STUDY ON SUPERRADIANT SMITH-PURCELL RADIATION

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Abstract

An analysis of superradiant Smith-Purcell radiation is carried out with the help of performing a three-dimensional simulation in GHz regime using a particle-in-cell code. The simulation model supposes a rectangular grating with limited length and width, to be driven by a single electron bunch, a train of periodic bunches and a continuous beam, respectively. Besides the Smith-Purcell radiation, the evanescent wave is clearly observed, which holds the frequency lower than the allowed minimum Smith-Purcell frequency. It is also shown that the superradiant radiations excited by periodic bunches are emitted at higher harmonics of the bunching frequency and at the corresponding Smith-Purcell angles. The distributions of the radiation intensity are presented and compared with a recently proposed theory. The "start current" for a continuous beam to make the device start oscillation is addressed as well.

INTRODUCTION

The superradiant Smith-Purcell (SP) radiation has attracted many attentions since Urata and co-workers observed this phenomenon in their experiments [1,2]. It is a promising alternative in the development of a compact, tuneable and high power THz sources. To better understand the physics of the superradiant SP radiation is necessary for improving the performance of such kinds of devices.

It is known that the SP radiation is emitted as an electron passes close to the surface of a periodic metallic grating [3]. The wavelength λ of the radiation observed at the angle θ measured from the direction of electron beam is given by

$$\frac{\lambda}{d} = \frac{1}{|n|}\left(\frac{1}{\beta} - \cos\theta\right), \tag{1}$$

where d is the grating period, βc the electron velocity, c the speed of light, and n the order of the reflection from the grating. The incoherent SP radiation has been analysed in many ways, such as diffraction theory, integral equation method and induced surface current model [4-8]. The experimentally observed superradiant effect is regarded as the result of the appearance of periodic electron bunches. Several theories have been proposed to reveal the physics of the superradiant phenomenon [9-13], and a three-dimensional simulation is supposed to be necessary.

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In this paper, we perform a three-dimensional particle-in-cell simulation for the coherent and superradiant SP radiation using MAGIC [14], a code for simulating processes involving interactions between space charge and electromagnetic fields.

SIMULATION MODEL

The simulation model involving a rectangular grating and a cylindrical electron beam is shown in Fig. 1, where d is the periodic length, s the groove width, h the groove depth and w the width of the grating. The main parameters are summarized in table 1, and we note that the grating period, groove width and depth and initial electron's energy are same as those in Ref. [15]. The length of the grating is set differently for particular simulation case, which will be mentioned lately. The grating, assumed to be a perfect conductor, is set in the

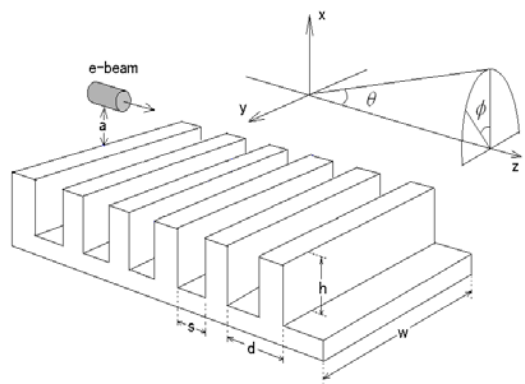

Figure 1: Three-dimensional simulation model of grating and electron beam.

centre of the bottom of a vacuum box, which is bounded by an absorption region. A perfect laminar beam produced from a cathode moves in the z-axis. The simulation area is divided into mesh with rectangular cell

Table I: Main Parameters for Simulation

Electron beam energy (injection)	E=100 keV
Beam radius	r=2.5 mm
Beam-grating distance	a=2 mm
Grating period	d=2 cm
Grating groove depth	h=1 cm
Grating groove width	s=1 cm
Grating width	w=10 cm

of very small size in the region of beam propagation and large in the rest.

