TRANSVERSE AND LONGITUDINAL COHERENCE PROPERTIES OF THE RADIATION FROM X-RAY SASE FELS

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Abstract

We present a comprehensive analysis of coherence properties of the radiation from X-ray free electron laser (XFEL). We consider practically important case when XFEL is optimized for maximum gain. Such an optimization allows to reduce significantly parameter space. Application of similarity techniques to the results of numerical simulations allows to present all output characteristics of the optimized XFEL as functions of the only parameter, ratio of the emittance to the radiation wavelength, $\hat{\epsilon} = 2\pi\epsilon/\lambda$. our studies show that optimum performance of the XFEL in terms of transverse coherence is achieved at the value of the parameter $\hat{\epsilon}$ of about unity. At smaller values of $\hat{\epsilon}$ the degree of transverse coherence is reduced due to strong influence of poor longitudinal coherence on a transverse one. At large values of the emittance the degree of transverse coherence degrades due to poor mode selection. Comparative analysis of existing XFEL projects, European XFEL, LCLS, and SCSS is presented as well.

INTRODUCTION

In the X-ray FEL the radiation is produced by the electron beam during single-pass of the undulator [1]. The amplification process starts from the shot noise in the electron beam. Any random fluctuations in the beam current correspond to an intensity modulation of the beam current at all frequencies simultaneously. When the electron beam enters the undulator, the presence of the beam modulation at frequencies close to the resonance frequency initiates the process of radiation. The FEL collective instability in the electron beam produces an exponential growth (along the undulator) of the modulation of the electron density on the scale of undulator radiation wavelength. The fluctuations of current density in the electron beam are uncorrelated not only in time but in space, too. Thus, a large number of transverse radiation modes are excited when the electron beam enters the undulator. These radiation modes have different gain. As undulator length progresses, the high gain modes start to predominate more and more. For enough long undulator, the emission will emerge in a high degree of transverse coherence.

The first analysis of the problem of transverse coherence has been performed in [2]. It has been found that even after finishing the transverse mode selection process the degree of transverse coherence of the radiation from SASE FEL visibly differs from the unity. This is consequence of the interdependence of the longitudinal and transverse coherence. First studies of the evolution of transverse coherence in the nonlinear regime of SASE FEL operation have been performed in [3]. It has been found that the degree of transverse coherence reaches maximum value in the end of the linear regime. Further increase of the undulator length leads to its decrease. Despite output power of the SASE FEL grows continuously in the nonlinear regime, maximum brilliance of the radiation is achieved in the very beginning of the nonlinear regime. Due to limited computing power available at that time we limited our study with a specific numerical example just illustrating the general features of coherence properties of the radiation produced by the SASE FEL operating in the nonlinear regime.

In this paper we present general analysis of the coherence properties (longitudinal and transverse) of the radiation from SASE FEL. The results have been obtained with time-dependent, three-dimensional FEL simulation code FAST [4] performing simulation of the FEL process with actual number of electrons in the beam. Using similarity techniques we present universal dependencies for the main characteristics of the SASE FEL covering all practical range of X-ray FELs.

BASIC RELATIONS

Design of the focusing system of XFEL assumes nearly uniform focusing of the electron beam in the undulator, so we consider axisymmetric model of the electron beam. It is assumed that transverse distribution function of the electron beam is Gaussian, so rms transverse size of matched beam is $\sigma = \sqrt{\epsilon\beta}$,where $\epsilon = \epsilon_{\mathrm{n}}/\gamma$ is rms beam emittance and β is focusing beta-function. In the case of negligibly small effects of the space charge and energy spread, operation of the FEL amplifier is described by the diffraction parameter B and the betatron motion parameter $\hat{k}_\beta$: [5]:

$$B = 2\Gamma\sigma^2\omega/c \, , \qquad \hat{k}_\beta = 1/(\beta\Gamma) \, , \tag{1}$$

