IMPACT OF LONGITUDINAL SPACE-CHARGE WAKE FROM FEL UNDULATORS ON CURRENT-ENHANCED SASE SCHEMES

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Abstract

We present a description of longitudinal wake fields in XFELs that is of relevance in relation with Enhanced Self-Amplified Spontaneous Emission (ESASE) schemes. We consider wakes in XFELs, in the limit when the electron beam has gone inside the undulator for a distance longer than the overtaking length (the length that electrons travel as a light signal from the tail of the bunch overtakes the head of the bunch). We find that the magnitude of the resulting energy chirp constitutes a reason of concern for the practical realization of ESASE schemes. **A more detailed report of our study is given in [1]**.

INTRODUCTION

This article presents a description of longitudinal wake fields in XFELs. Our study is of importance in connection with ESASE schemes, demanding for a detailed study of longitudinal wake fields arising after the dispersive section. For XFEL setups, the undulator parameter K obeys $K^2 \gg 1$. As a result, the average longitudinal Lorentz factor $\bar{\gamma}_z = \gamma/\sqrt{1+K^2/2}$ is such that $\bar{\gamma}_z^2 \ll \gamma^2$, γ being the Lorentz factor of the beam. Based on $\bar{\gamma}_z^2 \ll \gamma^2$, we demonstrate that the presence of the undulator strongly influences the space-charge wake. In contrast to this, in literature, wake calculations for the LCLS case are given in free-space, as if the presence of the undulator were negligible. In this paper we pose particular attention to the LCLS case, for which ESASE schemes have been first proposed. We thus restrict our attention to a specific region of parameters. First, the longitudinal size of the beam is much larger than the FEL wavelength, i.e. $\sigma_z \simeq \lambdabar \gg \lambdabar_r$. Second, we assume a long saturation length compared with the overtaking length, i.e. $L_s \gg 2\bar{\gamma}_z^2\lambdabar$. Third, effects of metallic surroundings can be neglected, i.e. $a \gg \bar{\gamma}_z\lambdabar$. We present a general theory based on these three assumptions. These are satisfied for the LCLS case, together with an extra-assumption on the transverse electron-beam size $\sigma_\perp$: $\sigma_\perp^2 \gg \lambdabar\lambda_w$, $\lambdabar$ being the reduced wavelength. Due to this last condition, major simplifications arise in the general theory. Radiation from the undulator is drastically suppressed and calculations of impedance and wake function can be performed considering a non-radiating beam, and thus accounting for space-charge interactions only. Space-charge impedance and wake function is found to reproduce the free-space case. Only, γ must be consistently substituted with $\bar{\gamma}_z$. We apply our theory to the ESASE setup referring to the LCLS facility. We calculate the energy chirp associated with wakes inside the undulator and between dispersive section and undulator. Subsequently, the magnitude of their effect is estimated by calculating the linear energy chirp parameter. We find that the gain of the FEL process is sensibly reduced, and that longitudinal wake fields constitute a reason of concern regarding the practical realization of ESASE schemes.

