VOLC: VOLUME FREE ELECTRON LASER SIMULATION CODE

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Abstract

First lasing of Volume Free Electron Laser (VFEL) in mm wavelength range was obtained recently [1]. Multi-wave volume distributed feedback (VDFB) where electromagnetic waves and electron beam spread angularly one to other in a spatially-periodic structure (resonator) is the VFEL distinctive feature [2]. Mathematical model and numerical methods for VFEL nonlinear stage simulation were proposed [3] and implemented in computer code VOLC that allows to simulate different geometries of two- and three-wave VFEL in amplifier and oscillator regimes. Electron beam is modelled by averaging over initial phases of electrons. VOLC dimensionality is 2D (one spatial coordinate and one phase space coordinate) plus time. A description of VOLC possibilities and representative numerical results are presented.

INTRODUCTION

Principles and theoretical foundations of VFEL operation based on VDFB [4] put the beginning of experimental developing of new type of electronic generators. Creation of VFEL solves the problem of current threshold reduction, frequency tuning in a wide range, miniaturization of resonators. All these problems present some difficulties for widely used schemes of free electron lasers (FEL), backward wave tubes (BWT), travelling wave tube (TWT) and other electronic devices. As a rule, these high efficient devices have optimally specified parameters (electron beam, waveguides, resonators and undulators periods etc.). Changing of one of these parameters to tune frequency leads to abrupt reduction of efficiency of the system. VFEL design features allow to move and rotate diffracting gratings, change the distance between gratings and the gap between gratings and electron beam as well as orientation of grating grooves with respect to electron beam velocity. These aspects provide possibility to tune conditions of diffraction. VFEL threshold current for an electron beam passing through a spatially-periodic structure in degeneration points decreases essentially in comparison with single-wave systems [4]. This is valid for all wavelength ranges regardless the spontaneous radiation mechanism and as a consequence this means the possibility to reduce significantly system sizes.

In VFEL operation the linear stage investigated analytically [5]-[7] quickly changes into the nonlinear one where the most part of the electron beam kinetic energy is transformed into electromagnetic radiation. Nonlinear regime of VFEL operation can be investigated only using methods of mathematical modelling. Experiments on VFEL go on [8]–[9] and need optimal geometry determination and result processing.

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MATHEMATICAL FORMULATION OF VFEL PHYSICAL MODEL

The scheme of VFEL resonator of the experimental installation [1] formed by two diffraction gratings with different periods and two smooth side walls the same as the scheme of the volume resonator (so-called ”grid” volume resonator) built from the metallic threads inside a rectangular waveguide of the installation [8]–[9] can be considered as the following scheme of VFEL. Here an electron beam with electron velocity u passes through a spatially periodic structure. Under diffraction conditions some strong coupled waves can be excited in the system. If simultaneously electrons of the beam are under synchronism condition, they emit electromagnetic radiation in directions depending on diffraction conditions in oscillator regime. In [10]–[12] different two-wave and three-wave schemes of VFEL were considered in amplifier and oscillator regimes. System of equations for all cases of VFEL is obtained from Maxwell equations in the slowly-varying envelope approximation. For two-wave VFEL it has the following form:

$$\begin{aligned} &\frac{\partial E}{\partial t} + \gamma_0 c \frac{\partial E}{\partial z} + 0.5 i\omega l E - 0.5 i\omega\chi_\tau E_\tau = \\ &= 2\pi j \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} (\exp(-i\Theta(t,z,p)) + \\ &+ \exp(-i\Theta(t,z,-p)))dp, \\ &\frac{\partial E_\tau}{\partial t} + \gamma_1 c \frac{\partial E_\tau}{\partial z} + 0.5 i\omega\chi_{-\tau} E - 0.5 i\omega l_1 E_\tau = 0. \end{aligned} \quad (1)$$

Here $E(t,z)$ and $E_\tau(t,z)$ are amplitudes of transmitted and diffracted waves with wave vectors $\mathbf{k}$ and $\mathbf{k}_\tau$ respectively. $l_0 = \left(k^2c^2 - \omega^2\varepsilon_0\right)/\omega^2$ and $l_1 = \left(k_\tau^2c^2 - \omega^2\varepsilon_0\right)/\omega^2$ are system parameters. $l = l_0 + \delta$. δ is a detuning from synchronism condition. $\gamma_0,\ \gamma_1$ are VDFB cosines. $\beta = \gamma_0/\gamma_1$ is an asymmetry factor. $\Phi = \sqrt{l_0 + \chi_0 - 1/(u/c\gamma)^2}$. γ is the Lorenz factor. $\chi_0,\ \chi_{\pm\tau}$ are Fourier components of the dielectric susceptibility of the target.