SIMULATION RESULTS

Single Bunch

We first perform the simulation of a single electron bunch. The grating is arranged to be 10 periods. The bunch length is chosen as 0.1 ps with the current of 1A. It is short compared to the radiation wavelength, so the radiation is coherent. In our simulation we focus on the first order SP radiation since the high orders are not evident. Fig. 2(a) illustrates the temporal behaviour of B_y, detected at the point of $\theta = 120^o$, $\phi = 0^o$. We notice that the SP radiation pulse consists of 10 periods and is separated in time from the evanescent wave. The corresponding FFT is given in Fig. 2(b), where one can find two clear peaks. The one peaked at 6.5 GHz is the SP radiation, while the other at 4.5 GHz is the evanescent wave. Not like the SP radiation, the evanescent wave frequency is angle independent. The dependency of the SP frequency and the amplitude of B_y on the angle θ is as shown in Fig. 3, observed at the same distance 36.35 cm to the centre of the grating. For comparison, we also plot the analytical result calculated from the theory of Andrews and co-workers [16]. It is seen that the simulation data for the SP radiation frequency agrees well to the theoretical curve as θ over 90^o, somehow the discrepancy appears for the rest. Our best guess is that the detector is not far enough for far-field detection. We also see that the maximum amplitude of B_y appears at 125^o. There are slight differences between the analytical and simulation results, which might be due to the fact that we use a broad electron beam that nearly reaches the grating, whereas they use a narrow electron beam model. In Fig.4, we give the distribution of field amplitude with respect to the azimuthal angle ϕ. The observation angle cannot vary in a large range due to the limit of the simulation geometry. The distribution shows maximum at the centre point $\phi = 0$ for cases of $\theta = 130^o$ and $\theta = 80^o$, but minimum for $\theta = 50^o$.

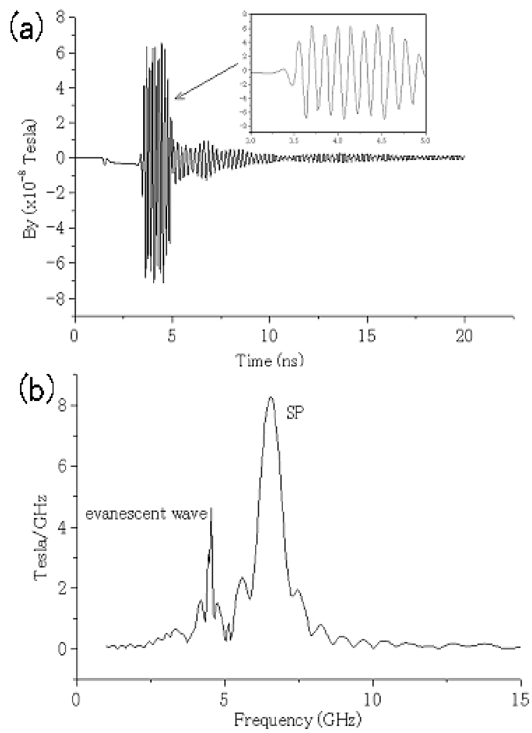

Figure 2: Time signal of B_y (a), and its corresponding FFT (b).

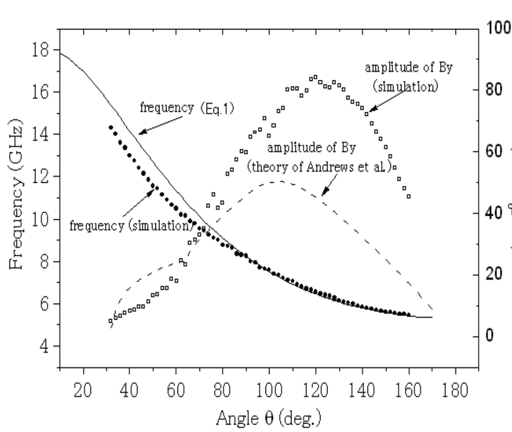

Figure 3: Distribution of SP frequency and its By amplitude.

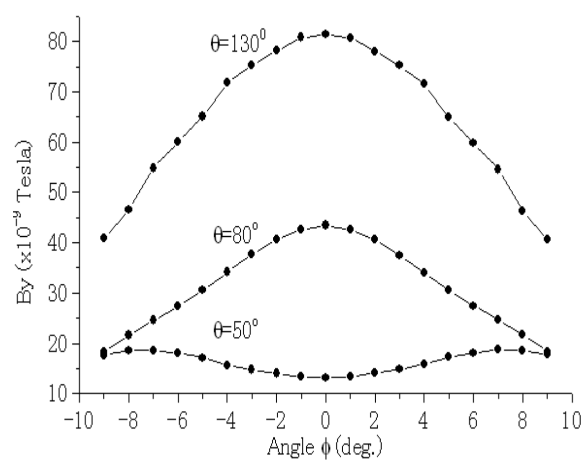

Figure 4: The dependency of By amplitude on azimuthal angle.