FIELD CALCULATION

Calculation of longitudinal wake field and impedance from an FEL undulator first demand characterization of the electric field generated at a given position by the bunch. We perform an analysis in terms of harmonics, i.e. $\vec{E} = \vec{\bar{E}}(\vec{r},\omega)\exp[-i\omega t] + C.C.$, the symbol "C.C." indicating complex conjugation[1]. The complex amplitude $\vec{\bar{E}}(\vec{r},\omega)$ can be considered as the electric field in the space-frequency domain, "the field". Transverse and longitudinal fields can be found by solving paraxial Maxwell's equation in the space-frequency domain: $\mathcal{D}[\vec{\tilde{E}}(z,\vec{r}_\perp,\omega)] = \vec{g}(z,\vec{r}_\perp,\omega)$. Here $\vec{\tilde{E}}_\perp = \vec{\bar{E}}_\perp\exp[-i\omega z/c]$ is the electric field envelope that does not vary much along z on the scale of the reduced wavelength $\lambdabar = \lambda/(2\pi)$. The differential operator $\mathcal{D}$ is defined by $\mathcal{D} \equiv ({\nabla_\perp}^2 + (2i\omega/c)\cdot\partial/\partial z)$, ${\nabla_\perp}^2$ being the Laplacian operator over transverse cartesian coordinates. The source-term $\vec{g}(z,\vec{r}_\perp)$ is specified by the trajectory of the source electrons, and can be written in terms of the Fourier transform of the transverse current density, $\vec{\bar{j}}(z,\vec{r}_\perp,\omega)$, and of the charge density, $\bar{\rho}(z,\vec{r}_\perp,\omega)$, as $\vec{g} = -4\pi\exp[-i\omega z/c](i\omega/c^2\vec{\bar{j}} - \vec{\nabla}\bar{\rho})$. Thus, we recognize current and gradient terms in the field. Here $\vec{\bar{j}}$ and $\bar{\rho}$ are regarded as given data. They will be treated as macroscopic quantities, and can be written as $\bar{\rho}(\vec{r}_\perp,z,\omega) = \rho_o(\vec{r}_\perp - \vec{r'}_{o\perp}(z))f(\omega)\exp[i\omega s_o(z)/v_o]$ and $\vec{\bar{j}} = \vec{v}_o\rho$. Here $f(\omega)$ is the Fourier transform of the longitudinal bunch-profile, while ρ_o is related with the transverse bunch-profile. $\vec{r'}_{o\perp}(z)$, $s_o(z)$ and v_o pertain a reference electron with Lorentz factor γ that is injected on axis with no deflection and is guided by the undulator field only. In particular, $r'_{ox}(z) = r_w\cos(k_w z)$ and

[1] For simplicity we will consider $\omega > 0$. Expressions for the field at negative values of ω can be obtained based on the property $\vec{\bar{E}}(-\omega) = \vec{\bar{E}}^*(\omega)$ starting from explicit expressions for $\vec{\bar{E}}$ at $\omega > 0$.

$r'_{oy}(z) = 0$, where the transverse amplitude of oscillations is $r_w = K/(\gamma k_w)$. The correspondent velocity is described by $\vec{v}_{o\perp}(z) = v_{ox}\vec{e}_x + v_{oy}\vec{e}_y$. Finally, $s_o(z)$ is the curvilinear abscissa measured along the trajectory of the reference particle. Solution of Maxwell's equation is performed with the help of a perturbation theory in the small parameter $\lambda_r/\lambda \ll 1$. Calculations yield [1]:

$$\begin{aligned}
\vec{\widetilde{E}}_\perp(z,\vec{r}_\perp) = &-\frac{i\omega\bar{f}(\omega)}{c}\int d\vec{r'}_\perp \rho_o\left(\vec{r'}_\perp\right)\exp\left[\frac{i\omega z}{2c\bar{\gamma}_z^2}\right] \\
\times\Bigg\{ &+\exp[+ik_w z]\frac{K\vec{e}_x}{i\gamma}K_0\left(\sqrt{2}\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) \\
&-\exp[-ik_w z]\frac{K\vec{e}_x}{i\gamma}K_0\left(-\sqrt{2}i\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) \\
&+\exp[+ik_w z]\frac{icr_w}{\omega}\Bigg[-\frac{\sqrt{2}K_1\left(\sqrt{2}\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)\vec{e}_x}{\sqrt{\bar{\lambda}\bar{\lambda}_w}\,|\vec{r}_\perp-\vec{r'}_\perp|} \\
&\quad+\frac{2(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{\bar{\lambda}\bar{\lambda}_w\,|\vec{r}_\perp-\vec{r'}_\perp|^2}K_2\left(\sqrt{2}\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)\Bigg] \\
&-\exp[-ik_w z]\frac{icr_w}{\omega}\Bigg[-\frac{\sqrt{2}iK_1\left(-\sqrt{2}i\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)\vec{e}_x}{\sqrt{\bar{\lambda}\bar{\lambda}_w}\,|\vec{r}_\perp-\vec{r'}_\perp|} \\
&\quad+\frac{2(x-x')(\vec{r}_\perp-\vec{r'}_\perp)}{\bar{\lambda}\bar{\lambda}_w\,|\vec{r}_\perp-\vec{r'}_\perp|^2}K_2\left(-\sqrt{2}i\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)\Bigg] \\
&-\left[\frac{ic}{\omega}\frac{\vec{r}_\perp-\vec{r'}_\perp}{|\vec{r}_\perp-\vec{r'}_\perp|}\right]\frac{2}{\bar{\gamma}_z\bar{\lambda}}K_1\left(\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\bar{\gamma}_z\bar{\lambda}}\right)\Bigg\} \qquad (1)
\end{aligned}$$

