COMPARISION BETWEEN KINETIC AND FLUID DESCRIPTION OF PLASMA-LOADED FREE-ELECTRON LASER

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Abstract

In the kinetic treatment of the plasma-loaded FEL single particle equation of motion for both beam and plasma electrons in the radiation fields are used. Therefore, interaction terms between the wiggler and the space-charge wave, in the transverse velocity of electrons, which are important elements in the fluid model, are neglected. A dispersion relation of a plasma-loaded FEL with kinetic theory is used that takes into account the velocity spread of both beam and plasma electrons. In the present analysis a dispersion relation is obtained, by the fluid theory, with the interaction terms between the wiggler and the space-charge wave in the transverse velocity of electrons taken into account. Since these interaction terms are inherently missing in the kinetic theory the two dispersion relation are compared to find out about the importance of these terms. It was found that although the absence of these terms has considerable effects on the growth rate, the general kinetic dispersion relation may be used to study the temperature effects of a warm beam/plasma on the instability of a free-electron laser with a plasma background.

INTRODUCTION

The effects of background plasma on the interaction of electrons with the radiation have been of considerable interest in devices for the generation of coherent electromagnetic radiation. There are several investigations on the plasma loaded FEL[1-11]. A kinetic dispersion relation (DR) of a plasma loaded FEL is derived in Ref. 1 that takes into account the velocity spread of both beam and plasma electrons

In all of the above fluid methods[1-9] and in the kinetic treatments,[1,12,13] of the plasma- loaded FEL, single particle equation of motion for beam or plasma electrons in the radiation fields are considered. Therefore, interaction terms between the wiggler and the space-charge wave, in the transverse velocity of electrons, which are important elements in the fluid model, are neglected.

In the present work, the kinetic theory of Ref. 1 and its DR of a plasma-loaded FEL is considered. Since in this kinetic model single particle treatment of electrons in the radiation field is inherent in the theory a fluid model is used to find a DR that takes into account the interaction terms between the wiggler and the space-charge wave in the transverse velocity of electrons. The fluid DR is compared with the kinetic DR to find the importance of these interaction terms. It is found that the absence of these interaction terms in the kinetic treatment has considerable effects on the growth rate. However, characteristic behaviour of the kinetic DR is found to be satisfactory and, consequently, the general kinetic DR may be used to study the temperature effects of a warm beam/plasma on the instability of a plasma–loaded FEL.

KINETIC DESCRIPTION

Consider a relativistic electron beam propagating in the z direction through background plasma and a helical wiggler magnetic field. A general DR for a plasma-loaded FEL, using kinetic theory, is derived in Ref. 1 as

$$c^2k^2D^L(k,\omega)D^T(k-k_0,\omega)D^T(k+k_0,\omega)$$
$$=\frac{1}{2}a_w^2[D^T(k-k_0,\omega)+D^T(k+k_0,\omega)]\times$$
$$\times\{[\chi^{(1)}(k,\omega)]^2-c^2k^2D^L(k,\omega)[\alpha_3\omega_b^2+\chi^{(2)}(k,\omega)]\}\,. \qquad (1)$$

In order to compare the kinetic DR with that of the fluid model a relatively weak wiggler is assumed. In this case, the resonant space-charge wave $D^L(k,\omega)\approx 0$ couples to the resonant right circular wave $D^T(k-k_0,\omega)\approx 0$, which leaves the left circular wave nonresonant, i.e., $D^T(k+k_0,\omega)\neq 0$. Moreover, terms containing a_w^2 are also neglected in the coupled equations and the cold beam and plasma is assumed to obtain the following DR

$$(\omega^2-c^2k_r^2-\frac{\omega_b^2}{\hat{\gamma}_b}-\omega_p^2)(1-\frac{\omega_b^2}{\hat{\gamma}_b\gamma_z^2[\omega-kV_\parallel]^2}-\frac{\omega_p^2}{\omega^2})$$
$$=\frac{1}{2}a_w^2\frac{\omega_b^4}{\hat{\gamma}_b^4}\frac{(\omega\beta_\parallel-ck)^2}{(\omega-kV_\parallel)^4}\,, \qquad (2)$$