Periodic Bunches

In all radiation sources using an intense electron beam, the mechanism leading to superradiance is beam bunching. The spectral intensity of the radiation is enhanced at the bunching frequency and its harmonics. Recently Korbly and co-workers have carried out a SP experiment at MIT with using a pre-bunched electron beam [17]. When certain conditions are satisfied, a continuous beam can be bunched by the interaction with the evanescent wave, which has been discussed by Donohue and Gardelle [15]. In order to demonstrate the properties of superradiant radiation more clearly, we avoid the problem of bunching from an initially continuous beam. Instead, we generate a train of bunches to drive the grating.

The repetition frequency of bunches is chosen as 3 GHz, and the parameters for grating length and each single bunch are same to those mentioned earlier. Within the time of code running, 30 bunches are generated and enter the simulation area. From the FFT of the temporal behaviour observed by B_y detectors we know that the radiation focused on three frequencies, the second, third and forth harmonic of bunching frequency, as shown in

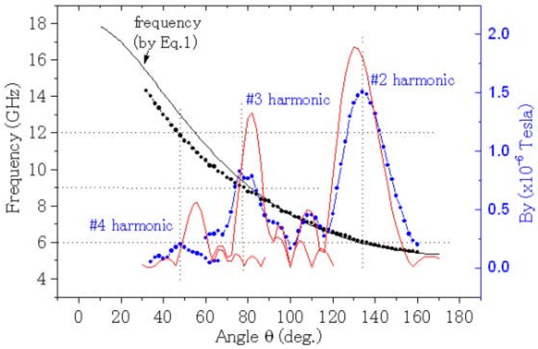

Figure 5: Distribution of superradiant SP radiation frequency and its B_y amplitude. The dotted blue line is the simulation results, and the red line the analytical result multiplied by 10.

Fig. 5. The dominant radiation is the second harmonic peaked at the angle 134°, which corresponds to Eq. 1. Also plotted In Fig. 5, is the analytical result according to Ref. [16], and we find the differences are smaller than one order of magnitude. Another evidence to show the fact that the radiations emit only at certain angles can be found in the contour plot of B_y, as shown in Fig. 6, where the second harmonic radiation is observed to radiate at the angle of about 134°, corresponding to what is shown in Fig. 5. These results strongly support the viewpoint of Andrews and co-workers.

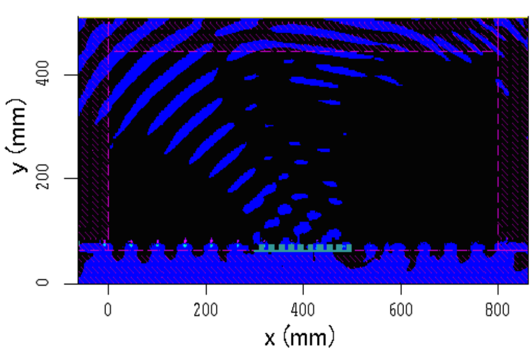

Figure 6: Contour plots of B_y.

Continuous beam

From the theoretical analysis in Ref. [15], we know that the beam line intersects the dispersion curve at a point representing a backward wave, which means the device operates in the mode of backward-wave oscillator (BWOs). Such a device is possible to start to oscillate without external feedback. When the beam current of an originally continuous beam is high enough to get the net gain, the beam will be bunched by the enhanced evanescent wave and the oscillation starts. The bunched beam consequently excites the superradiant SP radiation as discussed early. The value of the current starting the oscillation is called "start current", which needs careful analysis. In this section, we concentrate on determining the "start current" by three-dimensional simulation.

Considering the limit of the capacity of our computers, a grating consisting of 46 periods was employed in this simulation. The electron beam from the cathode is continuously generated, and an external magnetic field of 2T is introduced to prevent the beam from diverging. We vary the beam current and observe the amplitude of E_x of the evanescent wave. The appearance of exponential growth of E_x means that the oscillation happens.

