Fourier Optics treatment of Classical Relativistic Electrodynamics

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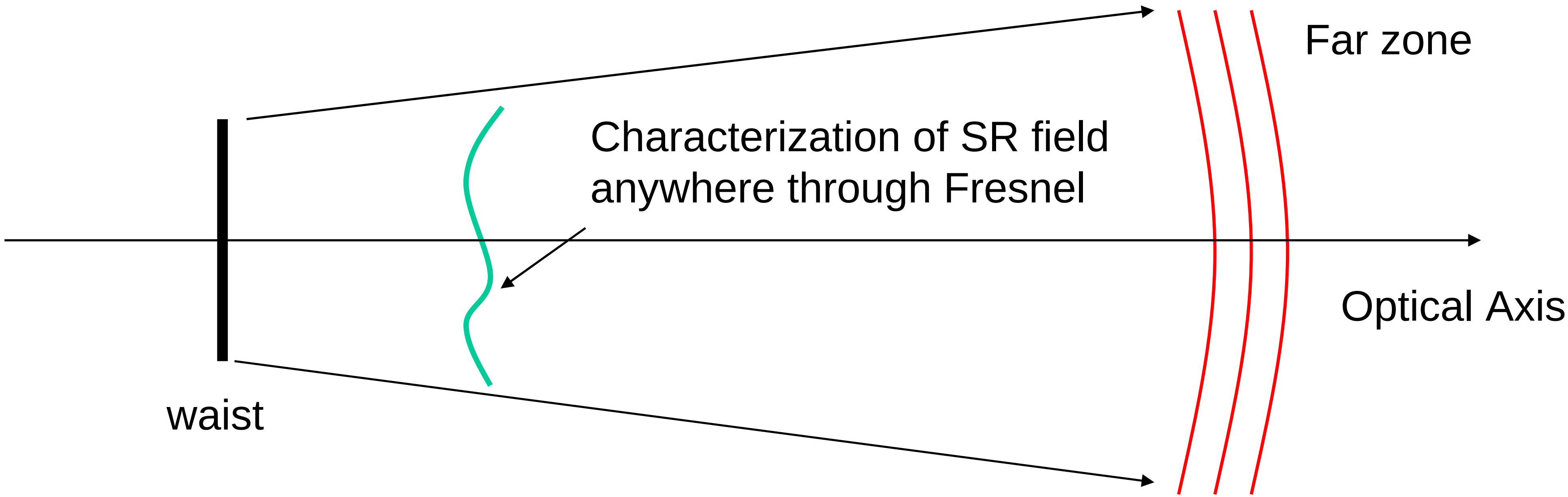
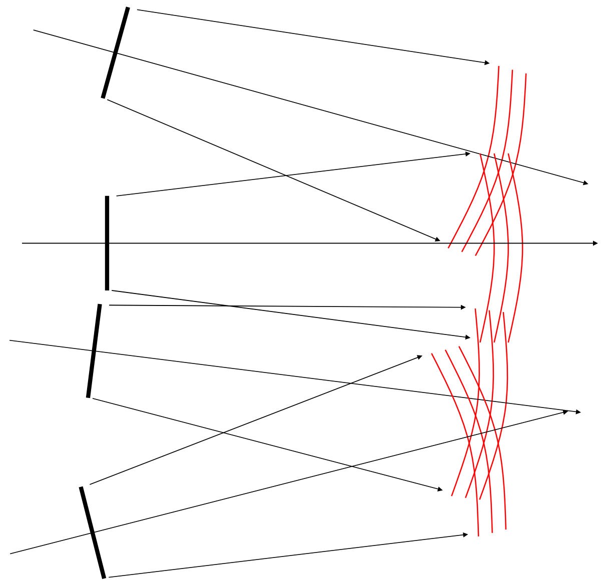
Space-frequency domain

SR beams from single electrons are similar to laser-beams

- **The far field completely characterizes radiation at a virtual source = "waist"**
- **Fresnel propagation formula propagates the field from the virtual source**



…and if many particles are present, with different propagation angles, offsets…



…tilt and move the virtual sources. Then sum up.

Intensities: incoherent superposition

Fields: Coherent superposition

In this talk: **single electron**

Very fundamental: any problem

NEW APPROACH TO SR IN NEAR-ZONE:

- **Calculate the far-zone field**
- **Calculate the field at the virtual source**
- **Propagate the field from the virtual source**

Technique yields analytical results

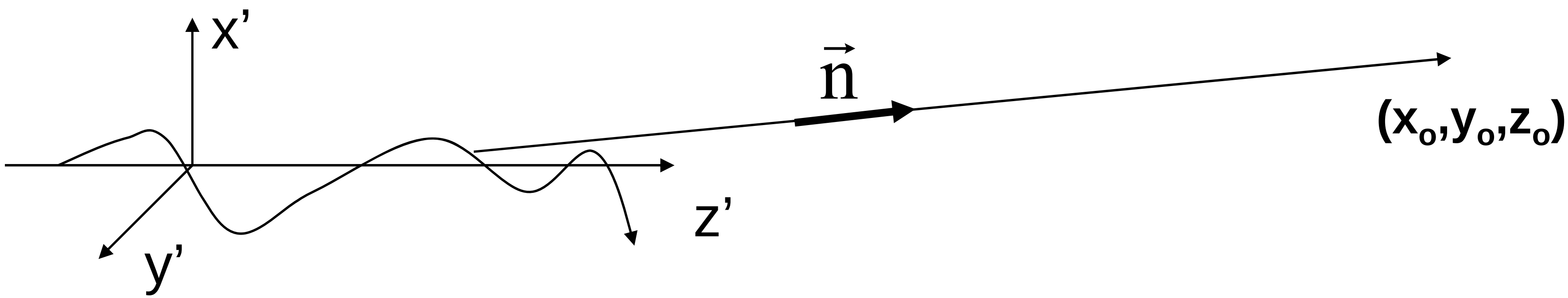
Fourier Optics is powerful:

Results from textbook expressions, far field-based

Looks paradoxical though!

$$\vec{E}(\vec{r}_o,t) = -e\frac{\vec{n}-\vec{\beta}}{\gamma^2(1-\vec{n}\cdot\vec{\beta})^2|\vec{r}_o-\vec{r}\,'|^2} - \frac{e}{c}\frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{n}\cdot\vec{\beta})^2|\vec{r}_o-\vec{r}\,'|}.$$

Near zone **Far zone**



… and still, from the far-zone data
we can reconstruct the near-zone field…

Why is it possible to have this picture?

Ultrarelativistic electron
Frequency domain → λ

$$\vec{E}(\vec{r}_o,\omega) = -e\int_{-\infty}^{\infty} dt' \frac{\vec{n}-\vec{\beta}}{\gamma^2(1-\vec{n}\cdot\vec{\beta})^2|\vec{r}_o-\vec{r}'|^2} \exp\left[i\omega\left(t' + \frac{|\vec{r}_o-\vec{r}'(t')|}{c}\right)\right]$$
$$-\frac{e}{c}\int_{-\infty}^{\infty} dt' \frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{n}\cdot\vec{\beta})^2|\vec{r}_o-\vec{r}'|} \exp\left[i\omega\left(t' + \frac{|\vec{r}_o-\vec{r}'(t')|}{c}\right)\right].$$

Add a full derivative to the integrand

↓

$$\vec{E}(\vec{r}_o,\omega) = -\frac{i\omega e}{c}\int_{-\infty}^{\infty} dt' \left[\frac{\vec{\beta}-\vec{n}}{|\vec{r}_o-\vec{r}'(t')|} - \frac{ic}{\omega}\frac{\vec{n}}{|\vec{r}_o-\vec{r}'(t')|^2}\right] \exp\left\{i\omega\left(t' + \frac{|\vec{r}_o-\vec{r}'(t')|}{c}\right)\right\}$$

Starting point for SRW, SPECTRA…

$$\vec{E}(\vec{r}_o,\omega) = -\frac{i\omega e}{c}\int_{-\infty}^{\infty} dt' \left[\frac{\vec{\beta}-\vec{n}}{|\vec{r}_o - \vec{r}'(t')|} - \cancel{\frac{ic}{\omega}\frac{\vec{n}}{|\vec{r}_o - \vec{r}'(t')|^2}}\right] \exp\left\{i\omega\left(t' + \frac{|\vec{r}_o - \vec{r}'(t')|}{c}\right)\right\}$$

Condition to neglect: d>> λ

Characteristic size a, Wavelength λ, Formation length L_f

FAR ZONE	→ d such that **n** = constant	→ d>>a
FORMATION ZONE		→ d<L_f
RADIATION ZONE	→ field disentangled from electron	→ d>>λ
1/R-ZONE	→ 1/R^2 term can be neglected	↑
RECONSTRUCTION ZONE	→ solution of the inverse problem	→ d>>λ

Note that for ultra-relativistic particle → $a \geq L_f >> \lambda$

→ **paraxial approximation**

Field propagation

Space-frequency domain; the field $\vec{\vec{E}}(\vec{r},\omega)$

Introduce the slowly varying envelope $\tilde{\vec{\vec{E}}}_\perp = \vec{\vec{E}}_\perp \exp\left[-i\omega z/c\right]$

From the paraxial Maxwell Equation $\left(\nabla_\perp{}^2 + \frac{2i\omega}{c}\frac{\partial}{\partial z}\right)\tilde{E} = 0$

Get the well-known propagation equation in free-space

$$\tilde{E}(z_o,\vec{r}_{o\perp}) = \frac{i\omega}{2\pi c(z_o - z)} \int d\vec{r'}_\perp\ \tilde{E}(z,\vec{r'}_\perp) \exp\left[\frac{i\omega\left|\vec{r}_{o\perp} - \vec{r'}_\perp\right|^2}{2c(z_o - z)}\right]$$

Perfect synergy with Fourier codes (SRW, PHASE, ZEMAX …)

Virtual source calculations

Introduce the spatial F.T. of the field envelope, F:

$$\mathrm{F}\left(z,\vec{u}\right) = \int d\vec{r'}_{\perp} \tilde{E}(z,\vec{r'}_{\perp}) \exp\left[i\vec{r'}_{\perp}\cdot\vec{u}\right]$$

Relation between F and the far-zone field ($\theta = r_o/z_o$)

$$\tilde{E}(\vec{\theta}) = \frac{i\omega}{2\pi c z_o} \exp\left[\frac{i\omega|\vec{\theta}|^2}{2c}(z_o+z)\right] \mathrm{F}\left(z, -\frac{\omega\vec{\theta}}{c}\right)$$

$$\rightarrow \quad \bar{E}_{\perp}(z_s,\vec{r}_{\perp}) = \frac{i\omega z_o}{2\pi c} \int d\vec{\theta} \exp\left[-\frac{i\omega|\vec{\theta}|^2}{2c}(z_o+z_s)\right] \bar{E}_{\perp}(\vec{\theta}) \exp\left[\frac{i\omega}{c}\vec{r}_{\perp}\cdot\vec{\theta}\right]$$

Far-field pattern calculations

Final step. Calculate the field in the far zone.

