(A_y^0+\delta A_y)^2]^{(1/2)}$$

$$-e\delta\varphi(z,t), \qquad (6)$$

which is the particle energy for $P_x = 0 = P_y$, and $\gamma_T mc^2$ is the kinetic energy. In the absence of perturbed fields, the energy is given by

$$\gamma mc^2 = (m^2c^4 + c^2p_z^2 + e^2B_w^2/k_0^2)^{1/2}. \qquad (7)$$

It is assumed that the distribution function $G(z,p_z,t)$, $H(z,p_z,t)$, and $\delta\varphi(z,t)$ are spatially periodic with periodicity length $2L$ so it is convenient to introduce the specially averaged distribution function $G_0(p_z,t)$ defined by

$$G_0(p_z,t) = \langle G\rangle = \frac{1}{2L}\int_{-L}^{L} dzG(z,p_z,t). \quad (8)$$

For small $\delta\mathbf{A}$ and $\delta\varphi$,

$$G(z,p_z,t) = G_0(p_z,t) + \delta G(z,p_z,t), \quad (9)$$

$$H(z,p_z,t) = H_0(p_z,t) + \delta H(z,p_z,t) = \gamma mc^2 + \delta H, \quad (10)$$

and therefore

$$\delta H(z,p_z,t) = (e^2/\gamma mc^2)(A_x^0\delta A_x + A_y^0\delta A_y) - e\delta\varphi(z,t). \quad (11)$$

Equation (11) shows that the space-charge wave constitute the electrostatic potential of the perturbed Hamiltonian or energy. By substituting $v_z = p_z/\gamma_T m = p_z c^2/(H + e\delta\varphi)$, Eq. (6) can be expressed as

$$\frac{\partial}{\partial t}G + p_z c^2\frac{\partial}{\partial z}(\frac{G}{H+e\delta\varphi}) - \frac{\partial}{\partial p_z}(G\frac{\partial}{\partial z}H) = 0,$$

which can be averaged over the spatial periodicity length 2L to obtain

$$\frac{\partial}{\partial t}G_0 = \frac{\partial}{\partial t}\langle G\rangle = \frac{\partial}{\partial p_z}\langle\delta G\frac{\partial}{\partial z}\delta H\rangle. \quad (12)$$

In the approximation where only linear wave-particle interactions are retained in the description, δG evolves according to [1]

$$\frac{\partial}{\partial t}\delta G + \frac{p_z}{\gamma m}\frac{\partial}{\partial z}\delta G - (\frac{\partial}{\partial z}\delta H)\frac{\partial}{\partial p_z}G_0 = 0. \quad (13)$$

Equation (13) will be recognized as the linearized Vlasov equation for perturbations about spatially uniform distribution function $G_0(p_z,t)$, which varies slowly with time according to Eq. (12).

TIME EVOLUTION OF DISTRIBUTION FUNCTION AND ENERGY

We introduce the Fourier series representations

$$G(z,p_z,t) = G_0(p_z,t) + {\sum_k}'\delta G_k(p_z,t)\exp(ikz), \quad (14)$$

$$H(z,p_z,t) = H_0(p_z,t) + {\sum_k}'\delta H_k(p_z,t)\exp(ikz), \quad (15)$$

$$\delta\psi(z,t) = \sum_k \delta\psi(k,t)\exp(ikz), \quad (16)$$

where $\delta\psi$ is δA_x, δA_y, or $\delta\varphi$ and $k = n\pi/L$ with n an integer and the summations run from $k=-\infty$ to $k=+\infty$. The prime on the summations denotes that the $k=0$ term is omitted. From Eqs. (11), and (14)-(16) it follows that (for $k \neq 0$)

$$\delta H_k = (\frac{e^2 B_w}{2\gamma mc^2 k_0})[\delta A_x(k+k_0) + i\delta A_y(k+k_0)$$
$$+\delta A_x(k-k_0) - i\delta A_y(k-k_0)] - e\delta\varphi(k). \quad (17)$$

