# **FEL-OSCILLATOR SIMULATIONS WITH GENESIS 1.3**

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## Abstract

Modeling free-electron laser (FEL) oscillators requires calculation of both the light-beam interaction within the undulator and the propagation of the light outside the undulator. We present a paraxial Optical Propagation Code (OPC) based on the Spectral Method and Fresnel Diffraction Integral, which in combination with Genesis 1.3 can be used to perform either steady-state or time-dependent FEL oscillator simulations. A flexible scripting interface is used both to describe the optical resonator and to control the codes for propagation and amplification. OPC enables modeling of complex resonator designs that may include hard-edge elements (apertures) or hole-coupled mirrors with arbitrary shapes. Some capabilities of OPC are illustrated using the FELIX system as an example.

## **INTRODUCTION**

Free-electron laser (FEL) oscillators are complex devices. They require simulation of both the amplification of the radiation field within the undulator and the propagation of the radiation field through the resonator, to correctly predict the spectral and spatial properties of the output of the laser. These properties are very important for the design of user experiments.

To date, several codes exist that can simulate an FEL amplifier or oscillator (see, e.g., [1]). However, the wellestablished Genesis 1.3 FEL code [2] is primarily used for FEL amplifier or SASE simulations. In this paper we present an optical propagation code (OPC) that works together with Genesis 1.3 to simulate FEL resonators. The full functionality of Genesis 1.3 is maintained and simulation of FEL oscillators can be done both in steady-state and time-dependent modes.

To propagate the radiation field between optical and gain elements, we have implemented three related paraxial methods: the Spectral Method [3], the Fresnel Diffraction Integral Method [4] and a modified Fresnel Diffraction Integral Method. The latter is based on the normal Fresnel Diffraction Integral, however, it includes the ABCD matrix of the optical system between input and output plane [5]. The radiation field produced by Genesis 1.3 at the undulator's exit is propagated using one of these methods to the first optical element. Then the action of that particular optical element is applied to the wave and one of the propagation methods is again used to propagate the wave to the next optical element. This procedure is repeated until the undulator's entrance is reached. Note that propagation through a cascaded set of optical elements can be done in a single step if this set can be represented by a single overall ABCD matrix and using the Modified Fresnel Diffraction Integral for the propagation.

We have chosen to separate the optical propagation model from the FEL simulation model for two main reasons. First, the optical propagation model can be used with different gain models and is, in principle, not limited to FELs. Second, the propagation model can then also be used to propagate the field outside the resonator and determine the field distribution, for example, in the far field or in a user area that can be located at a considerable distance from the laser in the case of FELs. The optical propagation code is available for download [10].

In the remainder of this paper, we first describe briefly the different propagation methods, then the OPC code and end with an example illustrating the capabilities of the combination of the OPC code with Genesis 1.3.

## PARAXIAL OPTICAL PROPAGATION

By applying a Fourier transform over the transverse coordinates, the paraxial wave equation can be written as [5]:

$$(k_x^2 + k_y^2)\tilde{u} - 2ik\frac{\partial\tilde{u}}{\partial z} = 0, \qquad (1)$$

where

$$\tilde{u}(k_x, k_y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y, z) e^{-i(k_x x + k_y y)} \mathrm{d}x \mathrm{d}y \quad (2)$$

is the Fourier transform of the complex wave amplitude u(x, y, z). Given a known optical field  $u_0(x, y)$  at z = 0, the optical field after a propagation over a distance z can be obtained from eq. 1:

$$\tilde{u}(k_x, k_y, z) = \tilde{u}_0(k_x, k_y) e^{-i\frac{2z}{k}(k_x^2 + k_y^2)}.$$
(3)

The propagation method described by eq. 3 is known as the Spectral Method and consists of applying the spatial Fourier transform eq. (2) to the input plane, propagate the field over a distance z (eq. 3) and applying the Inverse Fourier Transform to  $\tilde{u}(k_x, k_y, z)$ . This Inverse Fourier Transform is similar to eq. 2 with the roles of  $\tilde{u}$ ,  $k_x$  and  $k_y$  interchanged with those of u, x and y, respectively, and a factor  $1/4\pi^2$  added.