Using well-known result in far zone:

$$\vec{E}(\vec{r_o},\omega) = -\frac{i\omega e}{c r_o} \exp\left[\frac{i\omega}{c}\vec{n}\cdot\vec{r_o}\right] \int\limits_{-\infty}^{\infty} dt' \; \vec{n}\times(\vec{n}\times\vec{\beta}) \exp\left[i\omega\left(t' - \frac{1}{c}\vec{n}\cdot\vec{r'}\right)\right]$$

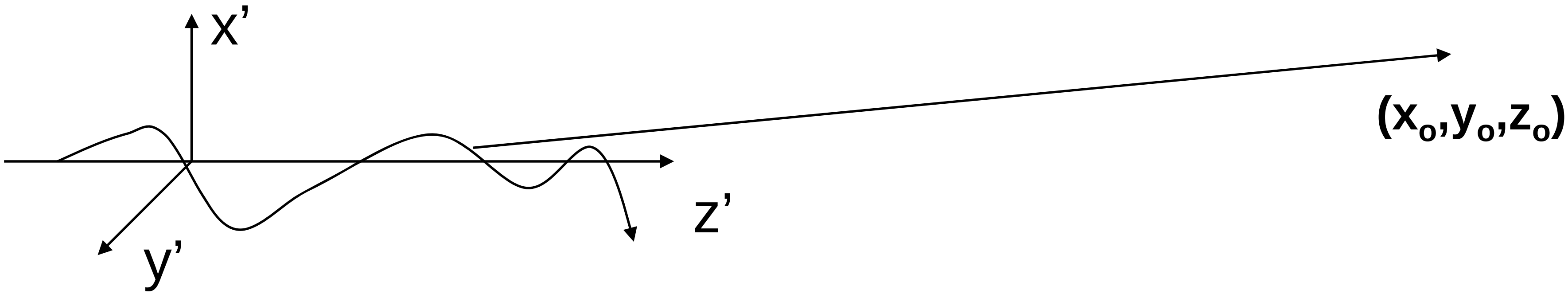
Far-field pattern calculations

We obtain an expression for the slowly varying envelope

$$\vec{\tilde{E}}_\perp = -\frac{i\omega e}{c^2 z_o}\int_{-\infty}^{\infty} dz' \exp\left[i\Phi_T\right]\left[\left(\frac{v_x(z')}{c}-\frac{x_o}{z_o}\right)\vec{x}+\left(\frac{v_y(z')}{c}-\frac{y_o}{z_o}\right)\vec{y}\right]$$

$$\Phi_T = \omega\left[\frac{s(z')}{v}-\frac{z'}{c}\right]+\omega\left(\frac{1}{z_o}+\frac{z'}{z_o^2}\right)\frac{\left[x_o-x'(z')\right]^2+\left[y_o-y'(z')\right]^2}{2c}$$

Where s(z') is the curvilinear abscissa



Summing up:

•Calculate the far-zone field
•Calculate the field at the virtual source
•Propagate the field from the virtual source

Application 1. Undulator at resonance

Calculate the far-zone field

Undulator length

Observation angle

$$\widetilde{E}_{\perp}(z_o, \theta) = -\frac{K \omega e L_w}{c^2 z_o \gamma} A_{JJ} \exp\left[i \frac{\omega z_o}{2c} \theta^2\right] \mathrm{sinc}\left[\frac{\omega L_w \theta^2}{4c}\right]$$

Observation distance

Spherical wave centred at $z_o=0$

$$A_{JJ} = J_o\left(\frac{K^2}{4+2K^2}\right) - J_1\left(\frac{K^2}{4+2K^2}\right)$$

Application 1. Undulator at resonance

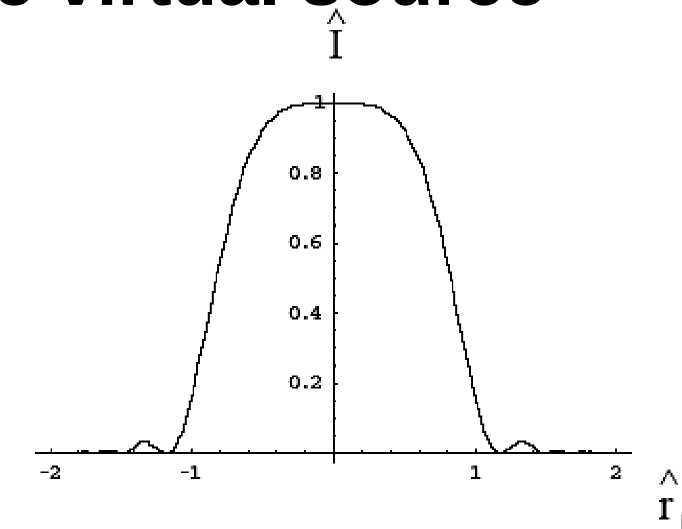
Fourier transform to get the virtual source

$$\rightarrow \quad \tilde{E}_\perp(0, r_\perp) = i\frac{K\omega e}{c^2\gamma} A_{JJ}\left[\pi - 2\mathrm{Si}\left(\frac{\omega r_\perp^2}{L_w c}\right)\right]$$

Guoy phase shift

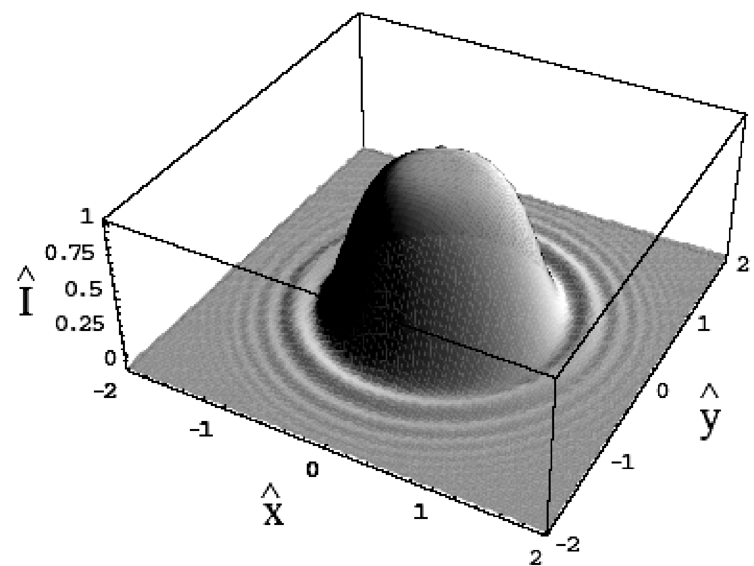
Sin Integral function

$\mathrm{Si}(z) = \int_0^z \sin(t)/t\, dt$



$$\rightarrow \quad \hat{I}(0, \hat{r}) \sim \left[\pi - 2\mathrm{Si}\left(\hat{r}^2\right)\right]^2 \longrightarrow$$

Having used normalization $\vec{\hat{r}} = \sqrt{\frac{\omega}{L_w c}}\vec{r}_\perp$



Application 1. Undulator at resonance

Propagate the field from the virtual source to get:

$$\widetilde{E}_\perp(z_o, r_\perp) = \frac{K\omega e A_{JJ}}{c^2\gamma}\left[\mathrm{Ei}\left(\frac{i\omega r_\perp^2}{2z_o c - L_w c}\right) - \mathrm{Ei}\left(\frac{i\omega r_\perp^2}{2z_o c + L_w c}\right)\right]$$

Exponential integral function

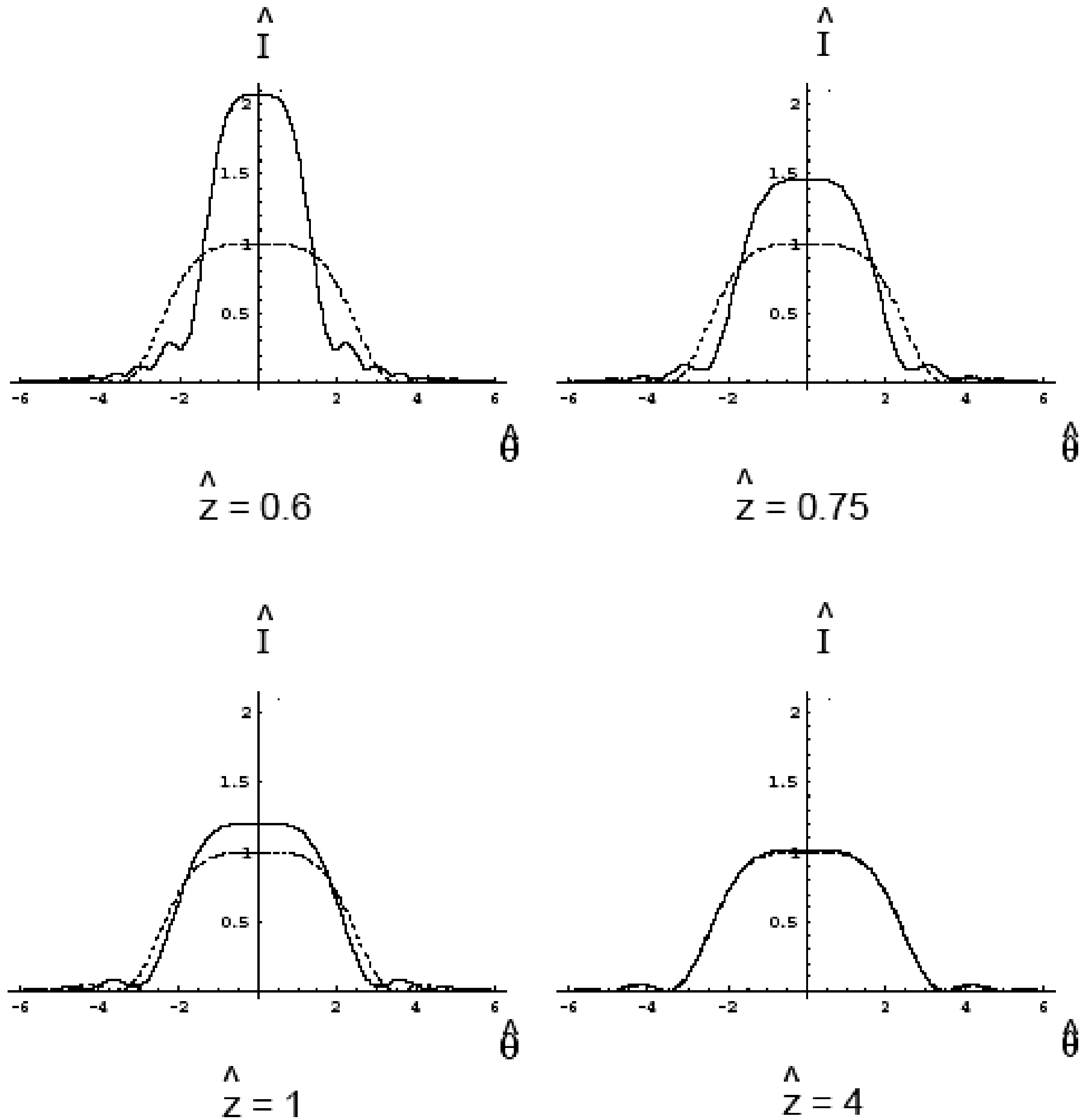
$$\mathrm{Ei}(z) = -\int_{-z}^{\infty} e^{-t}/t \, dt$$

$$\rightarrow \hat{I}\left(\hat{z}_o, \hat{\theta}\right) \sim \left|\mathrm{Ei}\left(\frac{i\hat{z}_o^2\hat{\theta}^2}{2\hat{z}_o - 1}\right) - \mathrm{Ei}\left(\frac{i\hat{z}_o^2\hat{\theta}^2}{2\hat{z}_o + 1}\right)\right|^2$$

Having used normalization $\quad \vec{\hat{\theta}} = \sqrt{\frac{\omega L_w}{c}}\vec{\theta} \qquad \hat{z} = \frac{z}{L_w}$