The simulation results are given in Fig. 7. It is shown that, the electric field shows no growth when the current is lower than 0.4 A, while it shows evident growth as the current is above 0.5 A. And the radiation can reach saturation if the current is higher that 0.6 A. From Fig. 7, we can roughly estimate that the "start current" is ~0.5 A. The precise value can be reached if more simulations are performed.

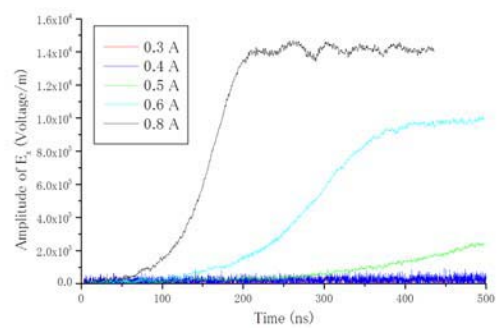

Figure 7: Evolution of amplitude of E_x.

CONCLUSION

In conclusion, we have studied the coherent and superradiant SP radiation through the three-dimensional simulation of an open grating system driven by different modes of electron beam. The single bunch simulation helps us to distinguish the true SP radiation from the evanescent wave. They are different in both frequency characteristics and generation mechanism. The amplitude of the SP radiation is angle dependent. The strongest radiation appears at 125° at the present parameters. The supperandiant effect is demonstrated with the simulation of a pre-bunched beam. We provide powerful evidence showing that the superradiant radiations are emitted at frequencies that are integer multiples of the bunching frequency, and at the corresponding SP direction. The simulation of an originally continuous beam determines the "start current" for the present parameters.

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REFERENCES

[1] J. Urata, M. Goldstein, M. F. Kimmitt, A. Naumov, C. Platt, and J. E. Walsh, Phys. Rev. Lett. **80**, 516 (1998).

[2] A. Bakhtyari, J. E. Walsh and J. H. Brownell, Phys. Rev. E **65**, 066503 (2002).

[3] S. J. Smith and E. M. Purcell, Phys. Rev. **92**, 1069 (1953).

[4] P. M. van den Berg, J. Opt. Soc. Am. **63**, 689 (1973).

[5] P. M. van den Berg and T. H. Tan, J. Opt. Soc. Am. **64**, 325 (1974).

[6] Y. Takaura and O. Haeberle, Phys. Rev. E 61,4441 (2000).

[7] J. Walsh, K. Woods and S.Yeager, Nucl. Instrum. Methods Phys. Res. A **341**, 277 (1994).

[8] Y. Shibata, S. Hasebe, K. Ishi, S. Ono, M. Ikezawa, T. Nakazato, M. Oyamada, S. Urasawa, T. Takahashi, T. Matsuyama, K. Kobayashi, and Y. Fujita, Phys. Rev. E **57**, 1061 (1998).

[9] L. Schachter and A. Ron, Phys. Rev. A **40**, 876 (1989).

[10] K. –J. Kim and S. –B. Song, Nucl. Instrum. Methods Phys. Res., sect. A **475**, 158 (2001).

[11] H. L. Andrews and C. A. Brau, Phys. Rev. ST Accel. Beams **7**, 070701 (2004).

[12] H.L.Andrews, C.H.Boulware, C.A.Brau and J.D.Jarvis, Phys. Rev. ST Accel. Beams **8**, 050703 (2005).

[13] V. Kumar and K.-J. Kim, Phys. Rev. E 73, 026501 (2006).

[14] D. Smithe and T.M. Abu-Elfadi, Comput. Phys. Commun. 106, **95** (1997).

[15] J.T.Donohue and J.Gardelle, Phys. Rev. ST-AB, **8**, 060702 (2005).

[16] H.L.Andrews, C.H.Boulware, C.A.Brau and J.D.Jarvis, Phys. Rev. ST Accel. Beams **8**, 110720 (2005).

[17] S.E.Korbly, A.S. Kesar, J.R.Sirigiri and R.J.Temkin, Phys. Rev. Lett. **94**, 054803, (2005).