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It can be shown that paraxial wave propagation over a distance z = L through a cascaded optical system can be realized in a single step using the elements of the overall ABCD matrix in Huygens' Integral for propagation [5]. If we use the following transform to remove the spherical portion of the wave at the input plane [5]:

$$v_0(\xi',\eta') \equiv \sqrt{a_1 a_3} u_0(\xi,\eta) e^{-i\frac{\pi(A_x - M_x)\xi^2}{B_x \lambda}} e^{-i\frac{\pi(A_y - M_y)\eta^2}{B_y \lambda}}$$
(4)

and at the output plane:

$$v(x',y') \equiv \sqrt{a_2 a_4} u(x,y,L) e^{+i\frac{\pi (D_x - M_x^{-1})x^2}{B_x \lambda}} e^{+i\frac{\pi (D_y - M_y^{-1})y^2}{B_y \lambda}},$$
 (5)

where  $x' = a_1 x$ ,  $\xi' = a_2 \xi$ ,  $y' = a_3 x$ ,  $\eta' = a_4 \eta$ , and  $A_{x(y)} \dots D_{x(y)}$  are the ABCD matrix coefficients for the x- and y direction respectively, then Huygens' Integral can be written as a modified Fresnel Diffraction Integral:

$$v(x',y') = i\sqrt{N_{c,x}N_{c,y}} \times \int_{-1}^{1} \int_{-1}^{1} K(x',y',\xi',\eta')v_0(\xi',\eta')d\xi'd\eta', \quad (6)$$

where the kernel K is given by

$$K(x',y',\xi',\eta') = e^{-i\pi N_{c,x}(x'-\xi')^2} e^{-i\pi N_{c,y}(y'-\eta')^2},$$
(7)

and the equivalent collimated Fresnel numbers  $N_{c,x(y)}$  are given by

$$N_{c,x(y)} = \frac{M_{x(y)}a_{1(3)}^2}{B_{x(y)}\lambda}.$$
(8)

The arbitrary scaling factors  $a_{1..4}$  in eqs. 4-5 define two magnification factors  $M_{x(y)} = a_{2(4)}/a_{1(3)}$ , if they correspond to either the size of a hard aperture or a size sufficiently large that the field is just negligible outside the area covered by that size. Note that for free-space propagation A = D = 1, C = 0 and B = L = z, and using  $M_x = M_y = 1$ , reduces eqs. 6-8 to the normal Fresnel Diffraction Integral.

Both the Spectral and the Modified Fresnel propagation methods are implemented using Fast Fourier Transforms and therefor their computation time scales as  $N^2 log_2(N^2)$ for a  $N \times N$  grid. For an equal grid, the Spectral method is the faster of the two because it requires less operations [6]. However, care has to be taken with the Spectral Method that the field remains zero at the border of the grid to avoid artificial reflections. The Modified Fresnel Diffraction Integral has the advantage that propagation through an optical system, described be a single overall ABCD matrix, is obtained in a single step. Another advantage is that the scaling applied to this method allows a magnification factor for the grid so that the mesh size in the input plane does not have to be the same as the mesh size in the output plane.



Figure 1: Flowchart of the simulation loop.

#### **IMPLEMENTATION**

Simulation of an FEL oscillator requires simulation of both the gain within the undulator and the propagation of the radiation through the remaining part of the resonator, that may contain multiple optical components. Genesis 1.3 [2] is used to propagate the optical wave through the undulator and calculate the amplification of the wave. The OPC receives the optical output from Genesis 1.3, and propagates it from optical element to optical element using one of the methods described above, until the undulator's entrance is reached and the next round trip can start. If a cascaded set of optical elements can be described by a single ABCD matrix, then propagation through the cascaded set can be done in a single step using the Modified Fresnel Diffraction Integral. Output can be produced at various positions, and by the optical elements, which is of advantage for beam diagnostics and for designing suitable optics for a further propagation of the output beam.

While the actual simulation algorithms are written in FORTRAN, the Perl scripting language is used to both control the program flow and define the resonator geometry. Fig. 1 shows schematically the program flow. The advantage of using a script to control the program instead of a config file is that it gives the user a lot of freedom to create complex models. Table 1 gives an example of a script for simulation of the FELIX system [7].