And defined for at any distance: $\vec{\theta} = \frac{\vec{r}_\perp}{z_o}$

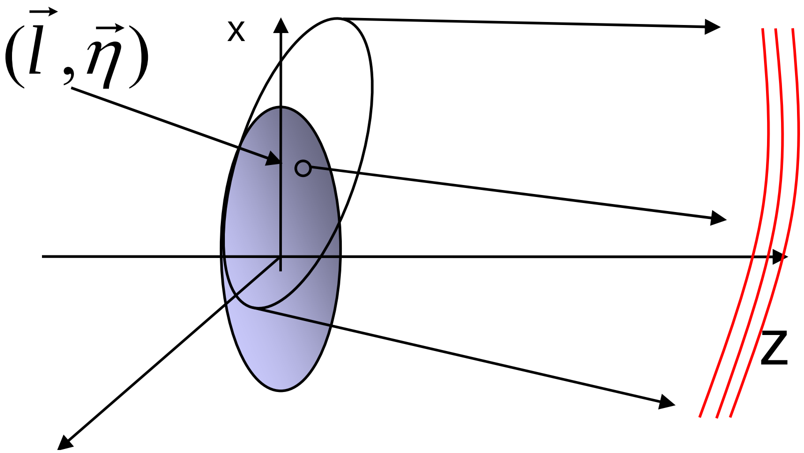
Application 1. Undulator at resonance



Application 1. Undulator at resonance

$$\widetilde{E}_\perp(0, r_\perp) = i\frac{K\omega e}{c^2\gamma}A_{JJ}\left[\pi - 2\mathrm{Si}\left(\frac{\omega r_\perp^2}{L_w c}\right)\right]$$

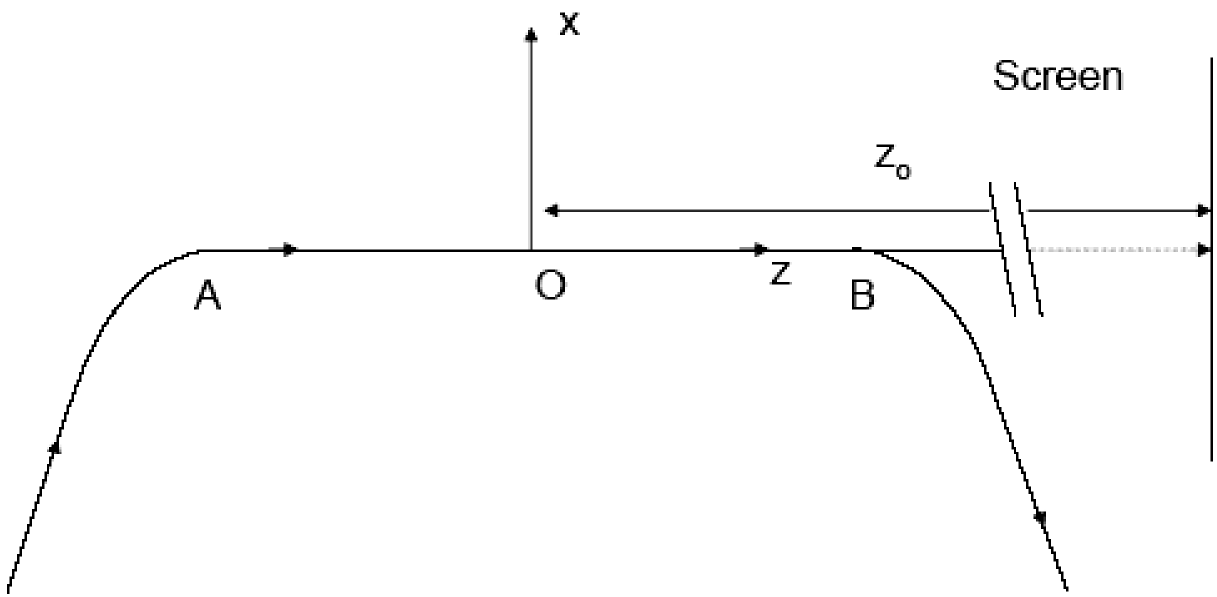
Can be generalized to account for offset and deflection



Shifts the virtual source

$$\widetilde{E}_\perp\left(0, \vec{\eta}, \vec{l}, r_\perp\right) = i\frac{K\omega e}{c^2\gamma}A_{JJ}\underbrace{\exp\left[i\frac{\omega}{c}\vec{\eta}\cdot\left(\vec{r}_\perp - \vec{l}\right)\right]}_{\textbf{Tilts the virtual source}}\left[\pi - 2\mathrm{Si}\left(\frac{\omega\left|\vec{r}_\perp - \vec{l}\right|^2}{L_w c}\right)\right]$$

Application 2. Edge radiation



$$\vec{\tilde{E}}_\perp(z_o,\vec{r}_{\perp o},\omega) = \vec{\tilde{E}}_{b1}(z_o,\vec{r}_{\perp o},\omega) + \vec{\tilde{E}}_{AB}(z_o,\vec{r}_{\perp o},\omega) + \vec{\tilde{E}}_{b2}(z_o,\vec{r}_{\perp o},\omega)$$

On the basis of this we will be studying the TUR case

Application 2. Edge radiation

ONLY contribution from AB Straight section:

Fundamental (due to superposition principle)

Can be directly applied when switcher contributions negligible

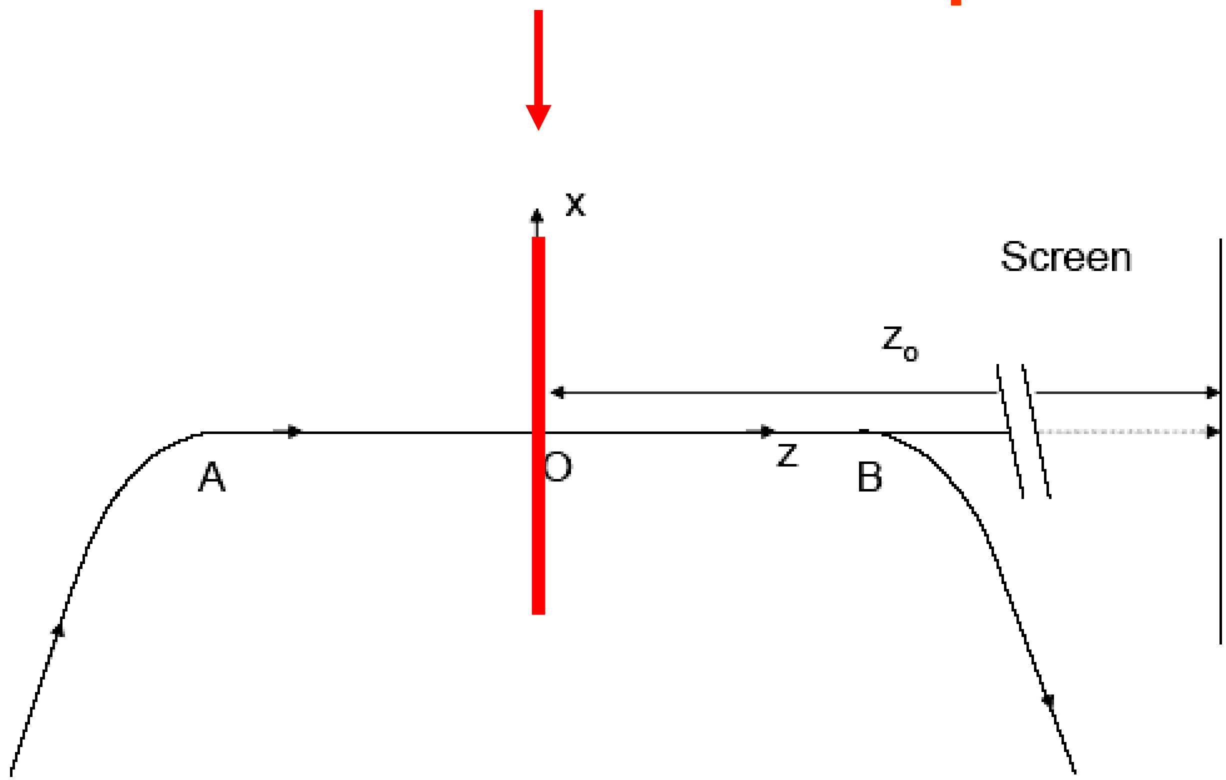
↓

e.g. $2\pi L \gg \gamma^2\lambda$: zero-length switchers approximation holds in some transverse region of observation when $\lambda \gg \lambda_c$

Application 2. Edge radiation

Two possibilities

1- Single virtual source in the middle of the setup



Application 2. Edge radiation

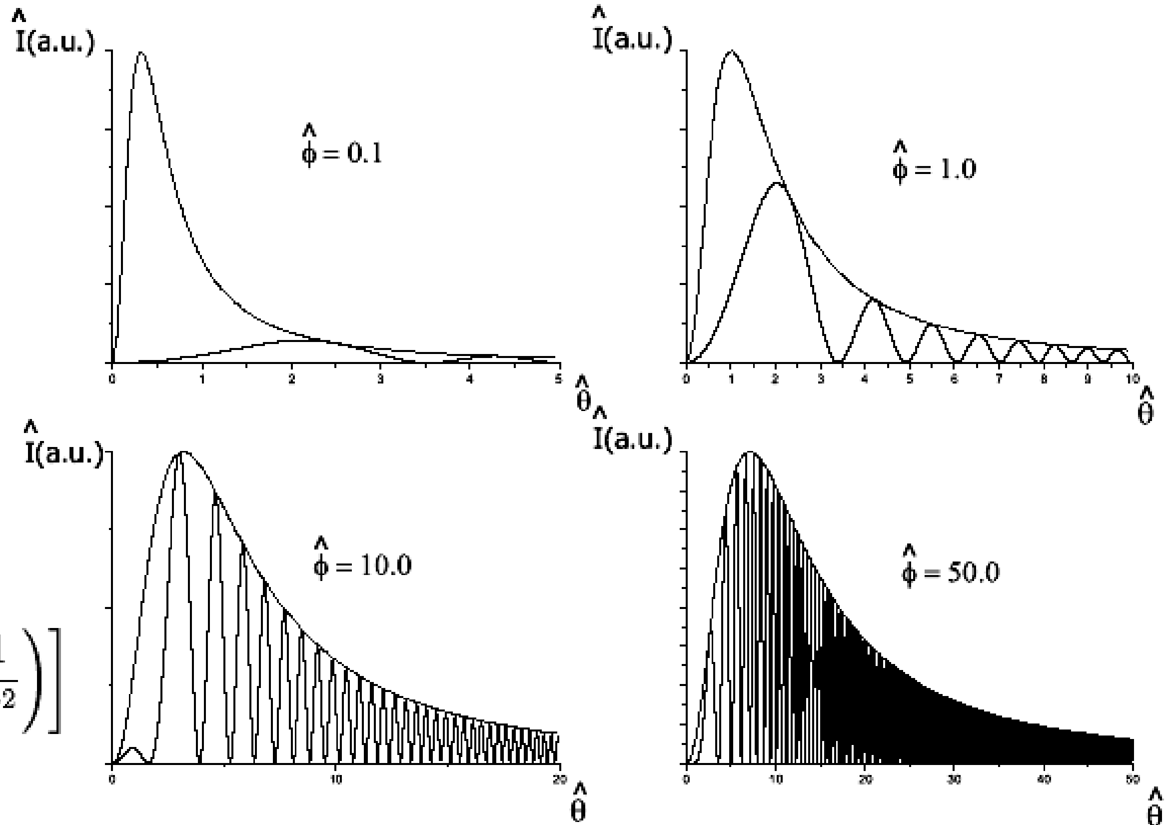
Single source - calculate the far-zone field

$$\vec{\tilde{E}}_{AB} = \frac{i\omega e}{c^2 z_o}\int_{-L/2}^{L/2} dz' \exp\left[i\Phi_{AB}\right]\left(\theta_x\vec{x}+\theta_y\vec{y}\right)$$

$$\Phi_{AB} = \omega\left[\frac{\theta_x^2+\theta_y^2}{2c}z_o + \frac{z'}{2c}\left(\frac{1}{\gamma^2}+\theta_x^2+\theta_y^2\right)\right]$$

$$\Downarrow$$

$$\vec{\tilde{E}}_{tot} = -\frac{i\omega e L}{c^2 z_o}\exp\left[\frac{i\omega\theta^2 z_o}{2c}\right]\vec{\theta}\,\mathrm{sinc}\left[\frac{\omega L}{4c}\left(\theta^2+\frac{1}{\gamma^2}\right)\right]$$