System (1) must be supplemented with proper initial and boundary conditions which can contain conditions for external reflectors. Equations for the phase dynamics of electron beam are written with respect to electron phase:

$$\frac{d^2\Theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2}\left(k_z - \frac{d\Theta(t,z,p)}{dz}\right)^3 \cdot$$
$$\cdot \mathrm{Re}\left(E\exp(i\Theta(t,z,p))\right), \qquad (2)$$
$$\frac{d\Theta(t,0,p)}{dz} = k_z - \omega/u, \quad \Theta(t,0,p) = p.$$

It was proposed in (2) that the electron beam is synchronous with the transmitted wave E only. The integral form of beam current in the right-hand side of the first equation of (1) is obtained by averaging over the following initial phases of electrons in the beam: entrance time of electron in interaction zone and transverse coordinate of its entrance point. Method of averaging over initial phases of electrons is well-known and widely used in simulation of BWT, TWB, FEL etc. Equations (1)-(2) are more complicated than usually used (see e.g. [14]). System (1)-(2) takes into account two-dimensional distributions with respect to spatial coordinate and electron phase p. So, these equations allow to simulate electron beam dynamics more precisely. This is very important when electron beam moves angularly to electromagnetic waves.

NUMERICAL METHODS AND DESCRIPTION OF VOLC POSSIBILITIES

At present time there are a wide number of different computer codes for FEL simulation. A comparison of existing FEL codes on dimensionality, time-dependent simulation, models of particle beam [15] shows that beam description by collective variables allows the faster calculation in time than by method of the Particle-in-Cell. We used the beam description by collective variables in VFEL simulation too.

Numerical algorithms proposed for solving (1)-(2) allow to use parallel processing and can be started up on standard symmetric multi-processor (SMP) computing systems with some parallel processors based on common memory. The big volume of processor work takes the computation of grid values of Θ in (2). In algorithm proposed their calculation is not locked-in with respect to spatial coordinate z, so it can be executed parallel. Finite difference system in the algorithm has the form AE = F. Matrix A is inverted once before the main computation and then it is multiplied to the vector of right-hand sides F recalculated in each point in time.

Numerical methods for all possible two-wave and three-wave geometries including external reflectors are implemented in computer code VOLC, version 1.0 [10], [11]. VOLC means "VOLume Code". It was developed on the basis of multiple Fortran codes, created in 1991—2005 years. Dimensionality is 2D (one spatial coordinate and one phase space coordinate) plus time. Theree-wave geometries were considered to confirm all main VFEL physical laws and mechanisms.

New version 2.0 of VOLC allows for two-wave VFEL geometries to obtain distributions of VFEL intensities with respect to current density j, resonator length L, diffraction asymmetry factor β, system parameters l_0 or l_1, detuning from Cherenkov condition δ, as well as dynamical regimes recognition and intensity Fourier transforms. Reduction of VOLC possibilities to two-wave case is connected with geometry of experimental installation [8] for which VOLC simulation was designed. Interface of VOLC, version 2.0 is presented in Fig.1. Its block-scheme is depicted in Fig.2.

Interface of VOLC is a standalone program written in Borland C++ Builder 6.0 for use in the Windows Operating environment. Interface allows to define input parameters, check their validity with corresponding messages, call the main routine for VFEL simulation and output some results in the window including 2D plots and summary table of results. The main routine for VFEL simulation was elaborated in Compaq Visual Fortran and can operate without VOLC interface. In this case only the file with input parameters should be filled. Numerical results are written in specified files.

VOLC interface uses the standard MS Windows dialog and are supplied by screen tips. All wrong user actions are stopped with corresponding messages. Computation of distributions of intensities with respect to some parameters can take much of computer time, so at any moment user can stop computation without losses of calculated data.

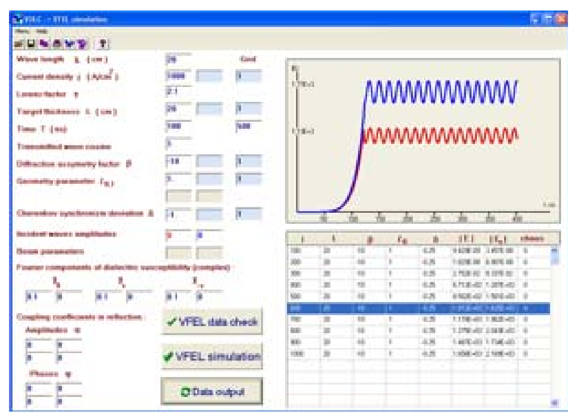

Figure 1: Interface of computer code VOLC

VOLC was tested with carping. Numerical results obtained were reported in [3], [10]–[13]. There were investigated different regimes such as oscillator and amplifier regimes, SASE (Self-Amplified Spontaneous Emission), VFEL as BWT, TWT, BWT-TWT and others. All results correspond to physical theory predictions. Some results of numerical experiments are presented below.