where, $k_r=k-k_0$ is the radiation wave number and all other quantities are defined in Sec. III and in Ref. 1. Kinetic model DR (2) has been solved numerically to find the imaginary part of the frequency. Figure 1 shows the variation of growth rate with radiation wave number for $B_w=1$ kG, $\hat{\gamma}_b=4$, and $\hat{n}_b=1\times10^{13}$ cm^{-3}. Curves 1, 2, 3, 4, and 5 corresponding to the density of background plasma at 0.8, 1.25, 1.75, 2, and 2.3, respectively (in units of 10^{13} cm^{-3}).

FLUID DESCRIPTION

We now consider the fluid theory description of a relativistic and cold electron beam that passes through a background plasma and a static helical wiggler magnetic

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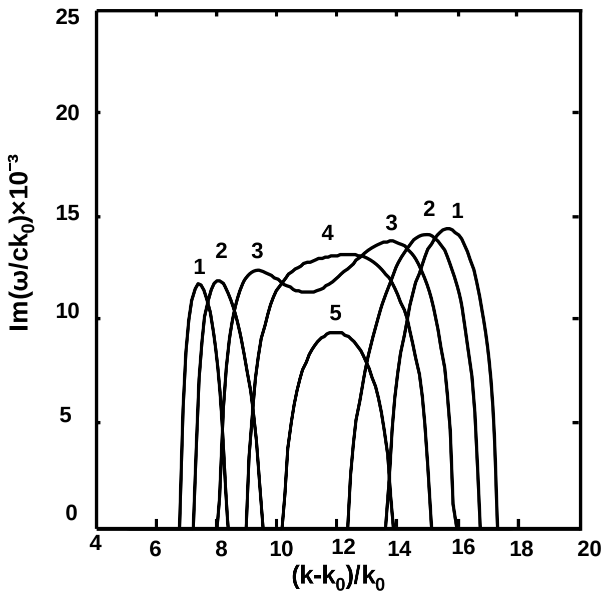

Fig. 1. Kinetic-model growth rate verses radiation wave number

field $\mathbf{B}_w$. The unperturbed state consists of

$$\mathbf{B}_w = B_w(\hat{\mathbf{e}}_x + i\hat{\mathbf{e}}_y)\exp(ik_0 z) + c.c., \quad (3)$$

$$\mathbf{v}_w = v_w(\hat{\mathbf{e}}_x + i\hat{\mathbf{e}}_y)\exp(ik_0 z) + c.c., \quad (4)$$

with equilibrium velocity given by $\mathbf{v}_w + \mathbf{v}_\parallel$, where $v_w = -eB_w / \hat{\gamma}_b mck_0$ is the transverse velocity and $\hat{\gamma}_b = [1-(v_w^2+v_z^2)/c^2]^{-1/2}$ is the total beam electron energy.

The perturbation quantities associated with the right circularly polarized electromagnetic wave (radiation) and the space-charge wave are

$$\delta\mathbf{E},\, \delta\mathbf{B},\, \delta\mathbf{A} \sim (\hat{\mathbf{e}}_x - i\hat{\mathbf{e}}_y)\exp[i(k_r z - \omega t)], \quad (5)$$

$$\delta\mathbf{E}_z,\, \delta\mathbf{v}_{bz},\, \delta\mathbf{v}_{pz},\, \delta n_b,\, \delta n_p \sim \exp[i(kz - \omega t)], \quad (6)$$

where $k = k_r + k_0$. In Eq. (5) subscripts minus, like δA_-, is removed in transverse field quantities for brevity. The relativistic momentum equation in the lab frame, for the electron beam,

$$m\gamma_b \frac{d\mathbf{v}}{dt} + m\mathbf{v}\frac{d\gamma_b}{dt} = -e\mathbf{E} - \frac{e}{c}(\mathbf{v}\times\mathbf{B}), \quad (7)$$

with

$$\frac{d}{dt}\gamma_b = -\frac{e}{mc^2}(\mathbf{v}.\mathbf{E}), \quad (8)$$

can be linearized using the linearized relativistic factor

$$\gamma_b = \hat{\gamma}_b + \frac{\hat{\gamma}_b^3}{c^2}(\mathbf{V}_\parallel.\delta\mathbf{v}_{bz} + \mathbf{v}_w.\delta\mathbf{v}_{b\perp}). \quad (9)$$