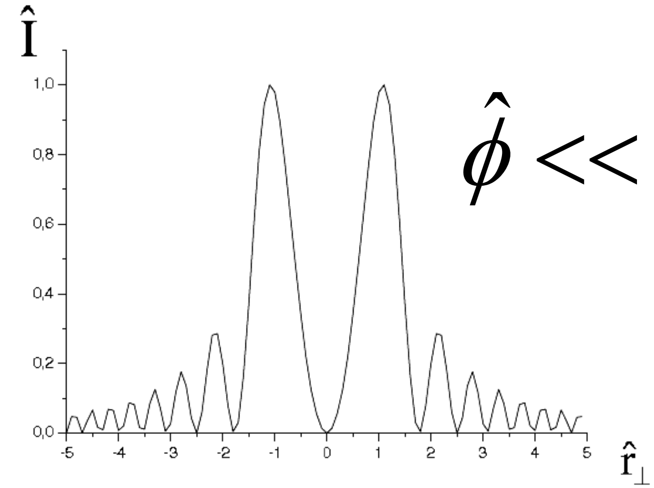
$$\rightarrow \hat{I} = \text{const.}\times\hat{\theta}^2\,\mathrm{sinc}^2\left[\frac{1}{4}\left(\hat{\theta}^2+\hat{\phi}\right)\right]$$

$$\boxed{\hat{\phi} = \frac{2\pi L}{\gamma^2\lambda}} \qquad \boxed{\vec{\hat{\theta}} = \sqrt{\frac{\omega L}{c}}\,\vec{\theta}}$$

Application 2. Edge radiation

Single source - get the virtual source:

$$\vec{\tilde{E}}(0,\vec{r}_\perp) = \frac{\omega^2 e L}{2\pi c^3}\int d\vec{\theta}\,\vec{\theta}\,\mathrm{sinc}\left[\frac{\omega L}{4c}\left(\theta^2+\frac{1}{\gamma^2}\right)\right]\exp\left[\frac{i\omega}{c}\vec{r}_\perp\cdot\vec{\theta}\right]$$



$$\hat{\phi} << 1$$

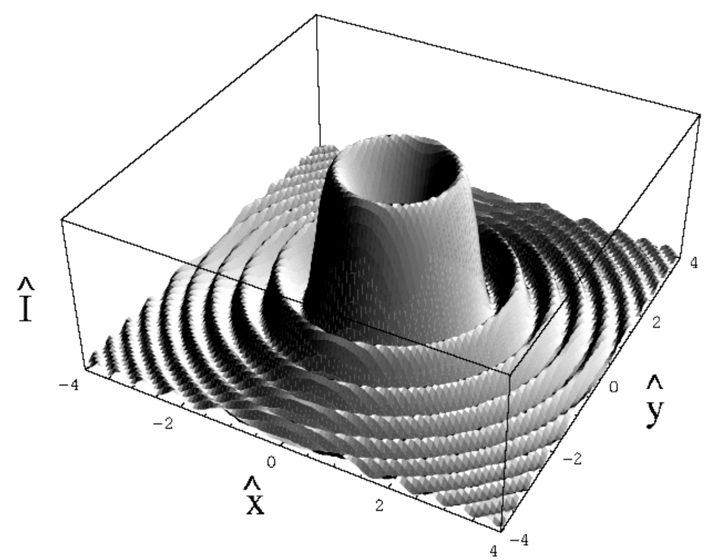
Convenient analytical expression for $\hat{\phi} = \dfrac{2\pi L}{\gamma^2\lambda} << 1$

$$\vec{\tilde{E}}(0,\vec{r}_\perp) = \frac{4\omega e}{c^2 L}\vec{r}_\perp \mathrm{sinc}\left(\frac{\omega}{Lc}\left|\vec{r}_\perp\right|^2\right)$$

$$\Downarrow$$

$$\hat{I}(\hat{r}_\perp) = \mathrm{const.}\times \hat{r}_\perp^2\,\mathrm{sinc}^2\left(\hat{r}_\perp^2\right)$$

$$\vec{\hat{r}}_\perp = \sqrt{\frac{\omega}{Lc}}\vec{r}_\perp$$



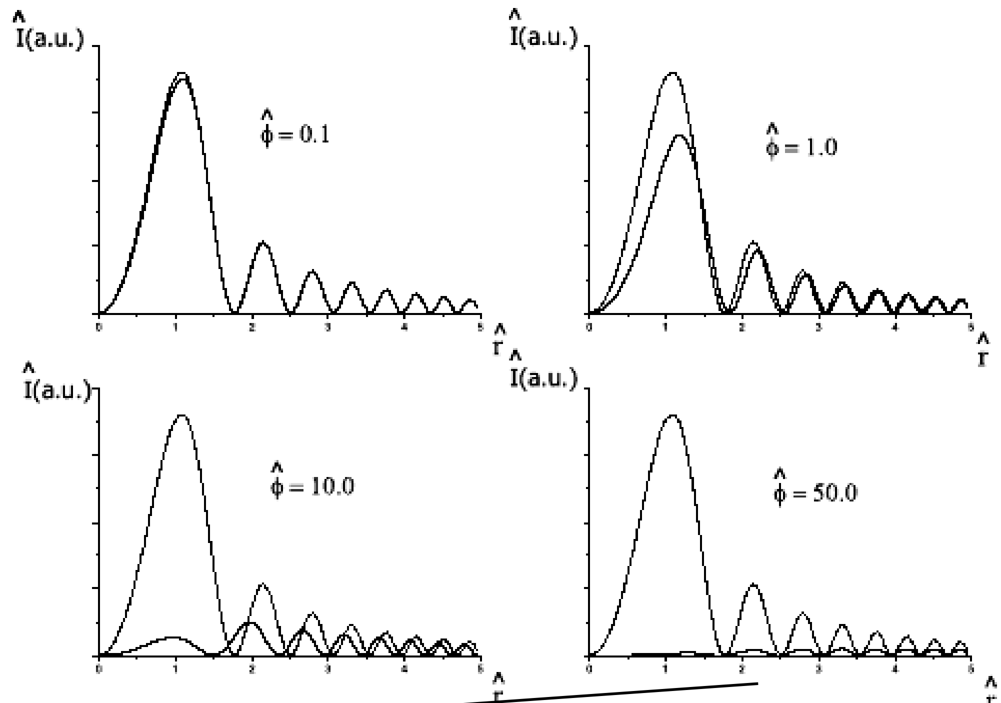
Application 2. Edge radiation

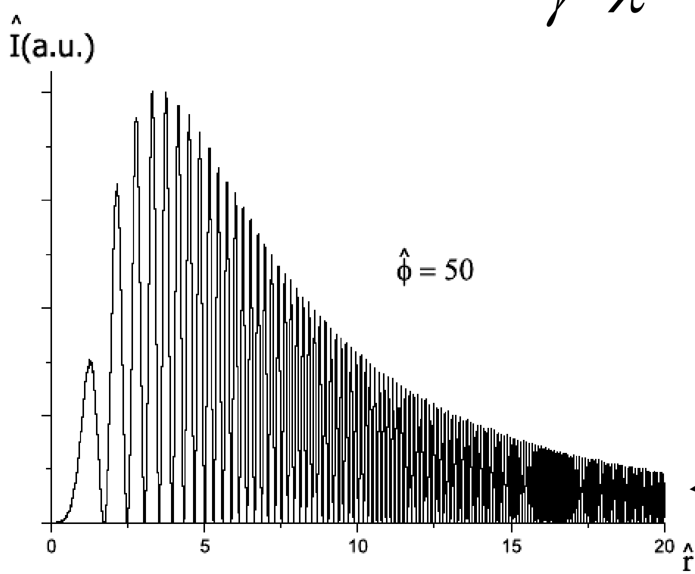
Single source - get the virtual source:

$$\vec{\bar{E}}(0, \vec{r}_\perp) = \frac{\omega^2 e L}{2\pi c^3} \int d\vec{\theta}\, \vec{\theta}\, \mathrm{sinc}\left[\frac{\omega L}{4c}\left(\theta^2 + \frac{1}{\gamma^2}\right)\right] \exp\left[\frac{i\omega}{c}\vec{r}_\perp \cdot \vec{\theta}\right]$$

How does it look like for

Other values of $\hat{\phi} = \frac{2\pi L}{\gamma^2 \lambda}$ **:**





$$\vec{\hat{r}} = \sqrt{\frac{\omega}{Lc}}\vec{r}_\perp$$

Application 2. Edge radiation

Single source – field propagation

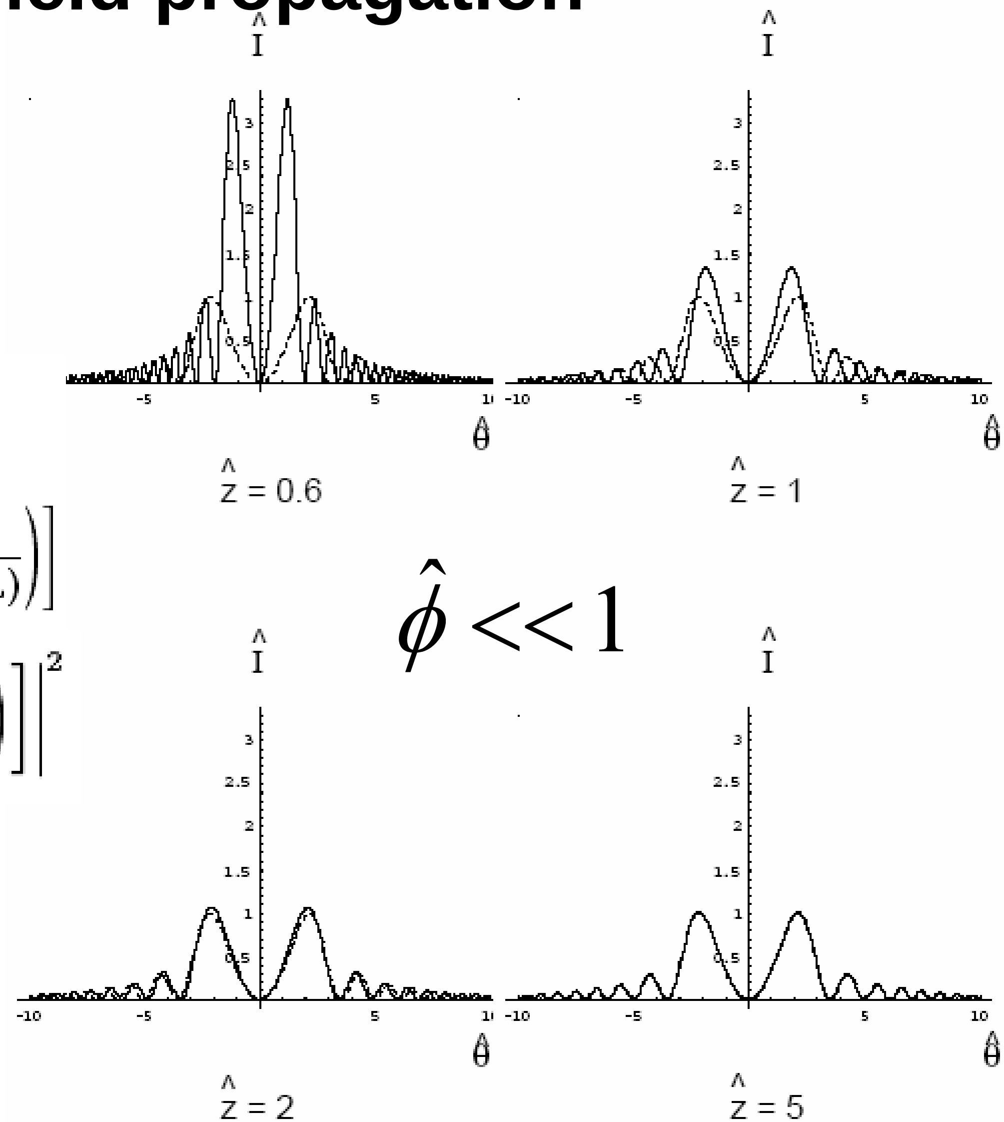
In principle, for any ϕ.
But: convenient analytical expression for $\hat{\phi} = \dfrac{2\pi L}{\gamma^2 \lambda} << 1$