NUMERICAL RESULTS

One of the main VFEL physical laws [4] is the following dependence of threshold current in degeneration points: $j_{th} \sim 1/((kL)^3(k\chi_\tau L)^{2s})$, where s is a number of surplus

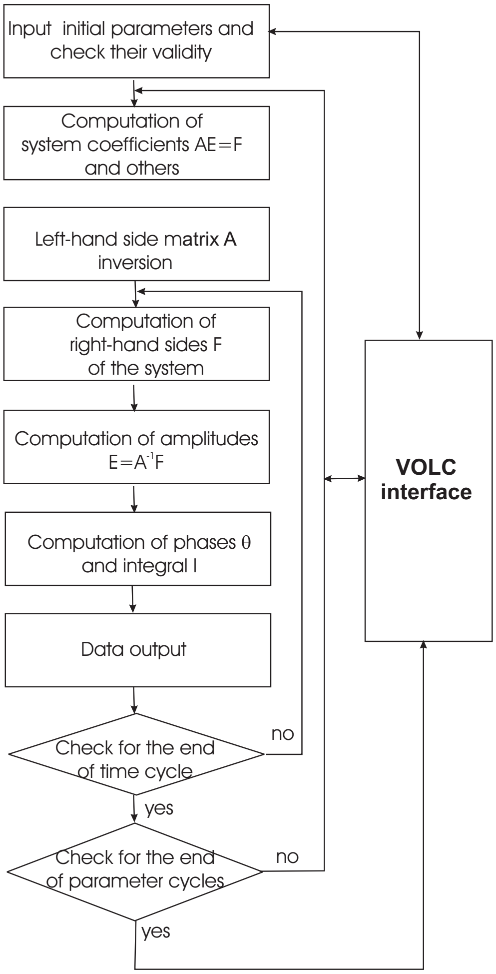

Figure 2: Block-scheme of VOLC

waves in diffraction. So, threshold current can be significantly decreased when modes are degenerated in multi-wave diffraction geometry if $k|\chi_\tau|L \gg 1$. On the other hand interaction length L can be reduced at given current value j. This was confirmed in numerical experiments and demonstrated in [11].

In [13] theoretically derived dependence of the threshold current on asymmetry factor of VDFB was presented. This relation confirms that VDFB allows to control the threshold beam current. Numerical results presented in [11] are in close agreement with the theory. Example of successful simulation with help of code VOLC of VFEL experiment with ”grid” volume resonator was proposed in [9].

In electronic devices such as FEL, TWT, BWT etc. self-oscillations are due to interaction of electron beam and electromagnetic field under distributed feedback. Investigation of chaos in such devices is of great interest in modern physics [14], [16]–[18]. In VFEL simulation we faced with chaotic behaviour of electromagnetic field intensities too. Here chaotic dynamics is induced by complicated interaction of electron beam bunches with electromagnetic field under VDFB. Investigation of chaos in VFEL is important in the light of its experimental development.

Two points of beam current threshold exist in VFEL theory. First threshold point corresponds to beginning of electron beam instability. Here regenerative amplification starts while the radiation gain of generating mode is less than radiation losses of the coupled feedback mode. Parameters at which radiation gain becomes equal to absorption correspond to the second threshold point. When electron current exceeds the second threshold value, generation progresses actively. In simulations carried out an important VFEL feature due to VDFB was shown. This is the initiation of quasiperiodic regimes at relatively small current near first and second threshold points that are bifurcation points. This is depicted via intensity plots in Fig.3 with corresponding intensity Fourier transforms in Fig.4. Curve 1 depicts regime under regenerative amplification. Current j is less than the value of the first threshold point. Value j for curve 2 is a little larger than this threshold. Lines 3 and 4 depict 1T periodic regimes that pass to quasiperiodic ones (lines 5-7).

Numerical investigations of chaotic lasing in VFEL show the possibility of complicated transitions between following chaotic regimes: stationary generation, self-modulation and periodicity, quasiperiodicity, intermittency, chaotic self-modulation or ”weak” chaos and chaotic self-oscillations or ”developed” chaos. Two-parameter analysis of VFEL chaotic lasing was carried out with respect to beam current density j and (1) diffraction asymmetry factor β, (2) detuning from synchronism condition δ, (3) length of the resonator L. One of possible roots to chaos in VFEL is presented in Fig.5 and Fig.6. Windows of periodicity and quasi-periodicity exist between chaos. Larger number of principle frequencies for transmitted wave can be explained by the fact that simultaneous generation at several frequencies is possible in VFEL. In the case considered electrons emit radiation namely in the direction of transmitted wave.