The transverse component of the momentum equation for the electron beam and plasma will be found as follows

$$\hat{\gamma}_b m(\frac{\partial}{\partial t} + V_\parallel\frac{\partial}{\partial z})\delta\mathbf{v}_{b\perp} + \hat{\gamma}_b m\delta v_{bz}\frac{\partial}{\partial z}\mathbf{v}_w^*$$
$$+ m\hat{\gamma}_b^3\frac{V_\parallel}{c^2}(\mathbf{V}_\parallel.\delta\mathbf{v}_{bz} + \delta\mathbf{v}_{b\perp}.\mathbf{v}_w)\frac{\partial}{\partial z}\mathbf{v}_w^* - \frac{e}{c^2}\mathbf{v}_w^*(\mathbf{v}_w.\delta\mathbf{E}_r)$$
$$= \frac{e}{c}(\frac{\partial}{\partial t}\delta\mathbf{A} - \mathbf{V}_\parallel\times\delta\mathbf{B} - \delta\mathbf{v}_{bz}\times\mathbf{B}_w^*), \quad (10)$$

$$m\frac{\partial}{\partial t}\delta\mathbf{v}_{p\perp} = \frac{e}{c}(\frac{\partial}{\partial t}\delta\mathbf{A} - \delta\mathbf{v}_{pz}\times\mathbf{B}_w^*). \quad (11)$$

Neglecting the terms proportional to a_w^2, Eqs. (10) and (11) will yield

$$\delta v_{b\perp} = \frac{e}{\hat{\gamma}_b mc}\delta A - \frac{\hat{\gamma}_b\beta_{ze}^2 ck_0 a_w^*}{\sqrt{2}(\omega - k_r V_\parallel)}\delta v_{bz}, \quad (12)$$

$$\delta v_{p\perp} = \frac{e}{mc}\delta A + \frac{ck_0 a_w^*}{\sqrt{2}\omega}\delta v_{pz}, \quad (13)$$

where $a_w = eB_w / mc^2 k_0$.

The wave equation for the radiation field is

$$(\omega^2 - c^2 k_r^2)\ \delta\mathbf{A} = 4\pi c\delta\mathbf{J}_\perp, \quad (14)$$

where $\delta\mathbf{J}_\perp = e\hat{n}_b\delta\mathbf{v}_{b\perp} + e\hat{n}_p\delta\mathbf{v}_{p\perp} + e\delta n_b\mathbf{v}_w$, δn_b is the electron beam density, and $\mathbf{v}_w$, $\delta\mathbf{v}_{b\perp}$ are transverse velocities of the electron beam due to the wiggler and radiation field, respectively. The longitudinal electrostatic field, i.e., space-charge wave, is produced by the perturbed electron density of the beam and background plasma and is given by

$$ik\delta E_z = -4\pi e(\delta n_b + \delta n_p). \quad (15)$$

The longitudinal component of the momentum equation for the electron beam and plasma will be found as follows

$$i\hat{\gamma}_b m(\omega - kV_\parallel)\delta v_{bz} = \frac{e}{\gamma_z^2}\delta E_z + \frac{e}{c}(\mathbf{v}_w\times\delta\mathbf{B}).\hat{\mathbf{e}}_z$$
$$+ \frac{e}{c}(\delta\mathbf{v}_{b\perp}\times\mathbf{B}_w).\hat{\mathbf{e}}_z - \frac{e}{c^2}V_\parallel(\mathbf{v}_w.\delta\mathbf{E}), \quad (16)$$

$$im\omega\delta v_{pz} = e\delta E_z + \frac{e}{c}(\delta\mathbf{v}_{p\perp}\times\mathbf{B}_w).\hat{\mathbf{e}}_z. \quad (17)$$