$$\vec{E}(z_o, \vec{r}_\perp) = -\frac{2e}{c}\frac{\vec{r}_\perp}{r_\perp^2}\exp\left[i\frac{\omega r_\perp^2}{2cz_o}\right] \times \left[\exp\left(-\frac{i\omega r_\perp^2}{2cz_o(1+2z_o/L)}\right) - \exp\left(\frac{i\omega r_\perp^2}{2cz_o(-1+2z_o/L)}\right)\right]$$

$$\hat{I}(\hat{\theta}) = \frac{1}{\hat{\theta}^2}\left|\left[\exp\left(-\frac{i\hat{\theta}^2\hat{z}_o}{2(1+2\hat{z}_o)}\right) - \exp\left(\frac{i\hat{\theta}^2\hat{z}_o}{2(-1+2\hat{z}_o)}\right)\right]\right|^2$$

where $\hat{\theta} = \hat{r}_o/\hat{z}_o$ and $\hat{z}_o = z_o/L$.

Far zone	$\hat{z}_o >> 1$
Near zone	$\hat{z}_o \leq 1$

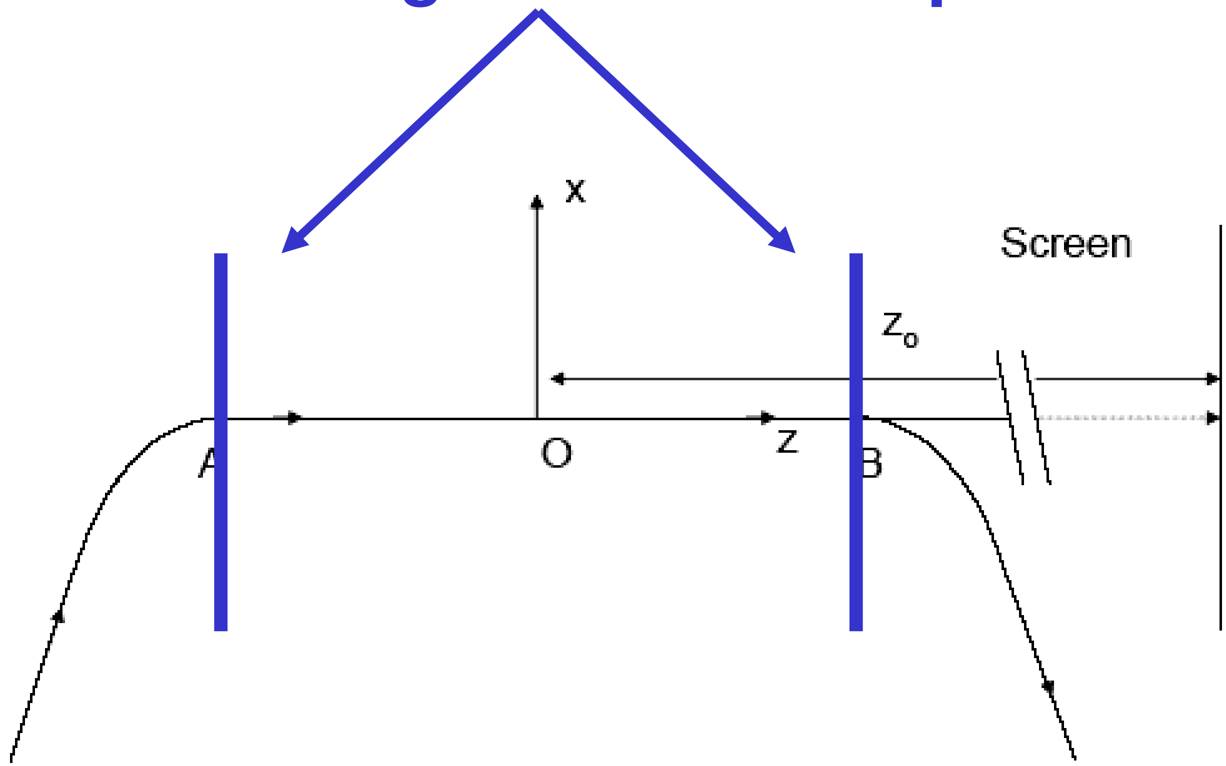


$$\hat{\phi} << 1$$

Application 2. Edge radiation

Two possibilities

Two virtual sources at the edges of the setup

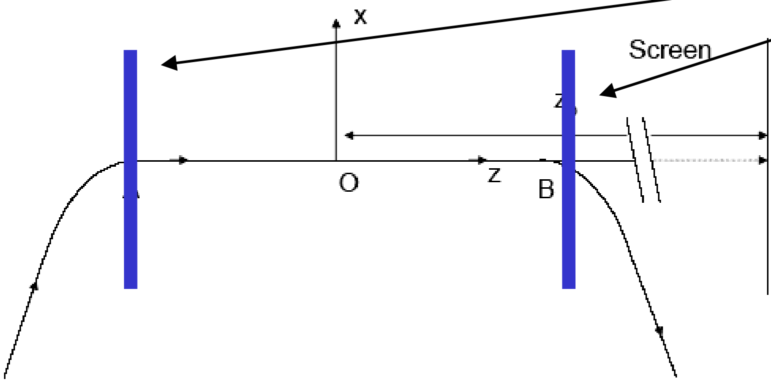


Application 2. Edge radiation

Two sources – Far zone field

$$\vec{\tilde{E}}\left(z_o,\vec{\theta}\right) = \vec{\tilde{E}}_1\left(z_o,\vec{\theta}\right) + \vec{\tilde{E}}_2\left(z_o,\vec{\theta}\right)$$

$$\vec{\tilde{E}}_{1,2}\left(\hat{z}_o,\vec{\theta}\right) = \mp\frac{2e\vec{\theta}}{cz_o(\theta^2+1/\gamma^2)}\exp\left[\pm\frac{i\omega L}{4c\gamma^2}\right]\exp\left[\frac{i\omega L\theta^2}{2c}\left(\frac{z_o}{L}\pm\frac{1}{2}\right)\right]$$



Spherical wave fronts centered at the edges (where virtual sources are)

Application 2. Edge radiation

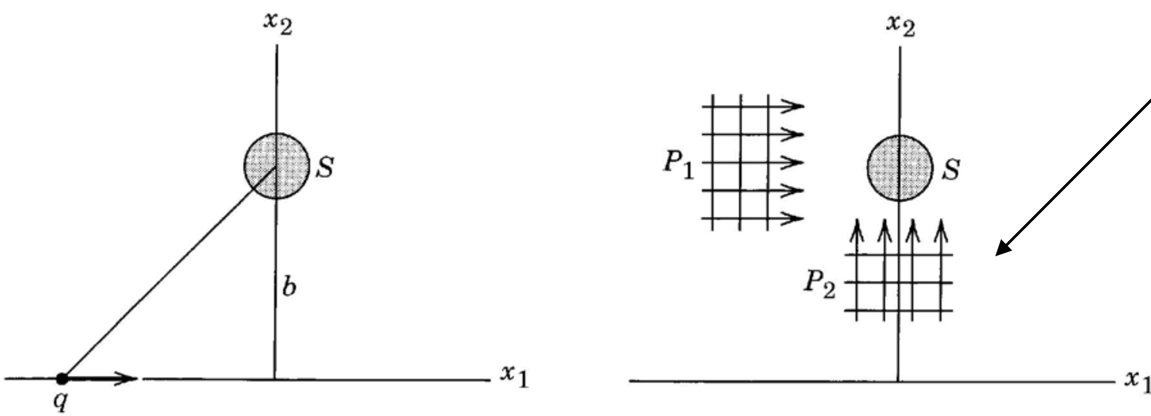
Two sources – Get the virtual sources:

$$\vec{E}\left(\pm\frac{L}{2},\vec{r}_\perp\right) = \mp\frac{2ie\omega}{c^2\gamma}\exp\left[\pm\frac{i\omega L}{4c\gamma^2}\right]\frac{\vec{r}_\perp}{r}K_1\left(\frac{\omega r_\perp}{c\gamma}\right)$$

Same result from Virtual quanta method $\left(\text{for } \hat{\varphi} = \frac{2\pi L}{\gamma^2\lambda} >> 1 \quad d << L\right)$

$$\left.\begin{matrix}\dfrac{dI_1(\omega, b)}{d\omega} \\ \dfrac{dI_2(\omega, b)}{d\omega}\end{matrix}\right\} = \frac{1}{\pi^2}\frac{q^2}{c}\left(\frac{c}{v}\right)^2\frac{1}{b^2}\left\{\begin{matrix}\left(\dfrac{\omega b}{\gamma v}\right)^2 K_1^2\left(\dfrac{\omega b}{\gamma v}\right) \\ \dfrac{1}{\gamma^2}\left(\dfrac{\omega b}{\gamma v}\right)^2 K_0^2\left(\dfrac{\omega b}{\gamma v}\right)\end{matrix}\right.$$

Transverse pulse
Out of paraxial
approximation



(From J.D. Jackson Classical Electrodynamics)

