INFLUENCE OF LINER FLUCTUATIONS ON LOW- AND HIGH-GAIN CHERENKOV FELS

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Abstract

Imperfections in the dielectric liner of a Cherenkov Free-Electron Laser (CFEL) result in fluctuations in the phase velocity of a radiation wave when it propagates through the lined waveguide. Random fluctuations in the phase velocity reduce the bunching of the electrons and consequently lower the gain of CFELs. Here we theoretically investigate the influence of these liner-induced phase fluctuations in the radiation field on the saturated power of low to high gain CFELs. To obtain different gain regimes, we keep the electron beam radius constant and vary the current density. As an example, we study a 50 GHz CFEL and quantify the reduction in the single-pass saturated power for different rms liner fluctuations when the CFEL is driven by an electron beam with current densities varying from 1 A/cm^2 (average gain of 0.43 dB/cm) to 25 A/cm^2 (average gain of 1.39 dB/cm).

INTRODUCTION

The increasing number of microwave applications in both research and industry [1, 2] has increased the interest in tuneable high-power microwave sources. The Cherenkov Free-Electron Laser (CFEL) is a promising candidate as a compact high-power microwave source for various applications. CFELs have been operated at 100 kW peak power level at 1 mm wavelengths, at 200 MW peak power level at 8 cm wavelengths and at wavelengths as short as in the far infrared [3]. The simplicity of its construction, the high efficiency and an affordable compact design have made the CFEL attractive for applications where high microwave frequencies and high powers are needed. However, simple imperfections in either the electron beam or the lined waveguide section, that are used to generate the laser gain, can seriously degrade the performance of the device [4].

In a CFEL, accelerated electrons are injected through a wave-guiding structure that slows the phase velocity of the electromagnetic wave to a sub-luminous value. An example is a metallic, cylindrical tube lined with a dielectric material, e.g., quartz. By choosing appropriate dimensions for the waveguide, i.e., the inner diameter for the tube, the thickness and dielectric constant of the liner, the phase velocity of EM waves can be matched to the velocity of the injected electron beam for a desired wave frequency [5]. A transversely magnetic (TM) wave can decelerate or accelerate co-propagating electrons. A net deceleration of electrons in the pump beam and, correspondingly, a net amplification of the wave, occurs only if the electrons are forming bunches, if these bunches travel with the phase velocity of the radiation wave, and if the bunches are located within a certain phase range of the radiation wave (when seen from the frame of that wave). A fluctuating phase velocity of the wave in the waveguide would appear as a fluctuating relative phase of the bunches; it could bring bunches out of the optimum driving phase, and this would reduce the amplification. Likewise, a spread in longitudinal electron velocities results in a spread in relative phases between wave and the electrons. This results in a reduced bunching within the electron beam and consequently in a lower amplification of the wave [3].

Although the variations in the phase velocity can have different origins, we will model only one type of imperfection, namely, fluctuations in the inner radius of the liner. Other types of fluctuations, such as a spatial inhomogeneity of the liner's dielectric constant, are not treated separately because we expect widely analogous results. In a previous study we have shown that in a low current CFEL, a typical manufacturing tolerance in the inner radius of the liner of 5 % can reduce the saturated power by a factor of 2 [4]. In this paper, we investigate the sensitivity of the CFEL to small random liner imperfections for different gain regimes. We therefore consider a 50 GHz CFEL with a fixed liner geometry, electron beam energy, and electron beam radius. The different gain regimes are obtained by varying the electron beam current density from 1 A/cm^2 to 25 A/cm^2.

In the remainder of this paper we first present a summary of the theoretical model that describes the dynamics of the CFEL in the presence of radii fluctuations. Next, we will show the numerical results for a CFEL system operating in different gain regimes. We end with a discussion and conclusions.

THEORETICAL MODEL

The theoretical model describing the dynamics of a CFEL in the presence of small and random fluctuations of the liner inner radius is presented in previous work [4] and here we present a summary.

The standard CFEL dynamical model [3, 6] assumes that the electromagnetic wave co-propagating with the electron beam can be described by a superposition of empty (i.e., without the electron beam) waveguide eigenmodes with amplitudes that vary slowly with longitudinal distance z. Using the power orthogonality property of the eigenmodes,

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and applying the slowly varying amplitude and phase approximation to Maxwell's equations, the dynamical equation for the mode amplitudes are derived. The system is closed by the Newton-Lorentz equations, that describe the motion of the electrons under influence of the electromagnetic wave.