where $\Gamma = \left[I\omega^2\theta_{\mathrm{s}}^2 A_{\mathrm{JJ}}^2/(I_{\mathrm{A}}c^2\gamma_{\mathrm{z}}^2\gamma)\right]^{1/2}$ is the gain parameter. When describing shot noise in the electron beam, one more parameter appears, the number of electrons in the volume of coherence: $N_{\mathrm{c}} = I/(e\omega\rho)$, where $\rho = c\gamma_{\mathrm{z}}^2\Gamma/\omega$ is the efiiciency parameter. The following notations are used here: I is the beam current, $\omega = 2\pi c/\lambda$ is the frequency of the electromagnetic wave, $\theta_{\mathrm{s}} = K_{\mathrm{rms}}/\gamma$, K_{rms} is the rms undulator parameter, γ is relativistic factor, $\gamma_z^{-2} = \gamma^{-2} + \theta_s^2$, $k_w = 2\pi/\lambda_w$ is the undulator wavenumber, $I_A = 17$ kA is the Alfven current, $A_{JJ} = 1$ for helical undulator and $A_{JJ} = J_0(K_{rms}^2/2(1+K_{rms}^2)) - J_1(K_{rms}^2/2(1+K_{rms}^2))$ for planar undulator. Here J_0 and J_1 are the Bessel functions of the first kind.

Target value of interest for XFEL optimization is the field gain length of the fundamental mode. For this practically important case the solution of the eigenvalue equation for the field gain length of the fundamental mode and optimum beta function are rather accurately approximated by [6]:

$$\begin{aligned} L_g &= 1.67 \left(\frac{I_A}{I}\right)^{1/2} \frac{(\epsilon_n \lambda_w)^{5/6}}{\lambda^{2/3}} \frac{(1+K^2)^{1/3}}{KA_{JJ}} \\ \beta_{opt} &\simeq 11.2 \left(\frac{I_A}{I}\right)^{1/2} \frac{\epsilon_n^{3/2} \lambda_w^{1/2}}{\lambda K A_{JJ}} , \end{aligned} \quad (2)$$

Accuracy of this fit is better than 5% in the range of parameter $\hat{\epsilon} = 2\pi\epsilon/\lambda$ from 1 to 5. It follows from (1) and (2) that diffraction parameter B and parameter of betatron oscillations, $\hat{k}_\beta$ are functions of the only parameter $\hat{\epsilon}$. FEL equations written down in the dimensionless form involve an additional parameter N_c defining the initial conditions for the start-up from the shot noise. Note that the dependence of output characteristics of the SASE FEL operating in saturation is slow, in fact logarithmic in terms of N_c.

SIMULATION ALGORITHM

Rigorous studies of the nonlinear stage of amplification is possible only with numerical simulation code. Typically FEL codes use an artificial ensemble of macroparticles for simulating of the FEL process when one macroparticle represents large number of real electrons. Thus, a natural question arises if macroparticle phase space distributions are identical to those of actual electron beam at all stages of amplification. Let us trace typical procedure for preparation of an artificial ensemble [7, 8]. The first step of particle loading consists in a quasi-uniform distribution of the macroparticles in the phase space. At this stage an ensemble of particles with random distribution is generated which occupies a fraction of the phase space. Then this ensemble is copied on the other parts of the phase space to provide pseudo-uniform loading of the phase space. Pseudo-uniformity means that initial microbunching at the fundamental harmonic (or for several harmonics) is equal to zero. Also, phase positions of the mirrored particles are correlated such that microbunching does not appear due to betatron oscillations, or due to the energy spread. Finally, artificial displacements of the macroparticles are applied to provide desired (in our case gaussian) statistics of microbunching at the undultor entrance. We note that it is not evident that such an artificial ensemble reflects actual physical situation for a short wavelength SASE FEL. Let us consider an example of the SASE FEL operating at the radiation wavelength of 0.1 nm. With the peak current of 5 kA we find that the number of electrons per wavelength is about 10^4. On the other hand, it is well known that properties of an artificial ensemble (even at the first step of pseudo-uniform loading) converge very slowly to the model of continuous media. In fact, even with the number of macroparticles per radiation wavelength 6.4×10^4 the FEL gain still visibly deviates from the target value. Introducing of an artificial noise makes situation with the quality of an ensemble preparation even more problematic. The only way to test the quality of an artificial ensemble is to perform numerical simulations with actual number of electrons in the beam. We constructed such a version of three-dimensional, time-dependent FEL simulation code FAST [4]. Comparison of the results with direct simulations of the electron beam and with an artificial distributions has shown that artificial ensembles are not adequate to the problem. Artificial effects are pronouncing especially when calculating such fine features as transverse correlation functions. Thus, all the simulations presented in this paper have been performed with code FAST using actual number of electrons in the beam.