Two sources – propagate the field

Single integration for any ϕ; more useful for $\phi >> 1$

Single-edge; near zone

$$0 \ll d_1 \lesssim \gamma^2 (c/\omega)$$

Single-edge; far zone

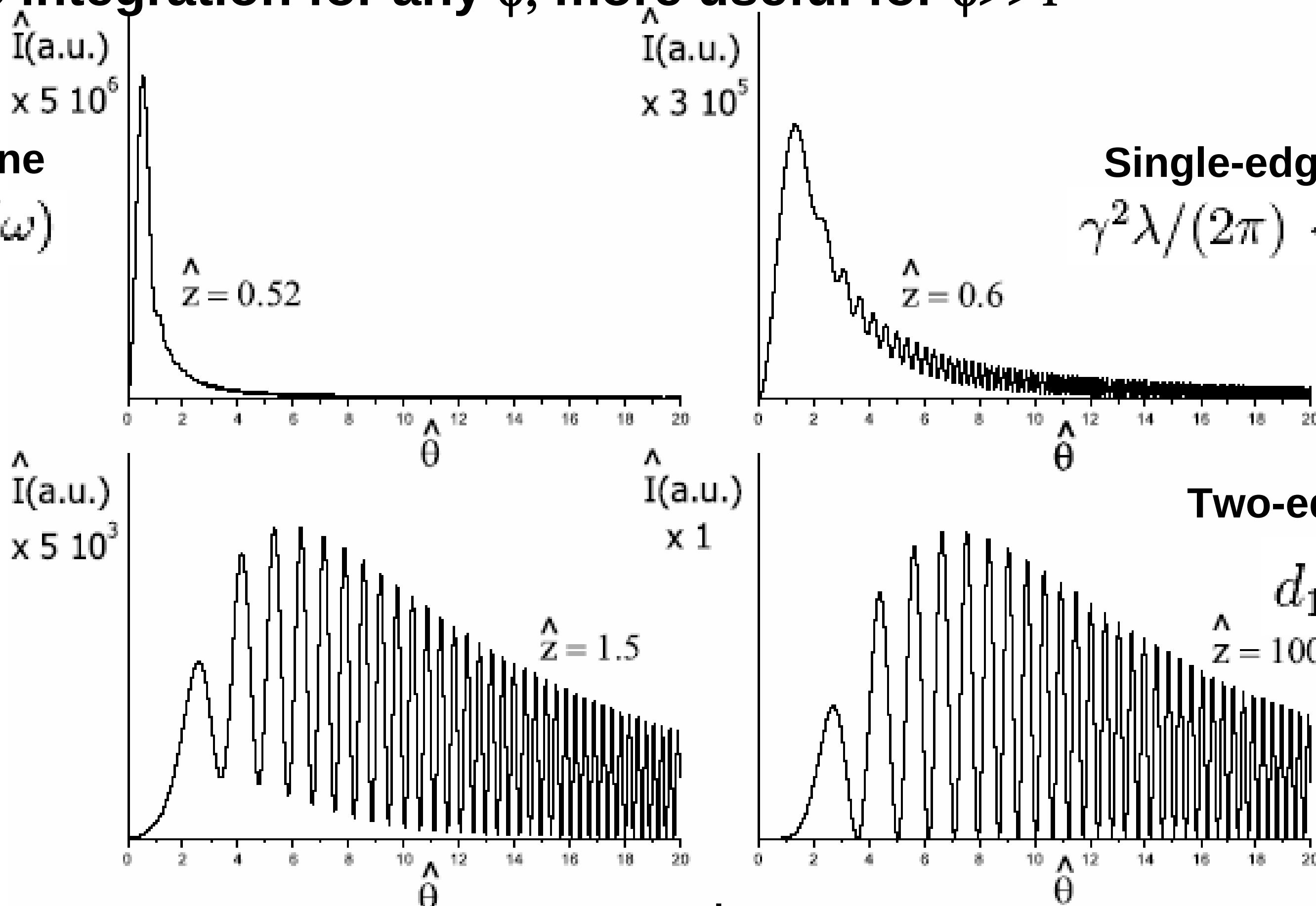
$$\gamma^2 \lambda / (2\pi) \ll d_1 \ll L$$

Two-edge; near zone

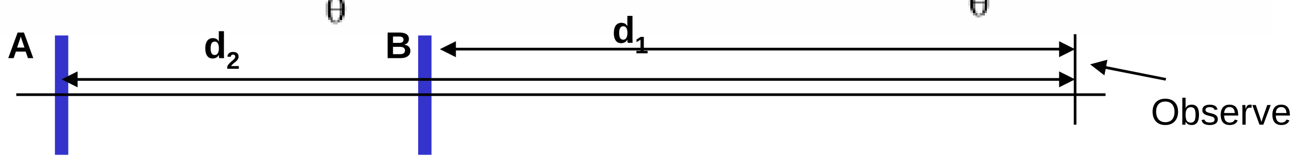
$$d_{1,2} \sim L$$

Two-edge; far zone

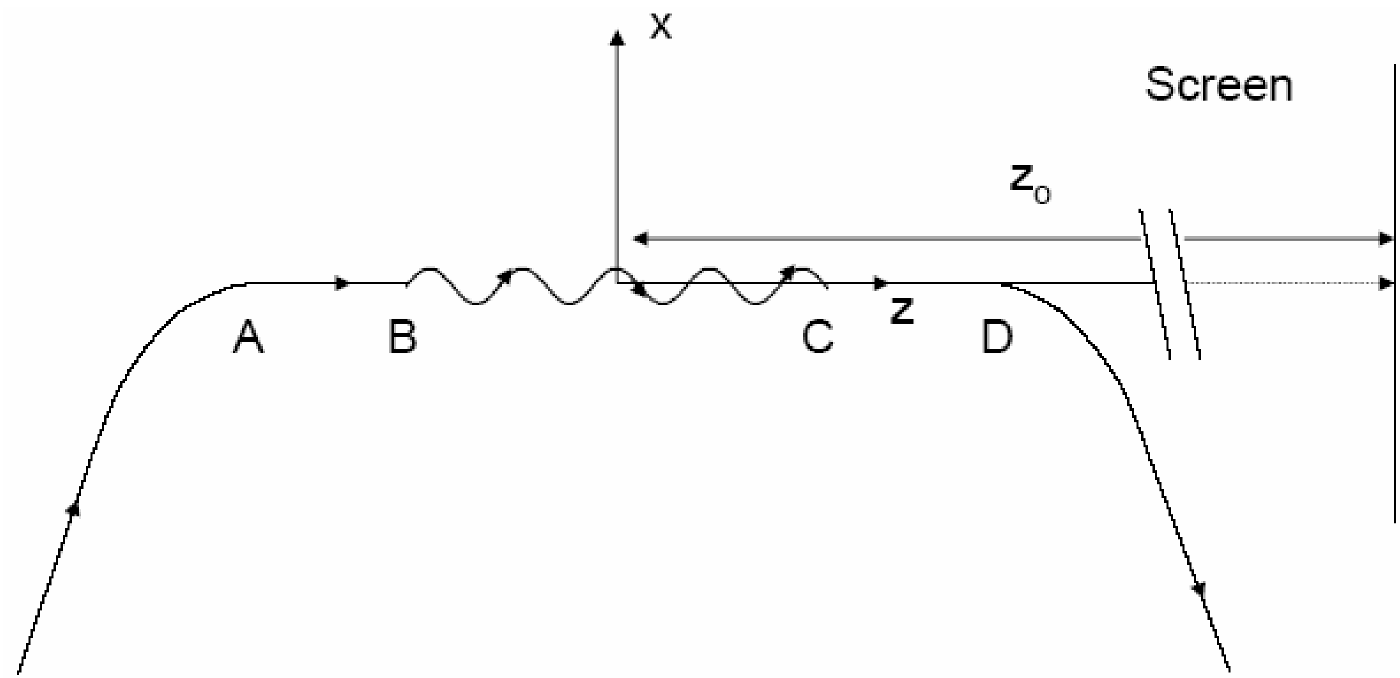
$$d_{1,2} \gg L$$



$$\hat{\phi} = 50$$



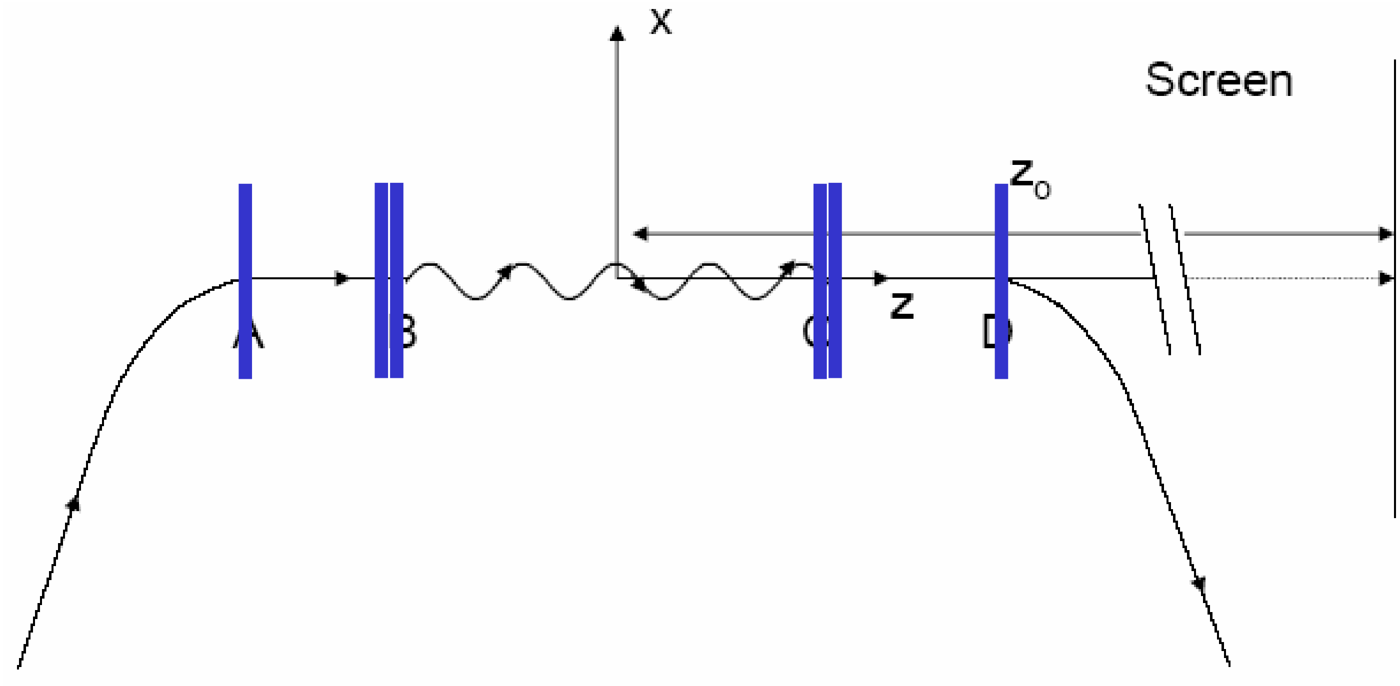
Application 3. Transition Undulator Radiation



$$\vec{\tilde{E}}_{\perp}(z_o, \vec{r}_{\perp o}, \omega) = \vec{\tilde{E}}_{b1}(z_o, \vec{r}_{\perp o}, \omega) + \vec{\tilde{E}}_{AB}(z_o, \vec{r}_{\perp o}, \omega) + \vec{\tilde{E}}_{BC}(z_o, \vec{r}_{\perp o}, \omega)$$
$$+ \vec{\tilde{E}}_{CD}(z_o, \vec{r}_{\perp o}, \omega) + \vec{\tilde{E}}_{b2}(z_o, \vec{r}_{\perp o}, \omega) \ ,$$

More sophisticated but no new physics.
Only a bigger number of virtual sources

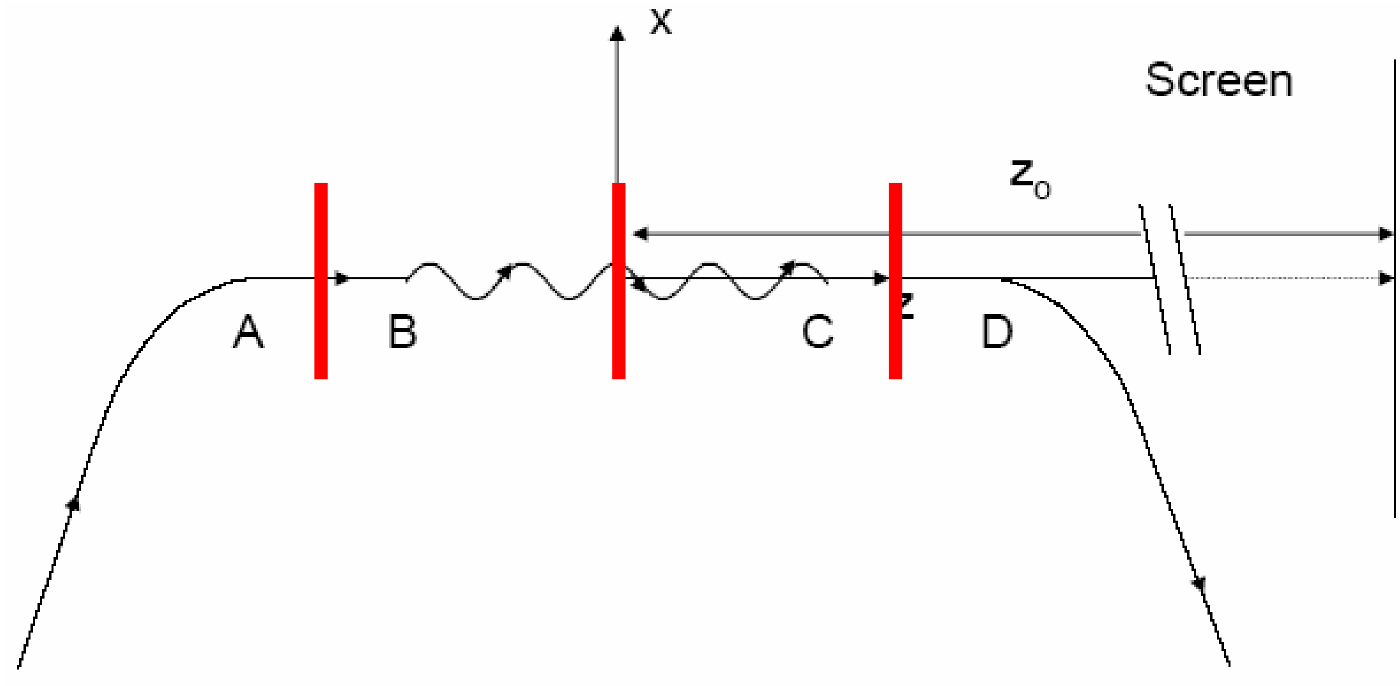
Application 3. Transition Undulator Radiation