The effect of a small random fluctuation in the inner radius $r_d(z)$ of the liner that varies slowly along its length is twofold. First, it varies the phase velocity of the wave. Second, it modifies, in principle, the transverse mode profile. However, our analysis shows that the liner fluctuations mainly affect the phase velocity of the wave and that the transverse mode profile remains approximately unchanged [4]. In our model, we allow the wave number to vary slowly along the length of the liner, and write for the longitudinal component of the electric field $E_{z,n}$ for mode TM_{0n}:

$$E_{z,n}(r,z,t) = a_{0n}(z)\frac{f_n(r)}{\sqrt{k_n(z)}}exp\left(i\int_0^z k_n(z')dz' - \omega t\right), \quad (1)$$

where $f_n(r)$ is the transverse mode profile of an empty lined waveguide [7], $a_{0n}(z)$ the slowly varying mode amplitude, and the slowly varying longitudinal wave number $k_n(z)$ is given by:

$$k_n(z) = k_{0n} + \int_0^z \frac{dk_n}{dz'}dz' \approx k_{0n} + \left.\frac{\partial k_n}{\partial r_d}\right|_{r_{d0}} \int_0^z \frac{dr_d}{dz'}dz', \quad (2)$$

where, for convenience, the inner radius of the liner at $z = 0$ is taken equal to the mean radius r_{d0} and k_{0n} is the wave number corresponding to a homogeneous waveguide with $r_d = r_{d0}$. The evolution of the normalized field amplitude $a'_{0n}(z) = \frac{e}{mc}a_{0n}(z)$ for the TM_{0n} modes is:

$$2\sqrt{k_n(z)}(1 - \frac{\kappa_n^2}{k_{0n}^2})\frac{\partial a'_{0n}}{\partial z} = -4\frac{\omega_p^2}{c_0^2}\frac{\beta_{z0}}{A_n r_{d0}^2}\times \int_0^{r_{d0}} drr\left[iI_1(\kappa_n r)\langle\frac{\beta_r}{|\beta_z|}e^{i\alpha_n}\rangle + \frac{\kappa_n}{k_{0n}}I_0(\kappa_n r)\langle e^{-i\alpha_n}\rangle\right] \quad (3)$$

Here A_n is a normalization constant, ω_p is the plasma frequency, β is the electrons velocity normalized to the speed of light in vacuum, I_0 and I_1 are the Bessel functions of second kind, and κ_n is the transverse wave number. The symbol $\langle \ldots \rangle$ represents an average over all electrons within one radiation wavelength. Undefined symbols are described in [4].

DESCRIPTION OF THE CFEL

This model has been applied to a particular design for a Cherenkov free-electron laser that can be operated with both low and high electron beam currents. In this analysis we choose Al_2O_3 with a dielectric constant of ϵ=9.8 as the liner material, and keep the geometry constant. Likewise, the outer radius of the cylindrically shaped electron beam is fixed at r_b=5 mm. However, to vary the single-pass gain and saturated power of this device, we consider different total beam currents I_b ranging from 0.8 A to 20 A. The remaining geometrical parameters are: an average inner liner radius of r_{d0}=5.5 mm and an average liner thickness of d=0.57 mm. This system requires an electron beam energy of 84.2 keV to operate at 50 GHz. We integrate the dynamic CFEL equation (eq. 3) up to the point where the laser saturates. Using a constant inner radius equal to r_{d0} and an initial seed power of 10 mW, these electron currents produce a saturated power P_0 of about 200 W at a distance z_0 of 100 cm and 20.8 kW at a distance of 45 cm for I_b=0.8 A and 20 A, respectively. The corresponding average gain is 0.43 dB/cm and 1.39 dB/cm, respectively.

The variation in the inner liner radius is modeled using a random fluctuation superimposed on the average radius r_{d0}. A spatial low-pass filter with cut-off distance z_c is used to remove fast fluctuations from the random distribution. Further, the fluctuations are scaled to obtain a certain standard deviation σ_{rd} of the distribution. For each particular realization of a fluctuating liner radius we numerically calculate the saturated power P_{sat} and the position z_{sat} at which this power is obtained. To obtain also statistical information on the output as a function of the experimental parameters, we generate 100 different realizations of liner fluctuations for each combination of the rms amplitude σ_{rd} and cut-off distance z_c. For each combination we determine the ensemble average $\overline{P_{sat}}$ and standard variation σ_p and corresponding values for the distance to saturation z_{sat}.