Substituting $\delta v_{b\perp}$ and $\delta v_{p\perp}$ from Eqs. (12) and (13) and $\mathbf{v}_w$ from Eq. (4) into Eqs. (16) and (17) we have

$$\hat{\gamma}_b m(\omega - kV_\parallel)\delta v_{bz}$$
$$= -ie\gamma_z^2\delta E_z + (ea_w\hat{\gamma}_b^{-1}/\sqrt{2})(k - V_\parallel c^{-2}\omega)\delta A, \quad (18)$$

$$m\omega\delta v_{Pz} = -ie\delta E_z + (ek_0 a_w/\sqrt{2})\delta A. \quad (19)$$

Using the continuity equation, the perturbed beam and plasma densities are given by

$$\delta n_b = \frac{k}{(\omega - kV_\parallel)}(\hat{n}_b\delta v_{bz} + V_\parallel\delta n_b), \quad (20)$$

$$\delta n_p = \frac{k}{\omega}\hat{n}_p \delta v_{pz} \, . \qquad (21)$$

With substitution of δv_{bz} and δv_{pz} from Eqs. (18) and (19) into Eqs. (20) and (21) δn_b and δn_p are obtained to be inserted in Eq. (15). This will give

$$(1-\frac{\omega_b^2}{\hat{\gamma}_b \gamma_z^2 (\omega - kV_\parallel)^2} - \frac{\omega_p^2}{\omega^2})\delta E_z$$

$$= i\frac{a_w}{\sqrt{2}}[\frac{\omega_b^2}{\hat{\gamma}_b^2(\omega-kV_\parallel)^2}(k-\beta_\parallel c^{-1}\omega) + \frac{\omega_p^2}{\omega^2}k_0]\delta A \, . \qquad (22)$$

Similarly, substitution of $\delta v_{b\perp}$, $\delta v_{p\perp}$ from Eqs. (12) and (13), v_w from Eq. (4), and δn_p from Eq. (21) into Eq. (14) will give

$$(\omega^2 - c^2k_r^2 - \frac{\omega_b^2}{\hat{\gamma}_b} - \omega_p^2)\delta A = i\frac{c^2k_0 a_w}{\sqrt{2}}(\frac{\beta_\parallel^2}{\gamma_z^2(\omega - k_r V_\parallel)}\frac{\omega_b^2}{(\omega - kV_\parallel)}$$

$$-\frac{\omega_b^2}{\gamma_z^2 \hat{\gamma}_b^2 (\omega - kV_\parallel)^2}\frac{k}{k_0} - \frac{\omega_p^2}{\omega^2})\delta E_z \, , \qquad (23)$$

where terms proportional to a_w^2 are neglected. Equations (22) and (23) will easily yield the DR as follows

$$(\omega^2 - c^2k_r^2 - \frac{\omega_b^2}{\hat{\gamma}_b} - \omega_p^2)(1 - \frac{\omega_b^2}{\hat{\gamma}_b \gamma_z^2 (\omega - kV_\parallel)^2} - \frac{\omega_p^2}{\omega^2}) =$$

$$\frac{1}{2}c^2a_w^2(\frac{\omega_b^2 k}{\gamma_z^2 \hat{\gamma}_b^2 (\omega - kV_\parallel)^2} + \frac{\omega_p^2}{\omega^2}k_0 - \frac{\beta_\parallel^2}{\gamma_z^2(\omega - k_r V_\parallel)}\frac{\omega_b^2 k_0}{(\omega - kV_\parallel)})$$

$$\times[\frac{\omega_b^2}{\hat{\gamma}_b^2}\frac{(k - V_\parallel c^{-2}\omega)}{(\omega - kV_\parallel)^2} + \frac{\omega_p^2}{\omega^2}k_0] \, . \qquad (24)$$

Equation (24) is the fluid-model DR to second order in wiggler amplitude.