…and different ways to organize them

$$\vec{\tilde{E}}_\perp(z_o, \vec{r}_{\perp o}, \omega) = \vec{\tilde{E}}_{AB}(z_o, \vec{r}_{\perp o}, \omega) + \vec{\tilde{E}}_{BC}(z_o, \vec{r}_{\perp o}, \omega) + \vec{\tilde{E}}_{CD}(z_o, \vec{r}_{\perp o}, \omega)$$

Application 3. Transition Undulator Radiation



…and different ways to organize them

$$\vec{\bar{E}}_\perp(z_o,\vec{r}_{\perp o},\omega) = \vec{\bar{E}}_{AB}(z_o,\vec{r}_{\perp o},\omega) + \vec{\bar{E}}_{BC}(z_o,\vec{r}_{\perp o},\omega) + \vec{\bar{E}}_{CD}(z_o,\vec{r}_{\perp o},\omega)$$

Application 3. Transition Undulator Radiation

Far zone contributions to the total field:

$$\vec{\tilde{E}}_{AB} = \frac{i\omega e}{c^2 z_o} \int_{z_A}^{z_B} dz' \exp\left[i\Phi_{AB}\right] (\theta_x \vec{x} + \theta_y \vec{y}) \qquad \Phi_{AB} = \omega \left[\frac{\theta_x^2 + \theta_y^2}{2c} z_o - \frac{L_w}{4c\gamma_z^2} + \frac{L_w}{4c\gamma^2} + \frac{z'}{2c}\left(\frac{1}{\gamma^2} + \theta_x^2 + \theta_y^2\right)\right]$$

$$\vec{\tilde{E}}_{\perp}(z_o, \vec{r_{\perp o}}, \omega) = \frac{i\omega e}{c^2 z_o} \int_{z_B}^{z_C} dz' \exp\left[i\Phi_{BC}\right] (\theta_x \vec{x} + \theta_y \vec{y}) \qquad \Phi_{BC} = \omega \left[\frac{\theta_x^2 + \theta_y^2}{2c} z_o + \frac{z'}{2c}\left(\frac{1}{\gamma_z^2} + \theta_x^2 + \theta_y^2\right)\right]$$

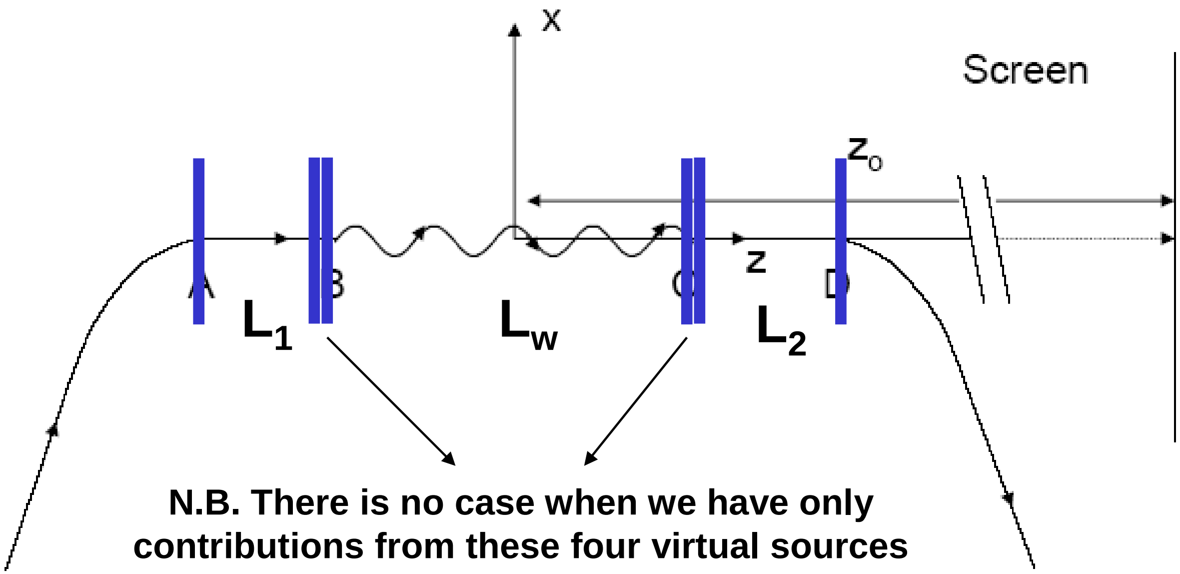
$$\vec{\tilde{E}}_{CD} = \frac{i\omega e}{c^2 z_o} \int_{z_C}^{z_D} dz' \exp\left[i\Phi_{CD}\right] (\theta_x \vec{x} + \theta_y \vec{y}) \qquad \Phi_{CD} = \omega \left[\frac{\theta_x^2 + \theta_y^2}{2c} z_o + \frac{L_w}{4c\gamma_z^2} - \frac{L_w}{4c\gamma^2} + \frac{z'}{2c}\left(\frac{1}{\gamma^2} + \theta_x^2 + \theta_y^2\right)\right]$$

Same structure as straight section Application 2.
Account for different longitudinal velocity in BC

Application 3. Transition Undulator Radiation

Far zone energy density:

$$
\begin{aligned}
\frac{dW}{d\omega d\Omega} = \frac{e^2}{\pi^2 c}\frac{\gamma^4\theta^2}{(1+\gamma^2\theta^2)^2}\Bigg| & -\exp\left[-i\frac{\omega L_w}{4c\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)\right] + \exp\left[-i\frac{\omega L_1}{2c\gamma^2}\left(1+\gamma^2\theta^2\right) - i\frac{\omega L_w}{4c\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)\right] \\
& + \frac{1/\gamma^2+\theta^2}{1/\gamma_z^2+\theta^2}\left\{-\exp\left[\frac{i\omega L_w}{4c\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)\right] + \exp\left[-\frac{i\omega L_w}{4c\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)\right]\right\} + \exp\left[i\frac{\omega L_w}{4c\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)\right] \\
& - \exp\left[i\frac{\omega L_2}{2c\gamma^2}\left(1+\gamma^2\theta^2\right) + \frac{i\omega L_w}{4c\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)\right]\Bigg|^2
\end{aligned}
$$



N.B. There is no case when we have only contributions from these four virtual sources In the expression for the photon flux!

Equivalent to Eq. (20) in R.A. Bosch, Il Nuovo Cimento 20, 4 (1998)

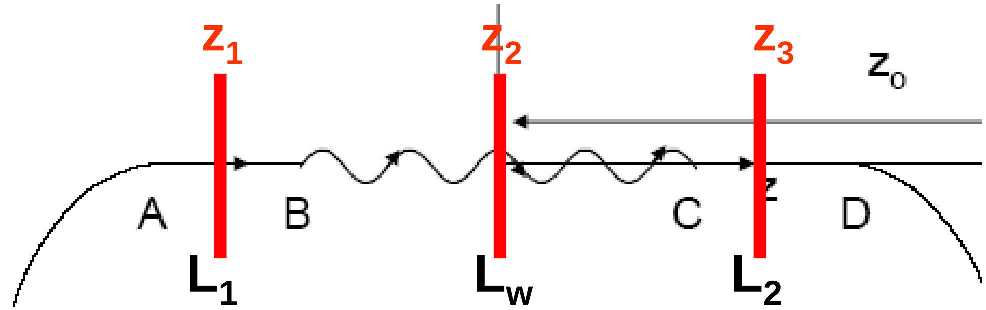
Virtual source characterization – three-source picture

$z_1 = -L_w/2-L_1/2$

$z_2=0$

$z_3=L_w/2+L_2/2$

→



I will treat only an example: $\hat{\phi} = \dfrac{2\pi L_{tot}}{\gamma^2 \lambda} << 1 \quad \hat{\phi}_w = \dfrac{2\pi L_w}{\gamma_z^2 \lambda} >> 1$

$$\vec{\hat{E}}_{AB}\left(\hat{z}_1,\vec{\hat{r}}_\perp\right) = -\hat{L}_1 \exp\left[-\frac{i}{4}\left(\hat{L}_w\hat{\phi}_w + \hat{L}_1\hat{\phi}\right)\right] \times \int d\vec{\hat{\theta}}\, \vec{\hat{\theta}}\, \mathrm{sinc}\left[\frac{\hat{L}_1}{4}\left(\hat{\theta}^2+\hat{\phi}\right)\right]\exp\left[i\vec{\hat{r}}_\perp\cdot\vec{\hat{\theta}}\right], \quad \Rightarrow \quad \vec{\hat{E}}(\hat{z}_1,\vec{\hat{r}}_\perp) = \frac{4\vec{\hat{r}}_\perp}{\hat{L}_1}\exp\left[-\frac{i\hat{L}_w\hat{\phi}_w}{4}\right]\mathrm{sinc}\left(\left|\vec{\hat{r}}_\perp\right|^2/\hat{L}_1\right)$$

$$\vec{\hat{E}}_{BC}(\hat{z}_2,\vec{\hat{r}}_\perp) = -\hat{L}_w \int d\vec{\hat{\theta}}\, \vec{\hat{\theta}}\, \mathrm{sinc}\left[\frac{\hat{L}_w}{4}\left(\hat{\theta}^2+\hat{\phi}_w\right)\right]\exp\left[i\vec{\hat{r}}_\perp\cdot\vec{\hat{\theta}}\right] \quad \Rightarrow \quad \vec{\hat{E}}(\hat{z}_2,\vec{\hat{r}}_\perp) \simeq 0$$

$$\vec{\hat{E}}_{CD}\left(\hat{z}_3,\vec{\hat{r}}_\perp\right) = -\hat{L}_2 \exp\left[\frac{i}{4}\left(\hat{L}_w\hat{\phi}_w + \hat{L}_2\hat{\phi}\right)\right] \times \int d\vec{\hat{\theta}}\, \vec{\hat{\theta}}\, \mathrm{sinc}\left[\frac{\hat{L}_2}{4}\left(\hat{\theta}^2+\hat{\phi}\right)\right]\exp\left[i\vec{\hat{r}}_\perp\cdot\vec{\hat{\theta}}\right] \quad \Rightarrow \quad \vec{\hat{E}}(\hat{z}_3,\vec{\hat{r}}_\perp) = \frac{4\vec{\hat{r}}_\perp}{\hat{L}_2}\exp\left[\frac{i\hat{L}_w\hat{\phi}_w}{4}\right]\mathrm{sinc}\left(\left|\vec{\hat{r}}_\perp\right|^2/\hat{L}_2\right)$$