and

$$\begin{aligned}
\widetilde{E}_z(z,\vec{r}_\perp) = &-\frac{i\omega\bar{f}(\omega)}{c}\int d\vec{r'}_\perp \rho_o\left(\vec{r'}_\perp\right)\exp\left[\frac{i\omega z}{2c\bar{\gamma}_z^2}\right] \\
\times\Bigg\{ &+\frac{\sqrt{2}}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\exp[+ik_w z]\left[\frac{cK}{\omega\gamma}\frac{x-x'}{|\vec{r}_\perp-\vec{r'}_\perp|}\right] \\
&\qquad\times K_1\left(\sqrt{2}\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) \\
&+\frac{\sqrt{2}i}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\exp[-ik_w z]\left[\frac{cK}{\omega\gamma}\frac{x-x'}{|\vec{r}_\perp-\vec{r'}_\perp|}\right] \\
&\qquad\times K_1\left(\sqrt{2}i\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) \\
&+\frac{2}{\bar{\gamma}_z^2}K_0\left(\frac{|\vec{r}_\perp-\vec{r'}_\perp|}{\bar{\gamma}_z\bar{\lambda}}\right)\Bigg\}. \qquad (2)
\end{aligned}$$

Terms not including $\exp[\pm k_w z]$ are entangled with the electron beam. We identify them as space-charge terms. The formation length of the space-charge field is $2\bar{\lambda}\bar{\gamma}_z^2$, while the correspondent diffraction size is $\bar{\gamma}_z\bar{\lambda}$. Terms including $\exp[\pm ik_w z]$ are indicative of radiation fields. Phase velocity of terms including $\exp[+ik_w z]$ is slower than that of the beam harmonic. Phase velocity of terms including $\exp[-ik_w z]$ is faster than that of the beam harmonic. The formation length of radiation field terms is $\bar{\lambda}_w$, while the correspondent diffraction size is $\sqrt{\bar{\lambda}\bar{\lambda}_w}$. The first and the second integral in the transverse field are (radiative) current density terms. The third and the fourth term are (radiative) gradient terms, while the last term is a (space charge) gradient term. As regards the longitudinal field instead, the third integral is a (space-charge) term originating from a mixture of gradient and current sources. It is possible to cross-check our expressions for the field with the help of Gauss law $\vec{\nabla}\cdot\vec{\bar{E}} = 4\pi\bar{\rho}$. In particular, it can be seen that $\vec{\bar{E}} = \vec{\bar{E}}_{\mathrm{rad}} + \vec{\bar{E}}_{\mathrm{sc}}$ separately verifies: $\vec{\nabla}\cdot\vec{\bar{E}}_{\mathrm{sc}} = 4\pi\bar{\rho}$ and $\vec{\nabla}\cdot\vec{\bar{E}}_{\mathrm{rad}} = 0$. This confirms that radiation field is not entangled with sources, while space-charge field is.

IMPEDANCE

The impedance is defined accounting for the transverse size of the beam as $Z(\omega,z) = 1/|\bar{f}(\omega)|^2\cdot\int_0^z dz'\int_A d\vec{r'}_\perp\vec{\bar{j}}^*\cdot\vec{\bar{E}}$. The integration volume is a cylinder of base A including the undulator up to position $z' = z$. Integration in z' is performed from 0 to z, because we are interested in impedance generated inside the undulator, that begins at $z = 0$. The wake function can be obtained by Fourier transformation of the impedance. Using Eq. (1) and Eq. (2) we obtain the total impedance $Z = Z_{\mathrm{r}} + Z_{\mathrm{sc}}$. The real part Z_R is given by

$$\begin{aligned}
Z_R = &-\frac{K^2\pi\omega z}{4\gamma^2}\int d\vec{r'}_\perp\int d\vec{r''}_\perp\rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) \\
&\times J_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp-\vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right). \qquad (3)
\end{aligned}$$

