DYNAMICS CONTROL OF THE ELETTRA STORAGE RING FREE-ELECTRON LASER WITH DIGITAL FEEDBACKS

E. Allaria#, G. De Ninno, Sincrotrone Trieste, 34012 Trieste, Italy
A. Antoniazzi, D. Fanelli, Dept. of Energetics, University of Florence, Florence Italy.

Abstract

The laser dynamics of a storage-ring free-electron laser has two main sources of instabilities. First of all, dynamical instabilities are developed as the free electron laser is moved away from the exact tuning between the period of the electron bunch(es) circulating into the ring and that of the photon pulse stored in the optical cavity. In addition, external (low-frequency) noise sources have a strong influence on the dynamical behavior of the system and can perturb its dynamics. Different feedback techniques have been proposed in order to control dynamical instabilities and stabilize the laser output. We present here a numerical and experimental investigation on the control of the Elettra storage ring free electron laser dynamics using different feedbacks techniques that can be experimentally implemented by means of a Field Programmable Gate Array.

INTRODUCTION

In a storage ring free electron laser (SRFEL) the electron bunch interacts with photons when passing through the optical klystron (Fig.1). The photons are stored in an optical cavity characterized by a traveling time ΔT and bounded by the two mirrors. The electron bunch, circulating in the storage ring, is characterized by the revolution period ΔT+ε which is determined by the storage ring radiofrequency.

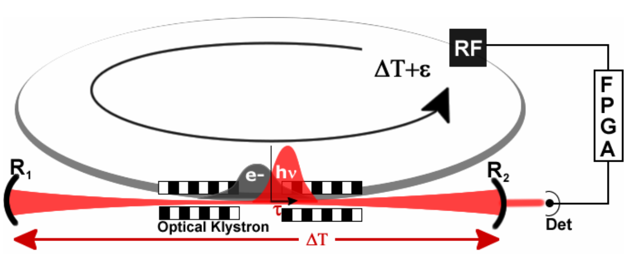

Figure 1: Layout of the storage-ring free-electron laser. The photon pulse (red) stored in the optical cavity interacts with the electron bunch (gray), circulating in the storage ring. The light intensity is acquired using an optical detector (Det). The resulting signal can be elaborated by an FPGA and used for slightly changing the electron revolution period (from ΔT to ΔT+ε) by varying the phase of the radio-frequency cavity (RF) of the ring.

Due to the impulsive character of the laser medium the laser intensity of a SRFEL is characterized by a sequence of micropulses whose duration is of the order of tens of picoseconds. Moreover its repetition rate is that of bunches in the ring (some MHz).

On a "slow" time scale (ms), the SRFEL behavior is strongly related to the temporal superposition of the photon and electron bunches inside the optical klystron. More precisely, the laser envelope displays a steady state regime for a perfect electron-photon tuning (ε=0 in Fig.1). Small light-electron detuning is sufficient to induce intensity oscillations on a slow time scale (Fig.2).

Due to these instabilities the quality of the laser temporal evolution is usually rather poor. Besides temporal detuning, the environmental noise (which is usually related to a residual 50Hz modulation coming from the power network) also perturbs the system and can strongly affect the SRFEL dynamics (Fig.2,5).

A simple model based on a recurrence map [1,2] can be used in order to describe the slow time-scale evolution of a SRFEL:

$$I_j(\tau) = R^2 I_{j-1}(\tau-\varepsilon)\cdot[1+g_{j-1}(\tau)] + i_s(\tau) \quad (1)$$

$$g_j(\tau) = g_0 \frac{\sigma_0}{\sigma_j} e^{\left(-\frac{\sigma_j^2-\sigma_0^2}{2\sigma_0^2}\right)} e^{\left(-\frac{\tau^2}{2\sigma_{t,j}}\right)} \quad (2)$$

$$\sigma_{j+1}^2 = \sigma_j^2 + \frac{2\Delta T}{\tau_s}\left(\gamma I_j + \sigma_0^2 - \sigma_j^2\right) \quad (3)$$