Application 3. Transition Undulator Radiation

Propagation of surviving contributions:

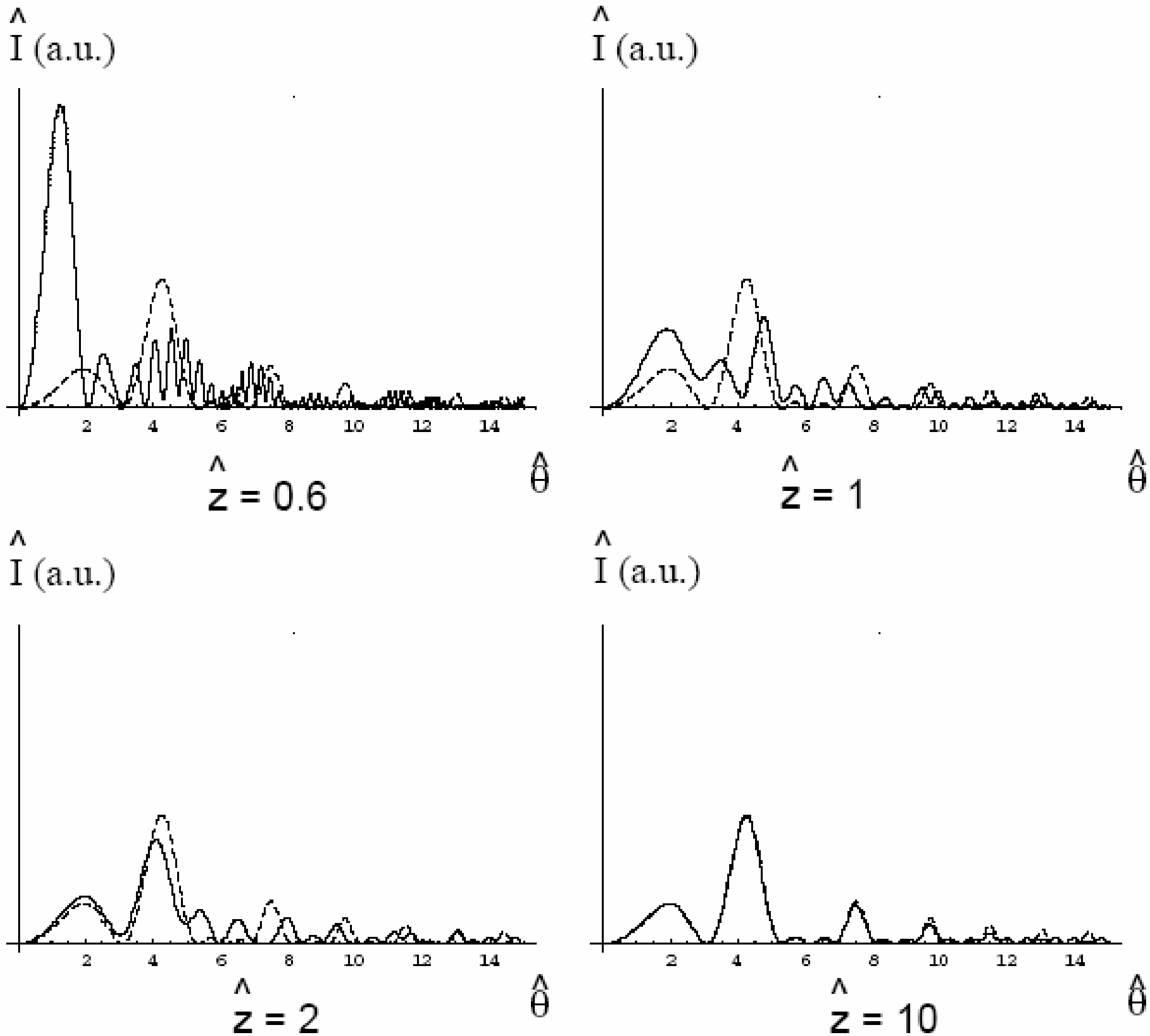
$$\vec{\hat{E}}_{AB}(\hat{z}_o,\vec{\hat{r}}_\perp) = -\frac{4\vec{\hat{r}}_\perp}{\hat{r}_\perp^2}\exp\left[-\frac{i\hat{L}_w\hat{\phi}_w}{4}\right]\exp\left[i\frac{\hat{r}_\perp^2}{2(\hat{z}_o-\hat{z}_{s1})}\right] \times\left[\exp\left(-\frac{i\hat{L}_1\hat{r}_\perp^2}{2(\hat{z}_o-\hat{z}_{s1})(\hat{L}_1+2\hat{z}_o-2\hat{z}_{s1})}\right) - \exp\left(\frac{i\hat{L}_1\hat{r}_\perp^2}{2(\hat{z}_o-\hat{z}_{s1})(-\hat{L}_1+2\hat{z}_o-2\hat{z}_{s1})}\right)\right]$$

$$\vec{\hat{E}}_{CD}(\hat{z}_o,\vec{\hat{r}}_\perp) = -\frac{4\vec{\hat{r}}_\perp}{\hat{r}_\perp^2}\exp\left[\frac{i\hat{L}_w\hat{\phi}_w}{4}\right]\exp\left[i\frac{\hat{r}_\perp^2}{2(\hat{z}_o-\hat{z}_{s3})}\right] \times\left[\exp\left(-\frac{i\hat{L}_2\hat{r}_\perp^2}{2(\hat{z}_o-\hat{z}_{s3})(\hat{L}_2+2\hat{z}_o-2\hat{z}_{s3})}\right) - \exp\left(\frac{i\hat{L}_2\hat{r}_\perp^2}{2(\hat{z}_o-\hat{z}_{s3})(-\hat{L}_2+2\hat{z}_o-2\hat{z}_{s3})}\right)\right]$$

Case $L_1 = L_2 = L_w = L$

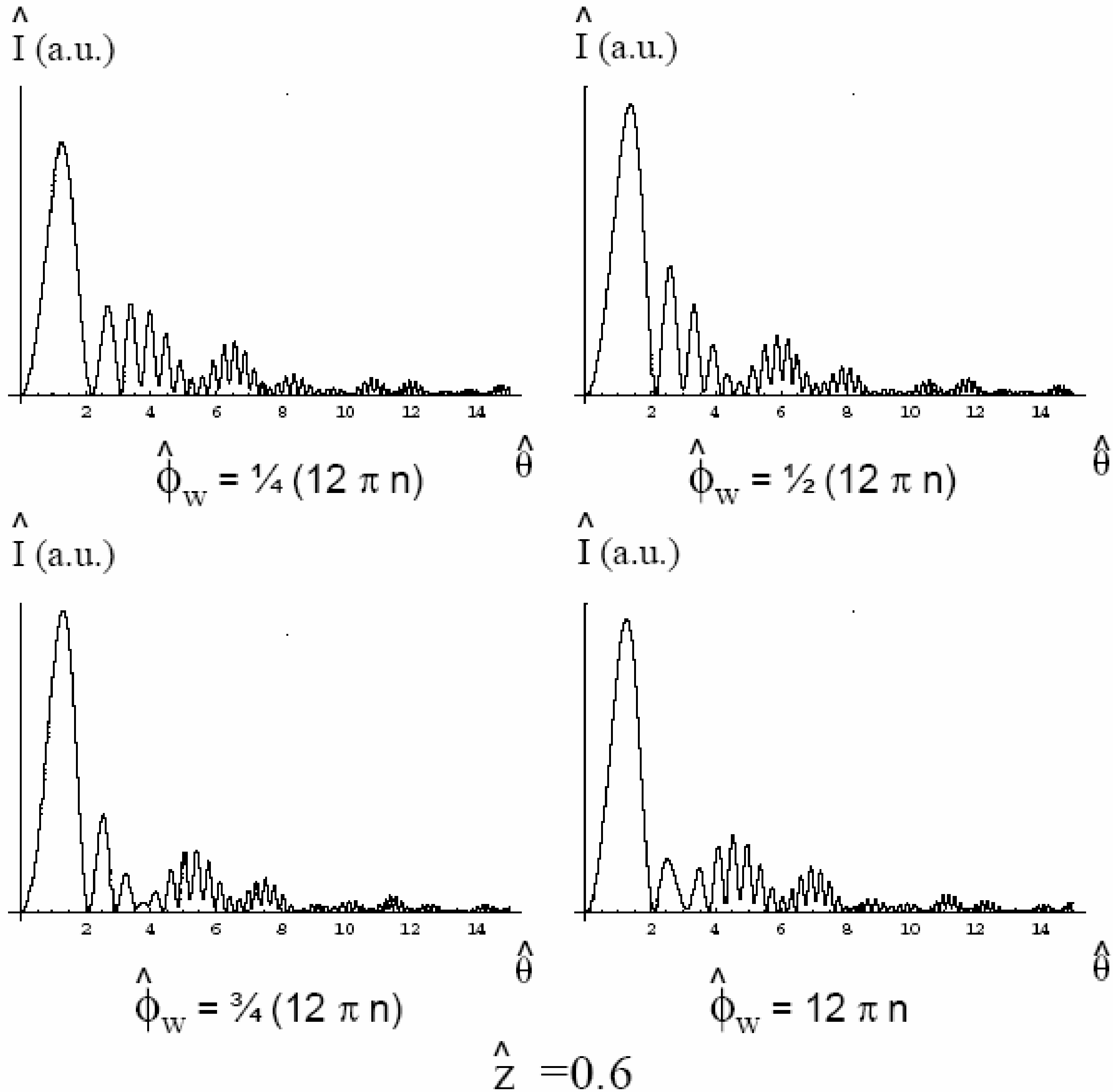
$$\hat{I}\sim\frac{1}{\hat{\theta}^2}\Bigg|\exp\left[i\frac{\hat{z}_o^2\hat{\theta}^2}{2(\hat{z}_o+1/3)}\right]\left[\exp\left(-\frac{i\hat{z}_o^2\hat{\theta}^2}{6(\hat{z}_o+1/3)(1+2\hat{z}_o)}\right)-\exp\left(\frac{i\hat{z}_o^2\hat{\theta}^2}{6(\hat{z}_o+1/3)(2\hat{z}_o+1/3)}\right)\right]\exp\left[-\frac{i\hat{\phi}_w}{12}\right] + \exp\left[i\frac{\hat{z}_o^2\hat{\theta}^2}{2(\hat{z}_o-1/3)}\right]\left[\exp\left(-\frac{i\hat{z}_o^2\hat{\theta}^2}{6(\hat{z}_o-1/3)(2\hat{z}_o-1/3)}\right)-\exp\left(\frac{i\hat{z}_o^2\hat{\theta}^2}{6(\hat{z}_o-1/3)(2\hat{z}_o-1)}\right)\right]\exp\left[\frac{i\hat{\phi}_w}{12}\right]\Bigg|^2$$

Application 3. Transition Undulator Radiation



$$\hat{\phi}_w = 12\,\pi$$

Application 3. Transition Undulator Radiation



Conclusions

➢ **We coupled F.O. and Classical Relativistic Electrodynamics**

- **Paraxial approximation**
- **Zone-classification**
- **Far-zone field**
  Source characterization
  Propagartion through Fresnel formula

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- ➢ **Application 2: edge radiation**

- •**Straight section contribution**
- •**Zero-switcher length approximation**
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- •**Single source / Two source picture**
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- **Application 3: Transition Undulator Radiation**

•Based on Application 2
•Different pictures (three/six sources)
•Analytic example discussed