This example script contains all essential elements needed to simulate the FELIX system. The first three lines are headers needed to load a library. Then two objects are created, one that corresponds to the genesis program (1.5) and one that corresponds to the optical propagation code (1.6). The genesis object requires the standard configuration file for Genesis 1.3 as input. A Perl script function is provided that gives the user complete control over the parameters in the configuration file. This allows, for example, the use of a Gaussian seed for Genesis 1.3 in the first round trip and the resonator feedback as input for Genesis 1.3 in consecutive round trips.

To define the optical propagation through the resonator, the optics object (1.6) accepts a simple script that describes the propagation methods and optical components used (1.7-16). The available propagation methods are described above, and the optical components so far include diaphragms, square apertures, curved mirrors and thin Table 1: Example script for simulating the FELIX FEL oscillator.

```
01:
      #!/usr/bin/perl
02:
      use lib './lib';
03:
      use Physics::OPC;
04:
      $genesis = genesis('./felix.itdp.in');
05:
      $optics = optics( "
06:
07:
         fresnel z=1.062
08:
         mirror r=3 R=98% # Downstream mirror
         fresnel z=5.89
09:
10:
         hole
               r=0.0015 ( # Out-coupling
11:
             dump var=output
12.
             fresnel z=1
                            # Far field distance
13:
             dump var=far )
14:
         mirror r=4 R=98% # Upstream mirror
15:
         dump var=reflected
         fresnel z=2.358");
16:
17:
18:
      for $i (1 .. 75) {
         run $genesis;
19:
         run $optics;
20:
21:
         move $output => "output.$i.dfl";
         move $far
                       => "far.$i.dfl";
22:
         move $reflected => "reflected.$i.dfl";
23:
         move $field
                       => "entrance.$i.dfl";
24:
25:
      }
```

lenses. Each optical component is defined by a set of parameters, for example the downstream mirror (1.8) for FELIX has a curvature ('r') of 3 m and a reflectivity ('R') of 98 %. Other possible parameters for a mirror are absorption ('A'), transmittance ('T'), offset ('xoff' and 'yoff'), and a small tilt ('xr' and 'yr'). The optical propagation starts with propagation over a distance z=1.062 m from the undulator's exit to the downstream mirror using the Fresnel Diffraction Integral (1.7). Then the downstream mirror is defined (1.8). After the downstream mirror the wave is propagated to the upstream mirror, again using the Fresnel Diffraction Integral (1.9). In FELIX, the output beam is extracted through a hole in the upstream mirror. In the model the "hole" component (1.10) is processed before the actual mirror component (1.14). The hole component creates two waves, one that is transmitted through the hole, and one that corresponds to the part outside the hole that remains within the resonator. In this case, the transmitted beam corresponds to the radiation extracted from the resonator. The output beam is propagated over a distance sufficiently large to be in the far field (1.11-13). At 1.14 the script continues with the part outside the hole and this is reflected by the upstream mirror. The combination of the "hole" and "mirror" commands represent the upstream mirror with the hole. The resonator is closed by propagation from the upstream mirror to the undulator's entrance (1.16).



Figure 2: Radiation intensity just after reflection on the upstream mirror (A) and just before the undulator's entrance (B) as a function of the horizontal cross-section (at y=0) and the position within the optical micro pulse s after 75 round trips and for  $\Delta L=-1.5\lambda$ .

The "dump" commands indicate positions where output is produced for further analysis.

The last part of the Perl script is the actual simulation loop where we run the two programs consecutively and save binary dumps (1.21-34) for each round trip at varies positions defined in the optical configuration. The binary files used by the propagation code have the same format as the field files produced by Genesis 1.3, so existing analysis tools can be re-used. Utilities to extract plain text data from these files are included with the OPC code. Using plotting tools such as Gnuplot, the user can create all kinds of views to analyze the optical pulse.

In steady-state mode Genesis 1.3 produces one field distribution, i.e., a single slice, as output, which is then processed by the propagation code. Genesis 1.3 can also run in time-dependent mode, which includes slippage effects. In this mode Genesis 1.3 produces a set of slices representing the radiation pulse as output. In time-dependent mode, the propagation code will process all slices produced by Genesis 1.3 consecutively. Each slice is propagated using the center wavelength of the optical pulse, which is valid only for a narrow bandwidth of the optical pulse. The slowly varying amplitude and phase approximation used by Genesis 1.3 limits the maximum bandwidth of the pulse. Furthermore, an analysis by Dattoli et.al. showed that the relative bandwidth is limit to about  $10^{-3}$  to  $10^{-2}$  for typical parameters of VUV, UV and IR FELs [8].