In the fluid DR (24) transverse velocity components of radiation, given by Eqs. (12) and (13), contain interaction terms between the wiggler and space-charge wave, which are given by the second terms in the right-hand sides (RHS) of Eqs. (12) and (13). In contrast, these interaction terms are absent in the derivation of the kinetic DR (2). In order to asses the importance of these interaction terms the fluid DR is derived with the second terms in the RHS of Eqs. (12) and (13) removed, which will yield

$$(\omega^2 - c^2k_r^2 - \frac{\omega_b^2}{\hat{\gamma}_b} - \omega_p^2)(1 - \frac{\omega_b^2}{\gamma_z^2 \hat{\gamma}_b (\omega - kV_\parallel)^2} - \frac{\omega_p^2}{\omega^2}) =$$

$$\frac{1}{2}\frac{c^2a_w^2\omega_b^2 k}{\gamma_z^2 \hat{\gamma}_b^2(\omega - kV_\parallel)^2}[\frac{\omega_b^2}{\hat{\gamma}_b^2}\frac{(k - V_\parallel c^{-2}\omega)}{(\omega - kV_\parallel)^2} + \frac{\omega_p^2}{\omega^2}k_0] \, . \qquad (25)$$

Equation (25) is the fluid model DR to second order in wiggler amplitude and with the interaction terms in the transverse velocities of radiation, i.e., Eqs. (12) and (13), neglected. This fluid-model DR is equivalent to the

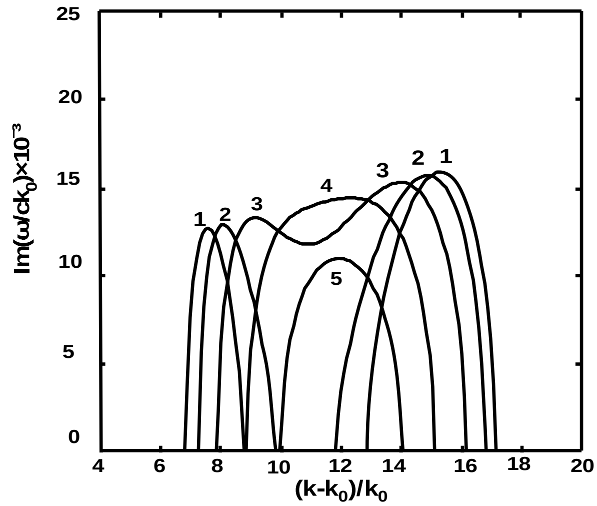

Fig. 2. Fluid-model growth rate verses radiation wave number with the interaction terms in the transverse velocities of radiation neglected.

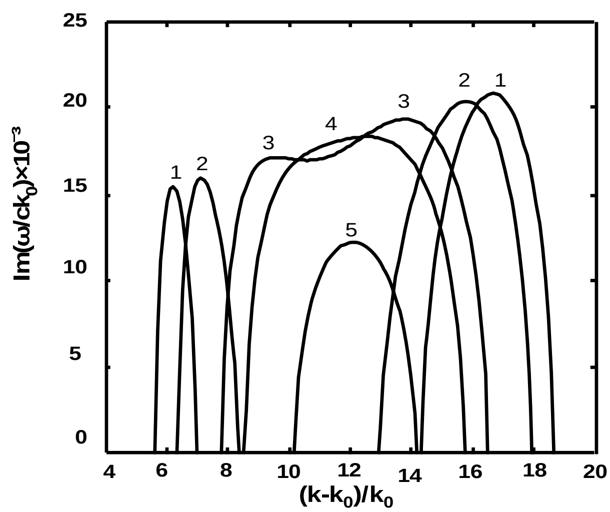

Fig. 3. Fluid-model growth rate verses radiation wave number with the interaction terms in the transverse velocities of radiation retained.