$$\varepsilon = \varepsilon + F(t) \quad (4)$$

$$F(t) = A\cdot\left[\frac{dI(t)}{dt}\right] \quad (5)$$

$$F(t) = A_1\cdot[I(t) - I(t-T_{d1})] + A_2\cdot[I(t) - I(t-T_{d2})] \quad (6)$$

Eq.1 describes the laser intensity at the j^{th} passage where τ is the temporal position with respect to the centroid of the electron bunch, R is the cavity mirror reflectivity, i_s stands for the spontaneous emission of the optical klystron, ε accounts for the detuning. The gain $g_j(\tau)$ is described by Eq.2 where g_0 and σ_0 respectively stand for the initial peak gain and energy spread respectively, $\sigma_{t,j}$ is the bunch length of the j^{th} interaction. The energy spread σ_j is described by Eq.3 where γ is the difference between equilibrium and initial energy spread, I_j the normalized laser intensity, ΔT the revolution period of electrons on the ring and τ_s the synchrotron damping time. For a more exhaustive description of the model we refer to Ref. [3].

Eqs.5,6 refers to the control signal *F(t)* for the case of derivative and delayed feedbacks. Those signals are used to modulate the detuning (ε) according to Eq. (4) where $t = j\Delta T$.

#enrico.allaria@elettra.trieste.it

Table 1: Parameters used for the simulations of the Elettra SRFEL.

$\Omega = 16 \quad kHz$	$\sigma_{\tau,j} = \frac{\alpha}{\Omega}\sigma_j$	$R = R_1 \cdot R_2 = 0.96$
$\Delta T = 216 \quad ns$	$\alpha = 1.6 \cdot 10^{-3}$	$\gamma = \sigma_e^2 - \sigma_0^2 \quad \sigma_e/\sigma_0 \approx 1.3$
$i_s = 4.3 \cdot 10^{-7}$	$\tau_s = 86 \quad ms$	$\varepsilon = \varepsilon_0 + \delta\varepsilon \cdot \sin(2\pi\nu t) \quad \nu = 50Hz$

The above model provides an ideal setting to investigate possible strategies for the stabilization of the SRFEL dynamics. The signal proportional to the laser output intensity extracted from the optical detector (Det in Fig.1) can be instrumental to the control of the system through dedicated feedback algorithms (Eqs. 4-6). More precisely, this signal can be used *ad hoc* to modify the electron revolution period (from ΔT to ΔT+ε) through the RF cavity of the ring (Fig.1).

Detuning and noise effects on SRFEL dynamics

By using the above model one can investigate the role of both the detuning and the external periodic noise modulation on the SRFEL dynamics.

Here we numerically simulate the Elettra SRFEL by using the model (Eq.1-3) assuming the values reported in Tab.1. We first assume an ideal case and neglect the external noise modulation at 50 Hz. In order to characterize the effect of a simple detuning ε_0 on the SRFEL dynamics, the bifurcations diagram of the laser output intensity (*I*) *vs* the detuning value (ε_0) has been reconstructed. Bifurcation diagrams are obtained by plotting the values of the laser intensity maxima and minima when dynamically varying the value of the photon-bunch detuning ε_0. In order to free our results from hysteresis effect we perform the scan both increasing and decreasing the values of ε_0.

Fig.2 (black curve) displays the results for the case of a non modulated detuning ε_0.

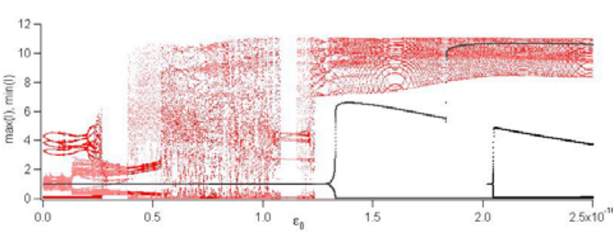

Figure 2: Bifurcation diagram of SRFEL intensity with respect of the detuning parameter ε_0. Black curve refers to the case without external perturbation of the detuning parameter ($\delta\varepsilon$ = 0, see table 1). Red points refers to the case where also an external periodic perturbation is present ($\delta\varepsilon$ = 0.18fs).