RESULTS AND DISCUSSION

First, we numerically study the sensitivity of this particular CFEL to an increasing amplitude of the random fluctuations in the liner inner radius while the spatial filter is kept constant at z_c=10 cm, corresponding to approximately 16 times the free-space wavelength. The ensemble average of the saturated power, $\overline{P_{sat}}$, normalized to the saturated power in absence of fluctuations, P_0, is shown in Figs. 1a and 1b as a function of the standard deviation of the fluctuations in the inner liner radius, σ_{rd}, for the two beam currents of 0.8 A and 20 A, respectively. Corresponding values for the distance to saturation z_{sat} are shown in Fig. 2. Fig. 1 shows that for this particular CFEL and a beam current of 0.8 A, a small rms liner fluctuation of 2.5 μm (=0.05 % of r_{d0}) is sufficient to reduce the saturated power by a factor of 2 on average. The rms liner fluctuation increases to 25 μm (=0.5 % of r_{d0}) to obtain a similar reduction when the CFEL is driven by a beam current of 20 A. At the same time we observe that the relative spread $\sigma_p/\overline{P_{sat}}$ is much larger for the 0.8 A beam current compared to the 20 A beam current. The variation of the distance to saturation with the rms liner amplitude (see Fig. 2) shows a different behavior for the two beam currents. For the 0.8 A current, the distance increase quickly from 100 cm to close to 150 cm and then remains approximately constant. The ob-

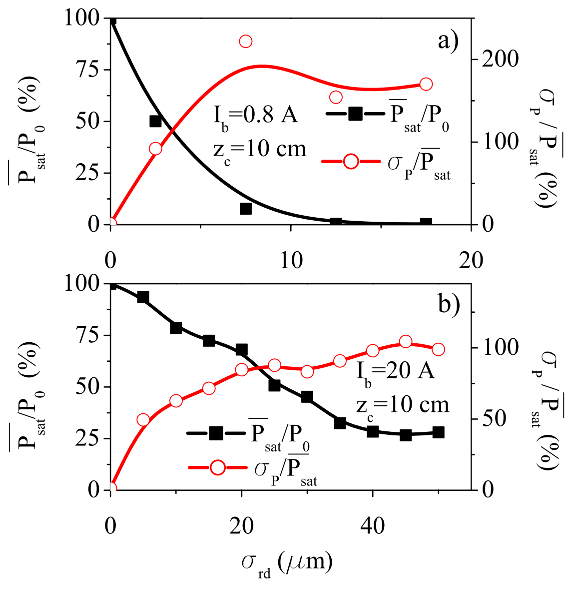

Figure 1: Normalized saturated power ($\overline{P_{sat}}/P_0$) and normalized standard deviation ($\sigma_p/\overline{P_{sat}}$) as a function of the standard deviation of the liner fluctuations σ_{rd} for a current of 0.8 A (a) and 20 A (b).

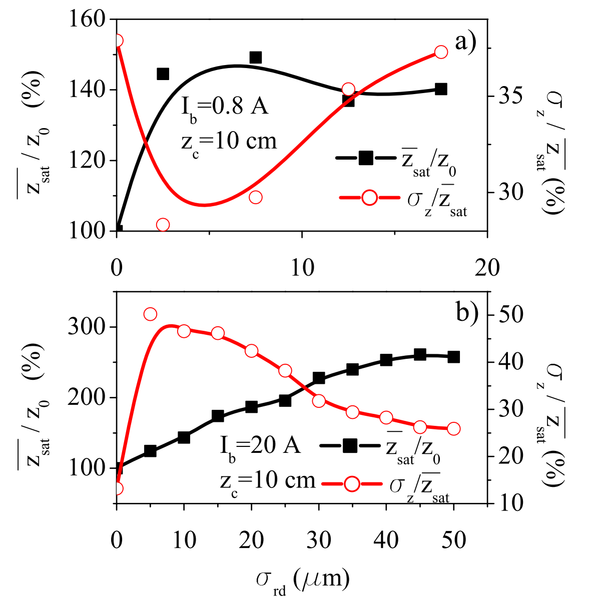

Figure 2: Normalized distance to saturation ($\overline{z_{sat}}/z_0$) and normalized standard deviation of z_sat ($\sigma_z/\overline{z_{sat}}$) as a function of the standard deviation of the liner fluctuations σ_{rd} for a current of 0.8 A (a) and 20 A (b).