The time dependence of perturbed quantities is assumed to be of the form $\exp(-i\int_0^t \Omega_k(t')dt')$ in circumstances where the time variation of $G_0(p_z,t)$ is sufficiently slow $\Omega_k = \omega_k + i\gamma_k = -\omega_{-k} + i\gamma_{-k}$. By use of $\bar{\gamma}mc^2 = const$ that denotes the characteristic mean energy of the unperturbed beam electrons we can write the perturbed amplitudes in terms of dimensionless amplitudes $\delta\Phi(k)$ and $\delta A^{\pm}_{k\pm k_0}$. In Fourier variables, Eq. (12) becomes

$$\frac{\partial}{\partial t}G_0 = -\frac{\partial}{\partial p_z}\sum_k ik\delta H_{-k}\delta G_k. \quad (18)$$

Solving Eq. (13) and neglecting free-streaming contributions to $\delta G_k(p_z,t)$, we obtain for the perturbed distribution function

$$\delta G_k(p_z,t) = -[(\frac{\bar{\gamma}eB_w}{2\gamma k_0})(\delta A^+_{k+k_0} + \delta A^-_{k-k_0}) - \bar{\gamma}mc^2\delta\Phi(k)]$$
$$\times(\frac{k\partial G_0/\partial p_z}{\Omega_k - kv_z})\exp(-i\int_0^t \Omega_k(t')dt'). \quad (19)$$

Substituting Eqs. (17) and (19) into Eq. (18) yields the quasilinear kinetic equation for $G_0(p_z,t)$:

$$\frac{\partial}{\partial t}G_0(p_z,t) = i\sum_k k^2\frac{\partial}{\partial p_z}(\frac{|\delta H_k|^2\partial G_0/\partial p_z}{\Omega_k - kv_z}). \quad (20)$$

Note that Eq. (20) has the form of a diffusion equation for $G_0(p_z,t)$ in momentum space.

Linear Dispersion Relation and Energy Equation

The complex oscillation frequency $\Omega_k(t)$ is obtained adiabatically in terms of $G_0(p_z,t)$ from the linear DR. For the present configuration the linear DR is derived in Ref. 9 and 10 as follows

$$c^2k^2 D_k^L(\Omega_k)D^T_{k-k_0}(\Omega_k)D^T_{k+k_0}(\Omega_k) =$$
$$(1/2)a_w^2[D^T_{k-k_0}(\Omega_k) + D^T_{k+k_0}(\Omega_k)]$$
$$\times\{[\chi_k^{(1)}]^2 - c^2k^2D_k^L(\Omega_k)[\chi_k^{(2)} + \alpha_3\omega_{pe}^2]\}, \quad (21)$$

where the dielectric functions D_k^L, $D^T_{k-k_0}$, $D^T_{k+k_0}$, and the effective susceptibility $\chi_k^{(n)}$ are defined by

$$D^T_{k\pm k_0}(\Omega_k) = \Omega_k^2 - c^2(k\pm k_0)^2 - \alpha_1\omega_{pe}^2, \quad (22)$$

$$\chi_k^{(n)}(\Omega_k) = \bar{\gamma}^{n+1}mc^2\omega_{pe}^2\int\frac{dp_z}{\gamma^n}\frac{k\partial G_0/\partial p_z}{\Omega_k - kv_z}. \quad (23)$$

$$D_k^L(\Omega_k) = 1 + \frac{\chi_k^{(0)}(\Omega_k)}{c^2k^2}, \quad (24)$$

Here $\omega_{pe}^2 = 4\pi n_0 e^2/\bar{\gamma}m$, $a_w = eB_w/\bar{\gamma}mc^2k_0$ and $\alpha_n = \bar{\gamma}^n \int \frac{dp_z}{\gamma^n} G_0$. We will now obtain the wave kinetic equation. The average energy density in the electromagnetic and electrostatic fields is given by

$$\frac{1}{8\pi}\sum_k [|\delta\mathbf{E}_k|^2 + |\delta\mathbf{B}_k|^2] =$$

$$(\frac{\bar{\gamma}mc^2}{2e})^2 \frac{1}{4\pi c^2}\sum_k \{|\delta A^+_{k+k_0}|^2[|\Omega_k|^2 + c^2(k+k_0)^2]$$