### **EXAMPLE**

The script shown in Table 1 is used to illustrate some of the capabilities of the OPC code. The script describes the resonator for the FELIX system [7]. The script produces four binary dump files for each iteration (that is, for each round trip). The first gives the field distribution just outside the outcouple mirror (1.21), 1.23 gives the field distribution just after reflection at the upstream mirror containing the hole for extraction of the radiation, and 1.24 gives the distribution at the entrance of the undulator. The far-field distribution (1.22) will not be used here. It should be noted that the same script is used for both steady-state and timedependent simulations, the only changes required are made in the genesis configuration file.

We performed a time-dependent simulation of the FELIX system. The horizontal cross-section (at y=0) of the radiation intensity just after reflection on the upstream mirror and just before the undulator's entrance are shown in Fig. 2 as a function the horizontal position x and the position s within the optical micro pulse after 75 round trips and for a cavity detuning  $\Delta L$ =-1.5 $\lambda$ . This figure clearly shows the effect of the hole in the mirror (Fig. 2A) on the radiation profile and how this profile has changed when propagated to the undulator's entrance (Fig. 2B), that shows a near maximum on-axis intensity. Note that the broad radiation profile at the undulator's entrance may be clipped by the electron beam transport tube. Although not present in the current script, this can be included by adding a diaphragm component just after propagation to the undulator's entrance (1.16) in Table 1.

If we integrate the intensity over the cross-section of the optical pulse, we obtain the micro-pulse optical power as a function of the position s within the micro-pulse. This is shown in Fig. 3 as a function of the round-trip number. The oscillation in the micro-pulse optical power as a function of time (=round trip #) is known as the so called limit-cycle oscillations [9]. It is more clearly visible in the total micro-pulse energy that is shown in Fig. 4 for three different values of the cavity detuning  $\Delta L$ .

#### CONCLUSION

We have developed an Optical Propagation Code that propagates an arbitrary radiation wave, in the paraxial approximation, through a complex optical system. In combination with a code to simulate the gain medium, such as Genesis 1.3 in case of FELs, this optical code can be used to model the output of a laser oscillator. The use of a flexible scripting interface allows a user to create both simple and complex resonator configurations. The OPC package is available for download at our website[10].

#### REFERENCES

 S. Biedron, Y. Chae, R. Dejus, B. Faatz, H. Freund, S. Milton, H.-D. Nuhn, and S. Reiche, Nucl. Instrum. Methods

Output power (W)



Figure 3: Micro pulse optical power as a function of both the position *s* within the pulse and the round trip number for a detuning  $\Delta L$ =-1.5 $\lambda$ .



Figure 4: Total energy of the micro pulse as a function of round-trip number for a detuning  $\Delta L$  of  $-0.5\lambda$ ,  $-1\lambda$  and  $-1.5\lambda$  respectively.

Phys. Res. A, Accel. Spectrom. Detect. Assoc. Equip. A445, 110 (2000).

- [2] S. Reiche, *pbpl.physics.ucla.edu/ reiche*.
- [3] E. Sziklas and A. Siegman, Appl. Opt. 14, 1874 (1975).
- [4] H.A. Hause, "Waves and fields in optoelectronics," 1984, Prentice-Hall, Englewood Cliffs.
- [5] A.E. Siegman, "Lasers," 1986, University Science Books, Mill Valley.
- [6] I.V. Volokhine, "Design and Numerical Analysis of TUE-FEL II", Ph.D. thesis, 2003 University of Twente.
- [7] D. Oepts, A. van der Meer, and P. van Amersfoort, Infrared Phys. Technol. 36, 297 (1995).
- [8] G. Dattoli, H. Fang, L. Giannessi, M. Richetta, A. Torre, and R. Caloi, Nucl. Instrum. Methods Phys. Res. A, Accel. Spectrom. Detect. Assoc. Equip. A285, 108 (1989).
- [9] D. Jaroszynski, R.J. Bakker, D. Oepts, A.F.G. van der Meer, and P.W. van Amersfoort, Nucl. Instrum. Methods Phys. Res. A, Accel. Spectrom. Detect. Assoc. Equip. A331, 52 (1993).
- [10] OPC design team, *lf.tnw.utwente.nl/opc.html*.