The imaginary part Z_I amounts to

$$\begin{aligned}
Z_I = &-\frac{K^2\omega z}{2\gamma^2}\int d\vec{r'}_\perp\int d\vec{r''}_\perp\rho_o^*(\vec{r'}_\perp)\rho_o\left(\vec{r''}_\perp\right) \\
&\times\Bigg\{\frac{\pi}{2}Y_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp-\vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right)-K_0\left(\frac{\sqrt{2}\left|\vec{r'}_\perp-\vec{r''}_\perp\right|}{\sqrt{\bar{\lambda}\bar{\lambda}_w}}\right) \\
&+\frac{4+2K^2}{K^2}K_0\left(\frac{\left|\vec{r'}_\perp-\vec{r''}_\perp\right|}{\bar{\gamma}_z\bar{\lambda}}\right)\Bigg\}. \qquad (4)
\end{aligned}$$

Z_R can be entirely traced back to the fast-wave part of the transverse radiative field. Z_I is composed of different contributions instead. The term in Y_0 follows from the fast-wave transverse radiative field. The second term (in K_0) follows from the slow-wave transverse radiative field. The last term (also in K_0) can be traced back to the longitudinal space-charge field.

Asymptotic case for $\sigma_\perp^2 \ll \bar{\lambda}\bar{\lambda}_w$

It is interesting to derive asymptotic limits of Eq. (3) and Eq. (4) in the case for $\sigma_\perp^2 \ll \bar{\lambda}\bar{\lambda}_w$. Bessel functions in Eq. (3) and Eq. (4) can be expanded for small argument values. In particular, using $J_0(x) \simeq 1$ for $x \ll 1$, the real part of the impedance becomes $Z_R = -K^2\pi z/(4c\bar{\lambda}\gamma^2)$, independently of the choice of ρ_o. Subsequently, we use $K_0(x) \simeq -\gamma_E - \ln(x/2)$ and $Y_0 \simeq 2/\pi[\gamma_E + \ln(x/2)]$, $\gamma_E \simeq 0.577216$ being the Euler Gamma constant in the imaginary part of the impedance, Eq. (4). We obtain $Z_I = -K^2 z/(c\bar{\lambda}\gamma^2)\ln(\sqrt{\bar{\lambda}/\bar{\lambda}_r}) + 2z/(c\bar{\lambda}\gamma^2)\ln(\sqrt{1+K^2/2}) + Z_{I\ \mathrm{free}}$, where $Z_{I\ \mathrm{free}} = 2z\gamma_E/(c\bar{\lambda}\gamma^2) + 2\omega z/\gamma^2 \int d\vec{r'}_\perp \int d\vec{r''}_\perp \rho_o^*(\vec{r'}_\perp)\rho_o(\vec{r''}_\perp)\ln[|\vec{r'}_\perp - \vec{r''}_\perp|/(2\bar{\lambda}\gamma)]$. $Z_{I\ \mathrm{free}}$ is the only model-dependent part of the impedance. Assuming a Gaussian transverse profile $\rho_o(r_\perp) = 1/(2\pi\sigma_\perp^2 c)\exp[-r_\perp^2/(2\sigma_\perp^2)]$, we obtain $Z_{I\ \mathrm{free}} = 2z\gamma_E/(c\bar{\lambda}\gamma^2) + 2z/(c\bar{\lambda}\gamma^2)\ln[\sigma_\perp/(\gamma\bar{\lambda})]$. Thus, $Z_{I\ \mathrm{free}}$, is logarithmically divergent on $\sigma_\perp$. This is the free-space impedance. The renormalized impedance, i.e. the difference $Z - i\,Z_{I\ \mathrm{free}}$ is independent of $\sigma_\perp$ and constitutes a result valid for any value of K. These results are in agreement with limiting cases discussed in literature.