GENERAL DEFINITIONS

The first-order transverse correlation function is defined as

$$\gamma_1(\vec{r}_\perp, \vec{r\prime}_\perp, z, t) = \frac{\langle \tilde{E}(\vec{r}_\perp, z, t) \tilde{E}^*(\vec{r\prime}_\perp, z, t)\rangle}{\left[\langle|\tilde{E}(\vec{r}_\perp, z, t)|^2\rangle \langle|\tilde{E}(\vec{r\prime}_\perp, z, t)|^2\rangle\right]^{1/2}} ,$$

where $\tilde{E}$ is the slowly varying amplitude of the amplified wave. For a stationary random process γ_1 does not depend on time, and the degree of transverse is:

$$\zeta = \frac{\int |\gamma_1(\vec{r}_\perp, \vec{r\prime}_\perp)|^2 I(\vec{r}_\perp) I(\vec{r\prime}_\perp) \,\mathrm{d}\,\vec{r}_\perp \,\mathrm{d}\,\vec{r\prime}_\perp}{[\int I(\vec{r}_\perp) \,\mathrm{d}\,\vec{r}_\perp]^2} , \quad (3)$$

where $I(\vec{r}_\perp) = \langle|\tilde{E}(\vec{r}_\perp)|^2\rangle$. The first order time correlation function, $g_1(t, t')$, is calculated in accordance with the definition:

$$g_1(\vec{r}, t - t') = \frac{\langle \tilde{E}(\vec{r}, t) \tilde{E}^*(\vec{r}, t')\rangle}{\left[\langle| \tilde{E}(\vec{r}, t) |^2\rangle \langle| \tilde{E}(\vec{r}, t') |^2\rangle\right]^{1/2}} , \quad (4)$$

For a stationary random process time correlation functions are functions of the only argument, $\tau = t - t\prime$. The coherence time is defined as

$$\tau_c = \int\limits_{-\infty}^{\infty} |g_1(\tau)|^2 \,\mathrm{d}\,\tau . \quad (5)$$

If one traces evolution of the brilliance of the radiation along the undulator length there is always the point, which we define as the saturation point, where the brilliance reaches maximum value [9]. In the following we present characteristics of the radiation at the saturation point which are universal functions of the only parameter, $\hat{\epsilon}$. In fact, the brilliance is proportional to the degeneracy parameter δ, i.e. the number of photons in a single pulse which are transversely and longitudinally coherent. We introduce a notion of normalized degeneracy parameter

$$\hat{\delta} = \hat{\eta}\zeta\hat{\tau}_{\mathrm{c}} \ .$$

Here normalized FEL efficiency is defined as $\hat{\eta} = P/(\rho W_{\mathrm{b}})$ where P is radiation power, and $W_{\mathrm{b}} = \gamma mc^2 I/e$ is electron beam power. Normalized coherence time is defined as $\hat{\tau}_{\mathrm{c}} = \rho\omega\tau_{\mathrm{c}}$. Parameter $\hat{\delta}$ and the degeneracy parameter δ are simply related as:

$$\delta = \frac{W_{\mathrm{b}}}{\hbar\omega^2}\hat{\delta} = 2.7\times 10^7 \times \lambda[\mathring{A}\,] \times I[\mathrm{kA}] \times E[\mathrm{GeV}] \times \hat{\delta} \ .$$

PROPERTIES OF THE RADIATION

Simulations of the FEL process have been performed for the case of a long bunch with uniform axial profile of the beam current. Such a model provides rather accurate predictions for the coherence properties of the XFEL, since typical radiation pulse from the XFEL is much longer than the coherence time. Calculations has been performed with FEL simulation code FAST using actual number of electrons in the beam. The value of parameter $N_{\mathrm{c}} = 8\times 10^5$ corresponds to the parameter range of XFEL operating at the radiation wavelength about 0.1 nm.

A series of simulation runs have been performed in the range of the parameter $\hat{\epsilon} = 0.25\ldots 4.5$. Application of similarity techniques described above allowed us to extract universal parametric dependencies of the main characteristics of the optimized XFEL operating in the saturation (see Figs. 1-3).