As we have more than ten control parameters (see explanations for (1)-(2)) it seams to be very difficult to investigate the full picture of possible chaotic behavior in VFEL. The aim of this investigation is to show the possibility to choose more precisely domains with periodic self-modulation instead of chaotic one.

CONCLUSION

The original software for VFEL simulation is released and allows to obtain all main VFEL physical laws and dependencies. VOLC overriding goal is to investigate the nonlinear stage of its operation. In simulation VFEL was considered as a dynamical system. Two-parameter analysis shows the complicated root to chaos. Author is grateful to Prof. V.Baryshevsky for permanent attention to her work.

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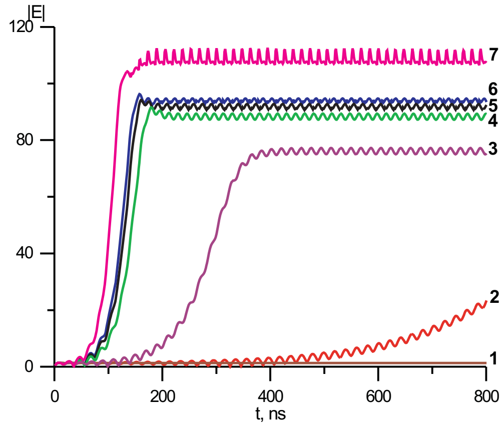

Figure 3: Transition from periodicity to quasiperiodicity for transmitted wave in amplification regime. j is equal to: 1) 350 A/cm^2, 2) 450 A/cm^2, 3) 470 A/cm^2, 4) 515 A/cm^2, 5) 525 A/cm^2, 6) 528 A/cm^2, 7) 550 A/cm^2.

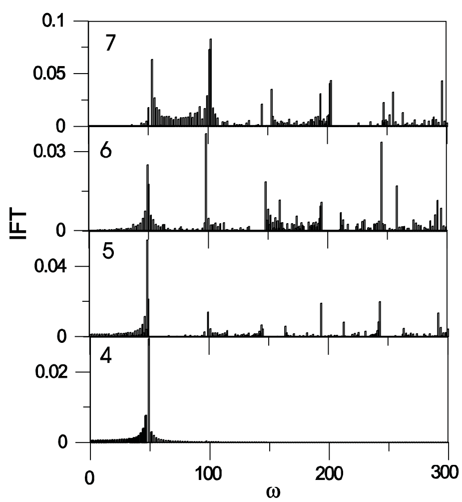

Figure 4: Intensity Fourier transform for curves 4-7 from Fig.3

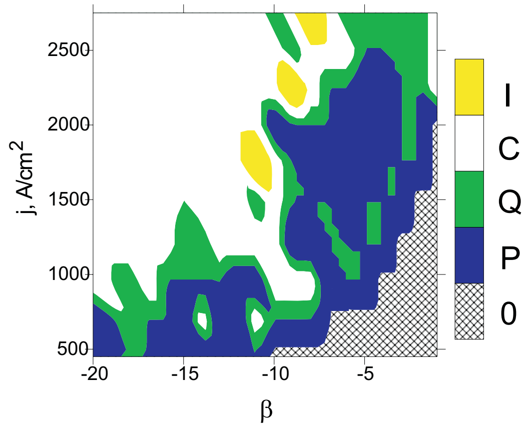

Figure 5: Dependence of the threshold current on asymmetry factor of VDFB for transmitted wave. 0 depicts a domain under beam current threshold. P, Q, C, I correspond to periodic regimes, quasiperiodicity, chaos and intermittency, respectively.

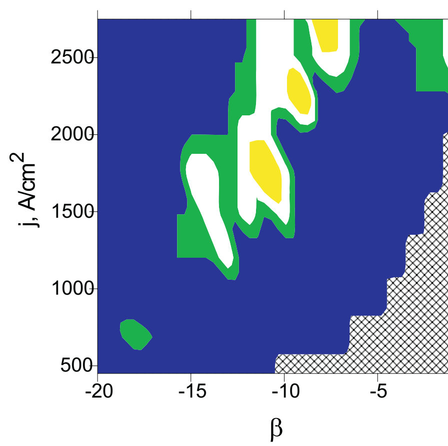

Figure 6: Dependence of the threshold current on asymmetry factor of VDFB for diffracted wave.

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