Asymptotic case for $\sigma_\perp^2 \gg \bar{\lambda}\bar{\lambda}_w$

Transverse scales pertaining radiation field and space-charge field are present in Eq. (3) and Eq. (4). The first two terms in Y_0 and K_0 in Eq. (4), as well as the entire real part of the impedance, are linked to the presence of transverse current density and to radiation field. The last term in Eq. (4) is due to the presence of longitudinal space-charge field, a combination of current and gradient terms. The corresponding Bessel functions yield different characteristic transverse scales. Bessel functions related with the radiation field yield $\sqrt{\bar{\lambda}\bar{\lambda}_w} \sim \sqrt{\bar{\lambda}\bar{\lambda}_r\bar{\gamma}_z^2}$. Those related with the longitudinal space-charge field yield $\bar{\lambda}\bar{\gamma}_z \sim \sqrt{\bar{\lambda}\bar{\lambda}\bar{\gamma}_z^2}$. Since $\bar{\lambda} \gg \bar{\lambda}_r$, it follows that $\sqrt{\bar{\lambda}\bar{\lambda}_w/2} \ll \bar{\lambda}\bar{\gamma}_z$. By inspection of Eq. (3) and Eq. (4) one can see that the value of $\left|\vec{r'}_\perp - \vec{r''}_\perp\right|$ is limited by $\sigma_\perp$, because of the presence of the exponential functions under the integration sign. Therefore, assuming constant total charge of the beam, when the electron beam transverse size $\sigma_\perp$ increases beyond $\sqrt{\bar{\lambda}\bar{\lambda}_w}$ the radiation contribution is suppressed with respect to the space-charge one. Summing up, when $\sigma_\perp^2 \gg \bar{\lambda}\bar{\lambda}_w$, we may neglect the real part of the impedance Z_R and approximate the total impedance with

$$Z = -i\frac{2\omega z}{\bar{\gamma}_z^2}\int d\vec{r'}\int d\vec{r''}\rho_o^*(\vec{r'})\rho_o\left(\vec{r''}_\perp\right)K_0\left(\frac{\left|\vec{r'} - \vec{r''}\right|}{\bar{\lambda}\bar{\gamma}_z}\right). \quad (5)$$

This means that, in the limit $\sigma_\perp^2 \gg \bar{\lambda}\bar{\lambda}_w$, the only field to be accounted for when calculating impedance (and wake), is the effective longitudinal space-charge field.

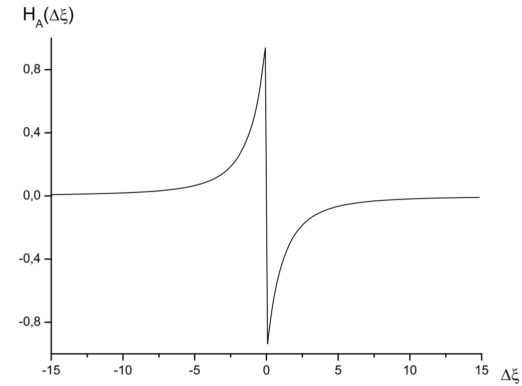

Figure 1: Plot of the universal function H_A.

STEADY STATE WAKE FOR $\sigma_\perp^2 \gg \bar{\lambda}\bar{\lambda}_W$

Our derivations drastically simplify for $\sigma_\perp^2 \gg \bar{\lambda}\bar{\lambda}_w$. We consider transverse and longitudinal gaussian profiles. When $\sigma_\perp^2 \gg \bar{\lambda}\bar{\lambda}_w$, an expression for the wake can be found by Fourier-transforming the impedance given in Eq. (5). Eq. (5) is mathematically identical to the free-space expression where γ is substituted by $\bar{\gamma}_z$. We find that the antisymmetric part of the wake G_A is given by $G_A(\Delta\xi) = \bar{\gamma}_z\eta\hat{z}/\sigma_\perp \cdot H_A(\Delta\xi)$, where $H_A(\Delta\xi) = -\Delta\xi/(2\sqrt{\pi})\ \{2\sqrt{\pi}/|\Delta\xi| - \pi\exp[(\Delta\xi)^2/4]\mathrm{erfc}[|\Delta\xi|/2]\}$. Here we defined $\Delta\xi = \bar{\gamma}_z(\Delta s)/\sigma_\perp$, $\eta = \bar{\gamma}_z\sigma_z/\sigma_\perp$ and $\hat{z} = z/(2\bar{\gamma}_z^2\sigma_z)$. A plot of the universal function H_A as a function of $\Delta\xi$ is given in Fig. 1.