kinetic DR (2) and, therefore, they can be compared. Figure 2 shows the growth rate found from fluid DR (28) with the same parameters as in Fig. 1. Since interaction terms in the transverse velocities of radiation are absent in both Figs. 1 and 2 they compare quite well with each other. The highest value for the kinetic growth rate at around 0.0144, in Fig.1, is about 10% below that of the fluid-model growth rate at around 0.016, in Fig. 2. Moreover, location of the left (right) peak, for curve 1, in Fig. 1 at $(k-k_0)/k_0 \approx 7.2$ (≈ 15.5) differs by only about 4% (1.3%) from the corresponding peak in Fig. 2 at $(k-k_0)/k_0 \approx 7.5$ (≈ 15.3). The critical density at around 2.41×10^{13} cm^{-3} is about 3% higher than in Fig.1. The widths of unstable spectrums in the two figures are also in satisfactory agreement. Therefore, due to the absence of interaction terms in the transverse velocities of radiation in the kinetic DR (2) as well as in the fluid DR (24) the two DRs compare quite well both quantitatively and characteristically.

Equation (24) is the fluid-model DR to second order in wiggler amplitude and with the interaction terms in the transverse velocities of radiation, i.e., Eqs. (12) and (13), retained. The fluid-model growth rate, found from this DR, is illustrated in Fig. 3. Due to the absence of interaction terms in the transverse velocities of radiation, in the kinetic-model growth rate in Fig.1, its peak value at around 0.0144 is about 31% lower than that of the fluid growth rate in Fig. 3 at around 0.021. This shows that the absence of these interaction terms in the kinetic DR has profound effects on its growth rate. Characteristics of the two figures can also be compared. Location of the left (right) peak, for curve 1, in Fig. 1 at $(k-k_0)/k_0 \approx 7.2$ (≈ 15.5) differs by about 18% (7%) from the corresponding peak in Fig. 3 at $(k-k_0)/k_0 \approx 6.1$ (≈ 16.7). The critical density at around 2.52×10^{13} cm^{-3} is about 7% higher than in Fig. 1. Therefore, characteristic behavior of the kinetic DR compares relatively better than its quantitative behavior with the fluid DR. Consequently, the general kinetic DR (1) may be used to study the temperature effects of a warm beam/plasma on the instability of a plasma-loaded FEL.

CONCLUSION

In the derivation of the kinetic DR, in Eq. 1, the transverse velocities of electrons in the radiation fields are found from the conservation of the transverse component of canonical momenta, which themselves are found from the single particle equations of motion. Therefore, no interaction with the wiggler field is present. On the other hand, in the fluid-model DR (24) the transverse velocity components of radiation, found from the fluid momentum equations and given by Eqs. (12) and (13), contain interaction terms between the wiggler and space-charge wave. To first order in a_w, these are the second terms in the RHS of Eqs. (12) and (13), which are missing in the kinetic treatment.

In order to study the importance of these interaction terms the kinetic-model DR has been compared with the fluid-model DR. For this purpose a weak coupling regime, i.e., small wiggler amplitude, is assumed under which the left circularly polarized electromagnetic wave is nonresonant, i.e., $D^T(k+k_0,\omega)\neq 0$, and terms containing a_w^2 in the coupled equations are neglected. It was found that the resulting kinetic DR (2) compares fairly well with the corresponding fluid DR (25), which does not contain the interaction terms in the transverse velocities of radiation. However, the kinetic DR (2) does not agree quantitatively with the fluid DR (24), which does contain the interaction terms. Nevertheless, characteristic behavior of the two DRs are of better agreement so that the general kinetic DR (1) may be used to study the temperature effect of a warm beam/plasma on the growth rate of a plasma-loaded FEL. This DR is full and contains both the Compton regime DR and the Raman regime DR. Therefore, a complete numerical investigation of the kinetic DR (1) should include a warm beam/plasma in both Compton and Raman regime.

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