served spread in z_{sat} remains approximately constant. On the other hand, for the 20 A current the distance to saturation increases approximately linearly with σ_{rd}, while the relative spread is slightly reduced.

These findings indicate that this particular CFEL is far more sensitive to fluctuations in the inner liner radius than the system studied in our previous study [4]. In that study we also considered a CFEL operating at 50 GHz. However, that system used a 0.8 A electron beam with a 1 mm radius (25 A/cm^2 current density) to drive the CFEL. We found that an rms fluctuation of about 5 % was required to reduce the saturated power by a factor of 2. For the device of this work, this is 0.5 % and 0.05 % for the same current density and total beam current respectively. To allow the CFEL of this work to be driven by a higher total current, we have increased the beam radius. As a consequence, the transverse dimensions of the CFEL have increased and we have chosen a different liner material. This leads to a larger variation of k_n with inner liner radius r_d (see eq. 2): $\partial k_n/\partial r_d$ is -4.6 10^6 m^{-2} for the device in this work, while it equals -0.46 10^6 m^{-2} for the device in our previous study. It is therefore not surprising that the current system is more sensitive to liner imperfections than the system considered in our previous study.

Second, we have investigated the influence of spatial distribution of the liner imperfections by varying the cut-off distance z_c of the low pass filter applied to the randomly generated liner fluctuations. The normalized ensemble average $\overline{P_{sat}}/P_0$ and the normalized spread $\sigma_p/\overline{P_{sat}}$ are shown in Fig. 3 for I_b=0.8 A and 20 A, and the corresponding values for z_{sat} are shown in Fig. 4. These figures show that both the saturated power P_{sat} and the distance to saturation z_{sat} vary only weakly with increasing z_c if z_c is sufficiently large ($z_c >> \lambda = c/f$). Note, that the latter condition is anyhow implicit in the approximations used in deriving the dynamical CFEL equation (eq. 3).

Last, we compare the sensitivity of this system to liner imperfections as a function of the total beam current I_b for a fixed σ_{r_d} =15 μm and a fixed cut-off distance z_c=10 cm. Fig. 5a shows the normalized ensemble average $\overline{P_{sat}}/\overline{P_0}$ and spread $\sigma_p/\overline{P_{sat}}$ as a function of I_b and Fig. 5b shows the corresponding values for z_{sat}. These figures show that I_b must be larger than 30 A to keep the reduction in $\overline{P_{sat}}$ to less than 15 %. We also observe that the relative spread $\sigma_p/\overline{P_{sat}}$ reduces with increasing beam current, while the spread $\sigma_z/\overline{z_{sat}}$ shows a slight increase.

CONCLUSION

We have studied the sensitivity to imperfections in the liner of a particular CFEL operating at 50 GHz as a function of total beam current. The system studied in this work is based on a previously studied system with increased transverse dimensions and a different liner. The increased dimensions allow a larger total current to pass through the lined waveguide and study the system in different gain regimes. As a consequence, the variation of longitudinal wave number with inner radius increased by a factor of 10 compared to the system we used in a previous study, that also operated at 50 GHz. Taking this difference into ac-