Results clearly show the presence of a steady state regime up to a detuning value of about 0.12fs where the transition to a pulsed regime occurs. Starting from ε_0 = 0.12fs, the laser is characterized by high-intensity short pulses followed by long periods where the laser is off. If the detuning is further increased, the peak of the laser intensity decreases up to a second transition point (ε_0 = 0.18fs). For larger detuning values the SRFEL achieves again a stable regime which falls outside the region considered in the subsequent analysis [4].

Results are instead different if one accounts for the presence of the external periodic noise signal. In Fig.2 (red curve) we report the bifurcation diagram of the SRFEL intensity *vs* the detuning value (ε_0) for a choice of the parameters which corresponds to the case of the Elettra SRFEL, i.e. $\delta\varepsilon_0$ = 0.18fs [3]. The used value for the noise strength is sufficient to destroy the initial (i.e. small detuning) steady state region and the bifurcation diagram now shows a cascade of transitions between periodic and chaotic behaviors. However, depending on the detuning value, there exists regions where the laser is always turned on.

In the following we shall consider the Elettra SRFEL to be represented by the model of Eq.s1-3 with ε_0 and $\delta\varepsilon$ respectively equal to 0.5fs and 0.18fs [10].

CONTROL ALGORITHMS

The sensitivity of the system to the detuning ε can be exploited to implement a feedback system. A signal proportional to the laser can be used as an input in a feedback loop in order to control the system. To this aim an appropriate change to the electron revolution period is applied through the RF cavity of the ring (Fig.1, Eqs.1-6).

Recently, encouraging experimental results have been reported for a derivative feedback based on a low-pass filter [5]. In the near future we plan to implement a more sophisticated feedback system, exploiting the intrinsic flexibility of a FPGA to design innovative control algorithm. In the following we shall provide a first theoretical insight into this issue by comparing a digital derivative feedback (Fig.3,4) and a digital delayed feedback.

Derivative feedback

A derivative feedback enables to reduce the chaotic oscillations of the SRFEL dynamics.

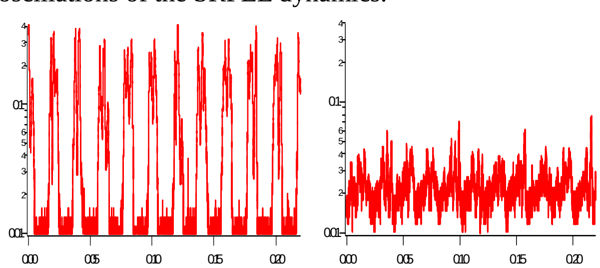

Figure 3: Time trace of the Elettra SRFEL showing the effect of the digital derivative feedback. a) Unstable behavior of the Elettra SRFEL. b) Controlled regime.

As appear evident from inspection of the experimental data reported in Fig.3, the digital derivative feedback control is able to prevent the laser to turn off. However, a small residual modulation at 50Hz is still present together with higher frequencies spurious contribution. In figure 4 we characterize the transition from the unstable pulsed regime to the controlled one as a function of the strength of the control loop.

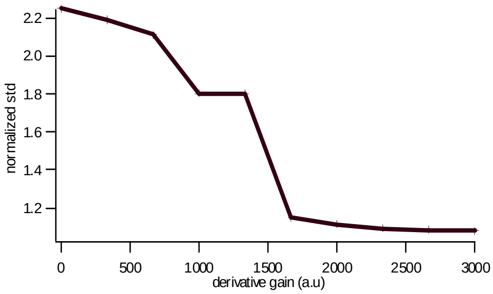

Figure 4: Transition from uncontrolled (Fig.3a) to controlled (Fig.3b) regimes: the normalized standard deviation of the SRFEL signal is reported as a function of the strength of the control signal.

One of the limitations of the derivative feedback is the fact that the sign of the controlling signal necessary for the stabilization of the SRFEL depends on the sign of the detuning [3]. For that reason in cases where the detuning (ε_0) is smaller or comparable to the perturbation of the external noise signal ($\delta\varepsilon$), a strong control signal cannot be employed. Otherwise, the control signal can induce detuning with the wrong sign and move the system away from the stability.