$$+|\delta A^-_{k-k_0}|^2[|\Omega_k|^2 + c^2(k-k_0)^2] + 2c^2k^2|\delta\Phi(k)|^2\}$$

$$\times exp(2\int_0^t \gamma_k(t')dt') = \sum_k \epsilon_k(t), \qquad (25)$$

where $\varepsilon_k(t)$ is the spectral energy density. From Eq. (25), it follows that $\epsilon_k(t)$ evolves according to

$$\frac{\partial}{\partial t}\epsilon_k(t) = 2\gamma_k(t)\epsilon_k(t), \qquad (26)$$

where the linear growth rate $\gamma_k(t)$ is determined adiabatically in terms of $G_0(p_z,t)$ from Eq. (21). Equations (20), (21), and (26) then form a closed quasilinear description of the system including the effects of linear wave-particle interactions.

CONSERVATION OF PARTICLE, MOMENTUM AND ENERGY

The fully nonlinear Vlasov-Maxwell equations possess three exact conservation relations. These are: average density,

$$\int_{-L}^{L}\frac{dz}{2L}\int d^3p f_b(z,\mathbf{p},t) = const, \qquad (27)$$

total average plasma kinetic energy density plus electromagnetic and electrostatic field energy density,

$$\int_{-L}^{L}\frac{dz}{2L}\{\int d^3p(\gamma_T - 1)mc^2 f_b(z,\mathbf{p},t)$$

$$+\frac{1}{8\pi}[(\delta\mathbf{E})^2 + (\mathbf{B}_0 + \delta\mathbf{B})^2]\} = const, \qquad (28)$$

and total average plasma momentum density plus electromagnetic field momentum density,

$$\int_{-L}^{L}\frac{dz}{2L}\{\int d^3p p_z f_b(z,\mathbf{p},t) + \frac{1}{4\pi c}(\delta\mathbf{E}\times\mathbf{B}_0$$

$$+\delta\mathbf{E}\times\delta\mathbf{B})_z\} = const. \qquad (29)$$

We now demonstrate that the conservation relations (27)-(29) are upheld by the quasilinear kinetic equations derived in Eq. (20). The distribution function f_b is taken to be of the form of Eq. (4). In Eqs. (27)-(29), we expand quantities such as $\gamma_T mc^2$ and retain up to second-order terms in perturbation amplitudes.

Particle conservation: Substituting Eq. (4) into Eq. (27), and making use of Eq. (9), we obtain

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty} dp_z n_0 G_0(p_z,t) = 0. \qquad (30)$$

Clearly this is true for $\partial G_0/\partial t$ given by Eq. (20) since Eq. (20) is in the form of a diffusion equation in momentum space and the integrand is an exact differential.

Energy conservation: To show energy conservation from quasilinear theory, the quantity $\gamma_T mc^2$ [Eq. (6)] is expanded to second order, by this we have

$$\langle\mathbf{KED}\rangle = \int_{-L}^{L}\frac{dz}{2L}\int_{-\infty}^{\infty} dp_z n_0((\gamma-1)mc^2 G_0$$

$$+\frac{e^2}{\gamma mc^2}[(A_x^0\delta A_x + A_y^0\delta A_y)\delta G - (\delta A_x^2 + \delta A_y^2)G_0]$$

$$-\frac{1}{2}\frac{e^4}{\gamma^3m^3c^6}(A_x^0\delta A_x + A_y^0\delta A_y)^2 G_0) \qquad (31)$$

In above equation $\langle\mathbf{KED}\rangle$ is the average plasma kinetic energy density. Then the quasilinear analog of the exact energy conservation relation in Eq. (28) can be expressed as

$$\frac{\partial}{\partial t}\langle\mathbf{KED}\rangle = -\frac{\partial}{\partial t}\langle\frac{1}{8\pi}[(\delta\mathbf{E})^2 + (\mathbf{B}_0 + \delta\mathbf{B})^2]\rangle, \qquad (32)$$