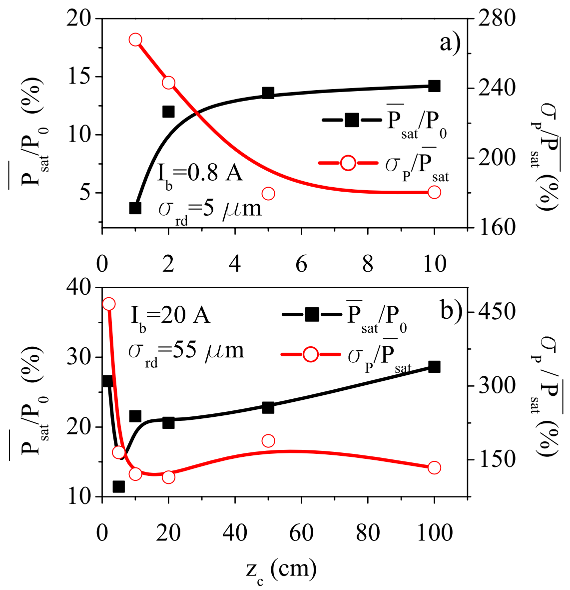

Figure 3: Normalized saturated power ($\overline{P_{sat}}/P_0$) and normalized standard deviation ($\sigma_p/\overline{P_{sat}}$) as a function of the cut-off distance z_c for a current of 0.8 A (a) and 20 A (b).

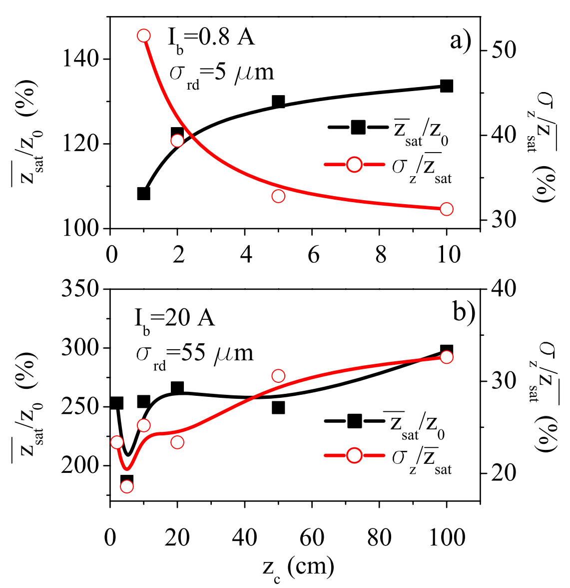

Figure 4: Normalized distance to saturation ($\overline{z_{sat}}/z_0$) and normalized standard deviation of z_sat ($\sigma_z/\overline{z_{sat}}$) as a function of the cut-off distance z_c for a current of 0.8 A (a) and 20 A (b).

count, we find that both systems show similar sensitivity to liner imperfections when the CFELs are driven by electron beams having the same current density of 25 A/cm^2. For the current system we find that a total current of at least 30 A is required to keep the reduction in ensemble average $\overline{P_{sat}}$ to less than ~15 % of P_0 (for σ_{rd}=15 μm). The same

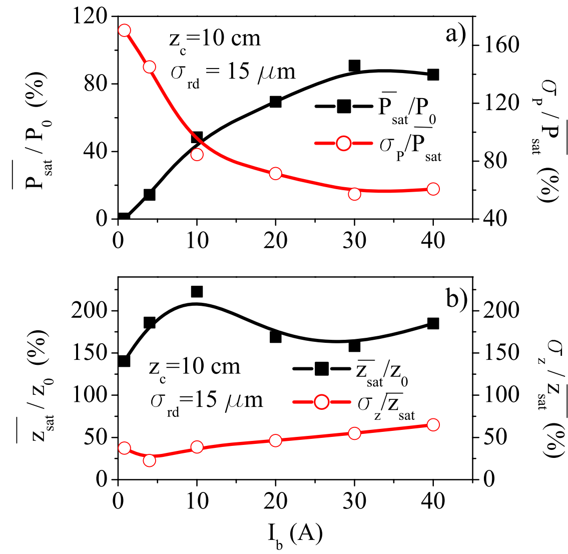

Figure 5: Normalized saturated power ($\overline{P_{sat}}/P_0$) and normalized standard deviation ($\sigma_p/\overline{P_{sat}}$) (a) and corresponding values for z_{sat} (b) as a function of the electron beam current for $\sigma_{rd} = 15$ μm and z_c=10 cm.

rms liner fluctuation reduces $\overline{P_{sat}}$ to less than 20 % of P_0 for I_b <5 A. A high gain also reduces the relative spread in P_{sat} for an ensemble of similar imperfections, while at the same time the relative spread in z_{sat} increases slightly. These findings show that systems with high gain are less sensitive to liner imperfections compared to systems with low gain. However, the actual allowable rms fluctuation depends both on the gain and on the particular geometry of the lined waveguide of the Cherenkov FEL.

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