The energy change of a single particle at position s within the bunch due to the reactive part of the wake (averaged over transverse coordinates) is given by $\Delta\mathcal{E}_A(s) = (-e)\int_{-\infty}^{\infty} G_A(\Delta s)f(s-\Delta s)d(\Delta s)$. An explicit expression for $\Delta\mathcal{E}_A/\mathcal{E}_o$, with $\mathcal{E}_o = \gamma m_e c^2$, as a function of $\xi = \bar{\gamma}_z s/\sigma_\perp$ is:

$$\frac{\Delta\mathcal{E}_A}{\mathcal{E}_o}\left(\frac{s}{\sigma_z};\eta\right) = \frac{I_{\max}\hat{z}}{\gamma I_A}F\left(\frac{s}{\sigma_z};\eta\right) \quad (6)$$

and

$$F\left(\frac{s}{\sigma_z};\eta\right) = \int_{-\infty}^{\infty} d(\Delta\xi)\eta H_A\left(\eta\frac{s}{\sigma_z} - \Delta\xi\right)\exp\left[-\frac{(\Delta\xi)^2}{2\eta^2}\right]. \quad (7)$$

A plot of F is given as a function of s/σ_z in Fig. 2 for different values of η.

APPLICATION TO ESASE SCHEMES

We now calculate the impact of longitudinal wake fields on ESASE schemes. We propose an analysis on a set of parameters referring to LCLS. Similar calculations may be performed on other parameter

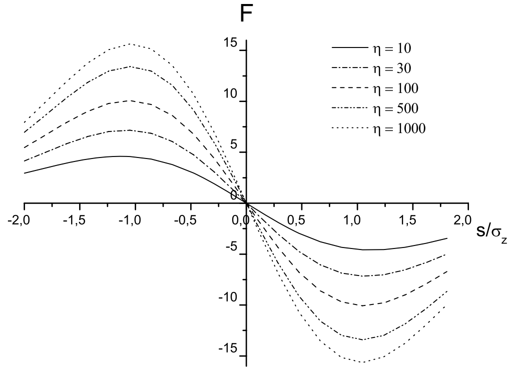

Figure 2: Plot of F for different values of η.