To verify Eq. (32), we proceed by taking the derivative of Eq. (31) with respect to time, and substituting Eq. (20) for $\partial G_0/\partial t$ into the first term on the right-hand side of Eq. (31). The perturbed distribution function δG appearing in second term is obtained from Eq. (19). Then by use of DR (21) we can eliminate the integrals over momentum in favor of dielectric functions $D^T_{k+k_0}$, $D^T_{k-k_0}$, and D^L_k. Relations between the perturbed amplitudes and dielectric functions are also used.

$$\frac{\partial}{\partial t}\langle\mathbf{KED}\rangle = (\frac{\bar{\gamma}mc^2}{2e})^2\frac{1}{2\pi c^2}$$

$$\sum_k\{(i\omega_k + \gamma_k)[D^T_{k+k_0}|\delta A^+_{k+k_0}|^2 + D^T_{k-k_0}|\delta A^-_{k-k_0}|^2]$$

$$+\gamma_k(\omega_{pe}^2\alpha_1)[|\delta A^+_{k+k_0}|^2 + |\delta A^-_{k-k_0}|^2]$$

$$+(i\omega_k - \gamma_k)[2c^2k^2|\delta\Phi(k)|^2]\}exp(2\int_0^t \gamma_k(t')dt'). \quad (33)$$

Substituting Eq. (22) into Eq. (33), and eliminating terms in the k summations, which are odd functions of k, yields

$$\frac{\partial}{\partial t}\langle\mathbf{KED}\rangle = -(\frac{\bar{\gamma}mc^2}{2e})^2\frac{1}{4\pi c^2}\frac{\partial}{\partial t}\sum_k exp(2\int_0^t\gamma_k(t')dt')$$

$$\times([|\delta A^+_{k+k_0}|^2(|\Omega_k(t)|^2 + c^2(k+k_0)^2)$$

$$+|\delta A^-_{k-k_0}|^2(|\Omega_k(t)|^2 + c^2(k-k_0)^2)] + [2c^2k^2|\delta\Phi(k)|^2]). \qquad (34)$$

The right-hand side of Eq. (32) can be evaluated

$$-\frac{\partial}{\partial t}\langle[\frac{1}{8\pi}(\delta\mathbf{E})^2 + (\delta\mathbf{B} + \mathbf{B}_0)^2]\rangle =$$

$$-(\frac{\bar{\gamma}mc^2}{2e})^2\frac{1}{4\pi c^2}\frac{\partial}{\partial t}\sum_k exp(2\int_0^t\gamma_k(t^{'})dt^{'})$$

$$([|\delta A^+_{k+k_0}|^2(|\Omega_k(t)|^2+c^2(k+k_0)^2)$$

$$+|\delta A^-_{k-k_0}|^2(|\Omega_k(t)|^2+c^2(k-k_0)^2)]+[2c^2k^2|\delta\Phi(k)|^2]). \quad (35)$$

Comparing Eqs. (35) and (34) completes the proof of Eq. (32).

Momentum conservation: We will now show that the average total axial momentum is conserved in the quasilinear theory. To obtain the quasilinear analog of Eq. (29), Eq. (4) is substituted into Eq. (29), the resulting equation is differentiated with respect to time, use is made of Eq. (12), and we integrate by part with respect to p_z:

$$-\frac{n_0e}{mc}\int_{-L}^{L}\frac{dz}{2L}[(\frac{e}{c}(A_x^0+\delta A_x)\frac{\partial}{\partial z}(A_x^0+\delta A_x)$$

$$+\frac{e}{c}(A_y^0+\delta A_y)\frac{\partial}{\partial z}(A_y^0+\delta A_y))\int_{-\infty}^{\infty}\frac{dp_z}{\gamma_T}G]$$

$$+\frac{\partial}{\partial t}\int_{-L}^{L}\frac{dz}{2L}\frac{1}{4\pi c}(\delta\mathbf{E}\times\mathbf{B}_0+\delta\mathbf{E}\times\delta\mathbf{B})_z=0. \quad (36)$$