Figure 1 shows the dependence of the saturation length $\hat{z}_{\mathrm{sat}} = \Gamma z_{\mathrm{sat}}$ on parameter $\hat{\epsilon}$. Analysis of the curve shows that the saturation length scales as $\hat{z}_{\mathrm{sat}} \propto \hat{\epsilon}^{5/6}$. Such dependence directly follows from the optimization procedure of the gain length given by (2). The normalized coherence

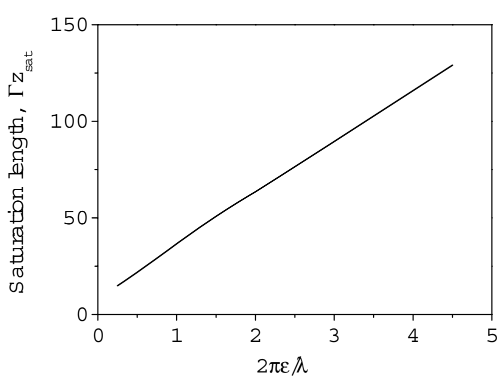

Figure 1: Saturation length Γz_{sat} versus parameter $\hat{\epsilon}$.

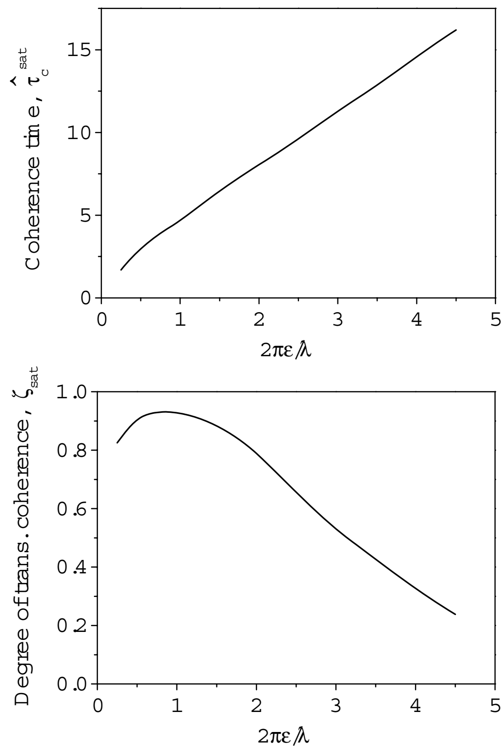

Figure 2: Degree of transverse coherence, ζ_{sat}, and normalized coherence time, $\hat{\tau}_{\mathrm{c}}^{\mathrm{sat}}$ in saturation versus parameter $\hat{\epsilon}$.

time in the saturation, $\hat{\tau}_{\mathrm{c}}^{\mathrm{sat}}$ is also proportional to $\hat{\epsilon}^{5/6}$ (see Fig. 2).

The dependence of the degree of transverse coherence in the saturation on the parameter $\hat{\epsilon}$ exhibits rather complicated behavior (see Fig. 2). It reaches maximum value in the range of $\hat{\epsilon}$ values about of unity, and drops at small and large values of $\hat{\epsilon}$. Actually, the degree of transverse coherence is formed due to two effects. The first effect takes place due to interdependence of the poor longitudinal coherence and transverse coherence [2]. Due to the start-up from shot noise every radiation mode is excited within finite spectral bandwidth. Actually this means that in the high gain linear regime the radiation of the SASE FEL is formed by many fundamental TEM_{00} modes with different frequencies. The transverse distribution of the radiation field of the mode is also different for different frequencies. Smaller value of the diffraction parameter (i.e. smaller value of $\hat{\epsilon}$) corresponds to larger deviation of the radiation mode from the plane wave. This explains a decrease of the transverse coherence at small values of $\hat{\epsilon}$. When the parameter $\hat{\epsilon}$ increases, the diffraction parameter increases as well thus leading to the degeneration of the radiation modes. Amplification process in the SASE FEL passes limited number of the field gain lengths, and starting from some value of $\hat{\epsilon}$ the linear stage of amplification becomes too short to provide mode selection process. When amplification process enters nonlinear stage, the mode content of