sets like those for the European XFEL. We consider a beam with normalized emittance after the dispersive section $\epsilon_n \simeq 1.2$ mm mrad. We take the average betatron function in the focusing lattice $\beta_f = 18$ m, and $\gamma = 2.8 \cdot 10^4$. This gives a transverse beam size $\sigma_\perp = (\epsilon_n \beta_f/\gamma)^{1/2} \simeq 30\ \mu$m. The longitudinal size of the bunch is $\sigma_z = 50$ nm. The maximal current is about the Alfven current $I_A \simeq 17$ kA; in fact, $I_{\mathrm{peak}} \simeq 18$kA. Finally, the undulator has a period $\lambda_w = 0.03$ m, $K = 3.7$, and the vacuum chamber dimension is $a = 2.5$ mm. We consider a wavelength $\lambdabar \simeq \sigma_z = 50$ nm. We can neglect the vacuum chamber influence, because $\overline{\gamma}_z \lambdabar = 500\ \mu$m, as $\overline{\gamma}_z \simeq 10^4$, and $\overline{\gamma}_z \lambdabar \ll a = 2.5$ mm. The overtaking length is $2\lambdabar\overline{\gamma}_z^2 \simeq 10$ m. The saturation length is about $L_s = 50$ m. Thus $\hat{z} = 5$, and we can use our asymptotic expression. Moreover $\eta = \overline{\gamma}_z \sigma_z/\sigma_\perp \simeq 16.7$. From fig. 2 (or from direct calculations) one can see that the maximal value assumed by $F(s/\sigma_z, \eta)$ for $\eta = 16.7$ is about $F_{\max} \simeq 6$. It follows that the energy-chirp peak-to-peak is given by $\Delta\mathcal{E}_{A,\mathrm{peak}} = 2m_e c^2 (I_{\max}/I_A) \cdot \hat{z} F_{\max} \simeq 30$ MeV. In contrast to this, estimations in literature indicate "a swing in energy of 2.4 MeV". The reason for this large discrepancy is due to the fact that, in literature, the Lorentz factor γ is incorrectly used in place of $\overline{\gamma}_z$. Energy chirp is also accumulated in the free-space between the dispersive section and the undulator, worsening the situation even more. In the LCLS case the dispersive section is a dogleg located about 200 m from the undulator. The overtaking length is now $2\lambdabar\gamma^2 \simeq 80$ m, so that $\hat{z} = 2.5$. Using the same procedure as for the wake inside the undulator (but considering γ instead of $\overline{\gamma}_z$), we obtain an extra energy chirp of about $\Delta\mathcal{E}_{A,\mathrm{peak}} \simeq 20$ MeV. The sum of contributions from the straight section after the dogleg and from the undulator amounts to about 50 MeV. In order to estimate the magnitude of the effect we can use the linear energy chirp parameter $\hat{\alpha} = -(\gamma\omega\rho_{1D}^2)^{-1} \cdot d\gamma/dt$, where ρ_{1D} is the one-dimensional ρ-parameter in FEL theory. For ESASE schemes at LCLS $I_{\mathrm{peak}} = 18$ kA, and we have $\rho_{1D} \simeq 10^{-3}$. Using an estimated peak-to-peak chirp of 50 MeV we obtain $\hat{\alpha} \simeq 1$. Thus, the saturation length is significantly modified. This is a reason of concern, because ESASE schemes are based on the assumption that the nominal saturation length of about 80 m is shortened to about 50 m, that is only 37.5% less.

CONCLUSIONS

We presented a theory of wake fields in an XFEL system. Specific constraints on parameters (fulfilled in XFEL setups) were considered. We derived expressions for the steady state impedance, that is composed of a radiative and a space-charge part. Radiation field and space-charge field are characterized by different formation lengths: the undulator period $\lambdabar_w$ and the overtaking length $2\lambdabar\overline{\gamma}_z^2$ respectively. As a result, the steady state radiative part of the impedance can be applied for any undulator system (with $N_w \gg 1$). The steady state space-charge part of the impedance can be used only assuming that the saturation length is long with respect to the overtaking length, which limits its practical region of applicability. After having dealt with a generic expression for the steady-state impedance, we specialized our theory to the case $\sigma_\perp^2 \gg \lambdabar\lambdabar_w$. Major simplifications arise: space-charge contributions to impedance and wake dominate with respect to radiative contributions. We showed that the (antisymmetric) wake can be given in terms of an asymptotic expression for the wake generated by a beam in uniform motion along the longitudinal axis provided that γ is consistently substituted with $\overline{\gamma}_z$. Final expressions are presented in the case of a planar undulator. However, there are no specific effects related with such choice, and our work may be straightforwardly extended to the case of a helical undulator. In the limit $\sigma_\perp^2 \gg \lambdabar\lambdabar_w$ radiation is suppressed, so that the beam can be considered as non-radiating, and only space-charge impedance is present. Such impedance amounts to the free-space impedance, where γ is consistently substituted with $\overline{\gamma}_z$. Eq. (5) gives the correct impedance at position z inside the undulator, as an asymptotic limit for $\sigma_\perp^2 \gg \lambdabar\lambdabar_w$ of our general theory. We used our theory to estimate the effect on ESASE schemes, and we found reason for concern. Our results are in contrast with literature, where the Lorentz factor γ is incorrectly used in place of $\overline{\gamma}_z$ in the calculation of the impedance.

For details on our work we refer the interested reader to [1].

REFERENCES

[1] G. Geloni, E. Saldin, E. Schneidmiller and M. Yurkov, DESY 07-087 (2007) at http://arxiv.org/abs/0706.2280, submitted to Elsevier Science