Where $\int_{-L}^{L}\frac{dz}{2L}\frac{1}{4\pi c}(\delta\mathbf{E}\times\mathbf{B}_0)_z = 0$. We can expand the transverse current density $-e\int d^3p(v_x\hat{\mathbf{e}}_x+v_y\hat{\mathbf{e}}_y)f_b$ in powers of perturbed quantities. Thus, for the form of f_0 given in Eq. (4) and by expanding G and $1/\gamma_T$ to first order perturbed amplitudes the time rate of change of the average plasma momentum density is given by

$$\frac{\partial}{\partial t}\langle\mathbf{PMD}\rangle=\int_{-L}^{L}\frac{dz}{2L}\int_{-\infty}^{\infty}dp_z(-\frac{n_0e^2}{mc^2})$$

$$\times[\frac{1}{\gamma}A_x^0\delta G\frac{\partial}{\partial z}\delta A_x+\frac{1}{\gamma}A_y^0\delta G\frac{\partial}{\partial z}\delta A_y$$

$$-\frac{e^2}{\gamma^3m^2c^4}(A_x^0\delta A_x+A_y^0\delta A_y)G_0(A_x^0\frac{\partial}{\partial z}\delta A_x+A_y^0\frac{\partial}{\partial z}\delta A_y)] \quad (37)$$

correct to second order in the perturbation amplitude. The analogue equation to Eq. (29) in quasilinear theory is

$$\frac{\partial}{\partial t}\langle\mathbf{PMD}\rangle=-\frac{\partial}{\partial t}\langle\frac{1}{4\pi c}(\delta\mathbf{E}\times\delta\mathbf{B})_z\rangle. \quad (38)$$

Rearranging the terms in Eq. (37) and by use of the DR (21) and the relation between perturbed amplitudes and dielectric functions we have

$$\frac{\partial}{\partial t}\langle\mathbf{PMD}\rangle=-(\frac{\bar{\gamma}mc^2}{2e})^2\frac{1}{2\pi c^2}\frac{\partial}{\partial t}\sum_k exp(2\int_0^t\gamma_k(t^{'})dt^{'})$$

$$\times\omega_k[(k+k_0)|\delta A^+_{k+k_0}|^2+(k-k_0)|\delta A^-_{k-k_0}|^2]. \quad (39)$$

We can now evaluate the second term of Eq. (29) by use of Eqs. (3), (14)-(16)

$$-\frac{\partial}{\partial t}\langle\frac{1}{4\pi c}(\delta\mathbf{E}\times\delta\mathbf{B})_z\rangle=\frac{1}{4\pi c}$$

$$-\frac{\partial}{\partial t}\sum_k\frac{ik}{c}(\delta A_x^*(k,t)\frac{\partial}{\partial t}\delta A_x(k,t)+\delta A_y^*(k,t)\frac{\partial}{\partial t}\delta A_y(k,t))$$

$$=-(\frac{\bar{\gamma}mc^2}{2e})^2\frac{1}{2\pi c^2}\frac{\partial}{\partial t}\sum_k exp(2\int_0^t\gamma_k(t^{'})dt^{'})$$

$$\times\omega_k[(k+k_0)|\delta A^+_{k+k_0}|^2+(k-k_0)|\delta A^-_{k-k_0}|^2], \quad (40)$$

Then, Eq. (38) is directly proved by comparing Eqs. (39) and (40).

CONCLUSION

In the Raman regime, due to the high density of the electron beam, the space-charge potential is not negligible compared to the pondermotive potential. In this paper we have derived conservation laws in the quasilinear analysis of an FEL in the presence of the space-charge field of electrons.

The assumption of a broad spectrum of waves ensures that the saturation takes place through the quasilinear diffusion of electrons in the momentum space rather than by the particle trapping.

The presence of space-charge wave, that is resonant in the Raman regime, with the real parts of the frequency and wave number satisfying the linear DR, modifies the problem considerably compared to the Compton regime in which space-charge potential is negligible in comparison to the pondermotive potential.

The quasilinear kinetic theory used to derive three exact conservation relations, corresponding to conservation of (average) particle density, total energy, and total axial momentum. These results may be used to study the quasilinear saturation of a FEL in the raman regime.

ACKNOWLEDGMENT

One of the authors (B.M.) would like to thank the Center of Excellence in Computational Aerospace Engineering for financial support.

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