Delayed feedback

Delay control feedback can avoid the aforementioned problem because it involves a low correlation between the values of the laser intensities used for the calculation of the control signal.

Such a method consists in applying to the system a control signal *F(t)* described by Eq. (6): the loop gain A and the delay times $T_{d1,2}$ are the parameters to be set in order to stabilize the laser evolution.

Delayed feedbacks have been originally proposed for the stabilization of unstable periodic orbits of chaotic oscillators [6]. Recently two different incommensurable delays has been proposed to be used to obtain the stabilization of a steady state [7]. We further showed the possibility of using such a strategy in a SRFEL [8]. We are here interested in testing the robustness of the method to small fluctuations of SRFEL and/or algorithm parameters.

Figure 5 analyzes the performances of the method as a function of the two delay times. Results clearly show the existence of a region (blue) where the standard deviation of the signal has been strongly reduced (<0.5) thus pointing to the stabilization of the SRFEL signal. It is important to emphasize that those regions are located out from the diagonal which in turn enables one to conclude that at least two delays are necessary for the method to effectively work.

The possibility, and advantages, of using additional delay times should be addressed [9]. The robustness of the proposed method has been verified with respect of the variation of several input parameters.

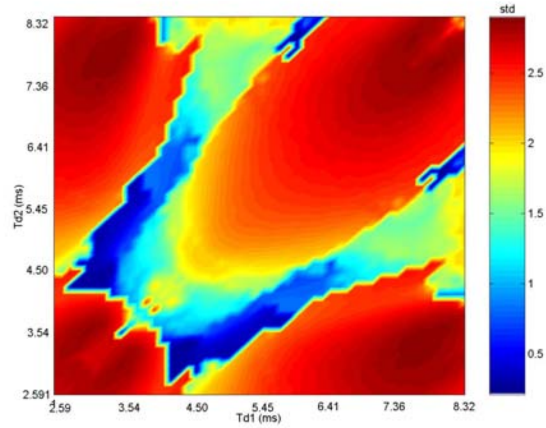

Figure 5: Color-scale plot of the standard deviation of the SRFEL output as a function of the delays used in the two delay lines of the control algorithm (A=1.9e-6). The plots clearly show the advantage of using two delays with respect to one (diagonal).

Numerical simulations indicate that the stable region (blue in Fig.4) is maintained when the input parameters (loop gain A, noise frequency and amplitude, electron-photons detuning …) are changed, as one would expect to occur in experimental conditions. This is a crucial observation in view of possible experimental realizations.

COMPARISON BETWEEN DERIVATIVE AND DELAYED FEEDBACK

In order to compare the performance of the delayed control algorithm and the derivative one we numerically tested both methods as a function of the strength of the external noise signal.

Figure 6 show the behavior of the SRFEL with ε_0=0.05fs as a function of the noise strength ($\delta\varepsilon$). The pulsed chaotic dynamics, which is usual for the Elettra SRFEL, is evident from the large range of fluctuation of the laser maxima for $\delta\varepsilon$ in the range 0.15-0.20fs.

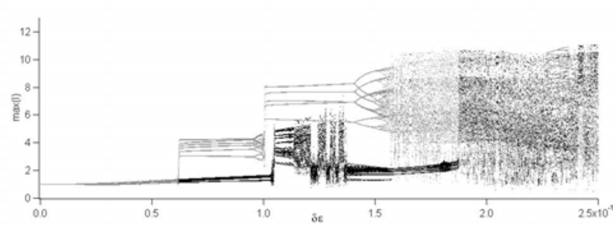

Figure 6: Bifurcation diagram of the SRFEL intensity as a function of the external noise modulation strength ($\delta\varepsilon$).

A proper setting of the derivative feedback [3] allows to stabilize the dynamics of the SRFEL in the region of $\delta\varepsilon \in$ (0.15-0.20fs) which is characteristic of the Elettra SRFEL. However, for larger values of $\delta\varepsilon$ the dynamics remains chaotic (Fig.7).