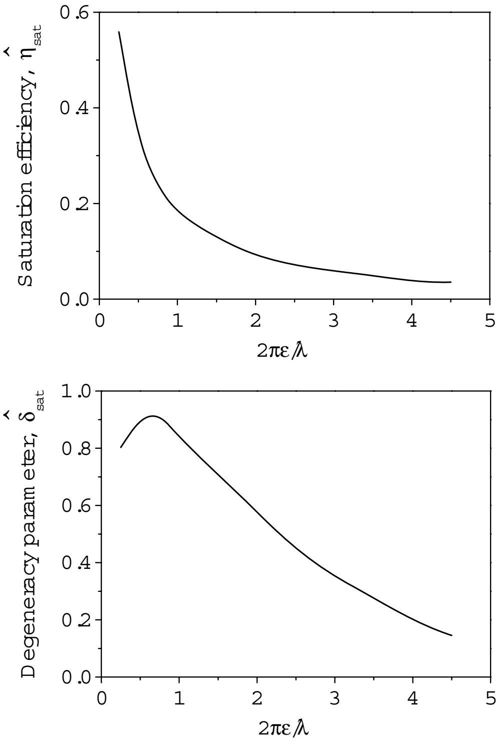

Figure 3: Efficiency, $\langle\hat{\eta}_{\rm sat}\rangle$, and normalized degeneracy parameter, $\hat{\delta}_{\rm sat}$, in the saturation versus parameter $\hat{\epsilon}$.

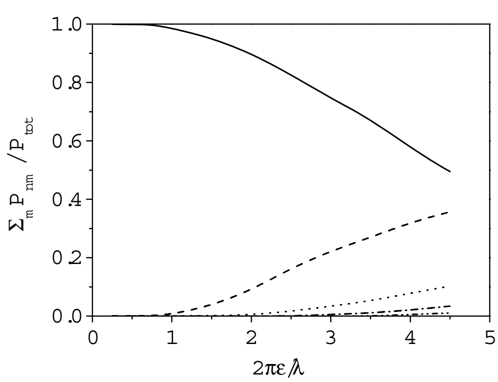

Figure 4: Partial contributions of the modes with azimuthal index $m = 0 \ldots 4$ into the total power versus parameter $\hat{\epsilon}$. SASE FEL operates in the saturation.

the radiation becomes even more rich due to independent growth of the radiation modes in the nonlinear medium (see Fig. 4). Thus, at large values of $\hat{\epsilon}$ the degree of transverse coherence is limited by poor mode selection. Analytical estimations presented in [9] show that in the limit of large emittance, $\hat{\epsilon} \gg 1$, the degree of transverse coherence scales as $1/\hat{\epsilon}^2$.

We present in Fig. 3 the plots for normalized efficiency and degeneracy parameter for optimized XFEL. Normalized efficiency in saturation has simple scaling, it falls inversely proportional to the parameter $\hat{\epsilon}$. Taking into account that the value of the coherence time $\hat{\tau}_{\rm c}^{\rm sat}$ scales proportional to $\hat{\epsilon}^{5/6}$, we find that the normalized degeneracy parameter of the radiation is nearly proportional to the degree of transverse coherence, $\hat{\delta} \propto \zeta/\hat{\epsilon}^{1/6}$.

Finally, in Table 1 we present comparison of existing XFEL projects, the European XFEL, LCLS and SCSS in terms of degree of transverse coherence [10–12]. We see that the European XFEL and LCLS are in the same range of parameter space. These projects assume conservative value of the emittance, and relatively high degree of transverse coherence is achieved by increasing the energy of the driving accelerator. Project SCSS assumes much smaller energy of the driving accelerator. However, despite much smaller value of the normalized emittance it falls in the range of parameters for the output radiation with poor transverse coherence.

Table 1: Parameter space of XFEL projects

	European XFEL		LCLS	SCSS
	SASE1	SASE2		
λ, nm	0.1	0.15	0.15	0.1
E, GeV	17.5	17.5	14.35	6.135
ϵ_n, mm-mrad	1.4	1.4	1.2	0.85
$\hat{\epsilon}$	2.6	1.7	1.8	4.5
ζ	0.65	0.85	0.83	0.24

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