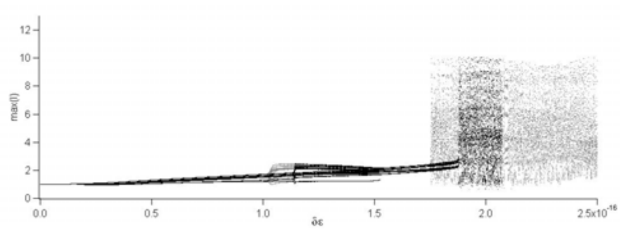

Figure 7: Bifurcation diagram of the SRFEL intensity with the use of the derivative feedback set for the stabilization (A=3.0e-2) of the unstable oscillations around the $\delta\varepsilon$ = 0.18fs case.

As clearly shown in figures 6,7,8 a dual delay algorithm results in a better stabilization of the SRFEL dynamics over a larger range of $\delta\varepsilon$.

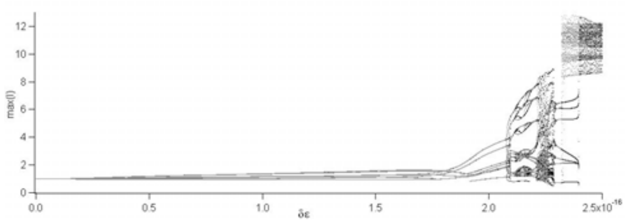

Figure 8: Bifurcation diagram of the SRFEL intensity with the use of the dual delay feedback set for the stabilization (A=1.9e-6, T_{d1}= 3.11ms, T_{d2}=4.24 ms) of the unstable oscillations around the $\delta\varepsilon$ = 0.18fs case.

Although the ultimate goal of obtaining a perfect steady state regime is beyond current possibilities, both methods are capable to stabilize the laser intensity for a large window of values of the noise strength $\delta\varepsilon$, a crucial quantity responsible for undesidered oscillations arising in the uncontrolled case (Fig. 6). A comparison between the proposed two methods in terms of extension of the allowed range of the noise strengthshow that the approach based on the multidelay performs better. This conclusion applies also as concerns the amplitude of the residual oscillations.

As previously anticipated, the reason for the above success is to be ascribed to the fact that the low correlation between delayed values in the case of chaotic signal allows us to implement a strong control term without facing the risk of producing opposite effects, reported instead for the case of the derivative feedback.

CONCLUSIONS

We presented a reliable model for the investigation of both detuned and noisy regimes in a SRFEL. The model has been applied to testing possible feedback algorithm to be developed with a FPGA. Preliminary experimental results have been reported concerning the stabilization of the Elettra SRFEL trough a digital derivative feedback. A proposed feedback method based on delayed signal has been presented and numerically investigated. The comparison of the delayed method with the derivative one shows the advantages of the former in terms of achieved stability and robustness to noise.

On the basis of these encouraging results, the experimental implementation of the delayed feedback control on the Elettra SRFEL is planned for the near future.

REFERENCES

[1] M. Billardon *et al.*, Phys. Rev. Lett. **69**, 2368 (1992).

[2] S. Bielawski *et al.*, Phys. Rev. **A**, **47**, 3276 (1993).

[3] G. De Ninno *et al.*, Phys. Rev. E **71**, 066504 (2005)

[4] M.E. Couprie *et al.*, Phys. Rev. E. **53**, 1871 (1996).

[5] S. Bielawski *et al.*, Phys. Rev. **E**, **69**, 045502 (2004).

[6] K. Pyragas, Physics Letters A, **170**, 421-428 (1992).

[7] A. Ahlborn and U. Parlitz, Phys. Rev. Lett. **93**, 264101 (2004).

[8] E. Allaria *et al.*, FEL05, JACoW/eConf C0508213, THPP005 (2005)

[9] Work in preparation.

[10] In the case of Elettra a detuning of 0.4fs correspond to a variation of 1Hz of the 500MHz RF frequency which is the limit of the accuracy of the instrument. Moreover to the time jitter of the master oscillator, that depending of the working conditions can be of the order of 1ps, can be associated a detuning of 0.02fs if we simplify the jitter to a 50Hz signal.