OPTIMIZATION OF PARAMETERS OF SMITH-PURCELL BWO *

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Abstract

We study the dependence of start current in Smith-Purcell backward wave oscillator (SP-BWO) on grating parameters and electron beam parameters. The attenuation due to finite conductivity of the grating material is taken into account and three-dimensional effects are included in an approximate way in the analysis. We find that the start current can be significantly reduced by optimizing the grating parameters.

INTRODUCTION

The Smith-Purcell free-electron laser (FEL) is a backward wave oscillator (BWO) for low energy electron beam [1,2]. In a BWO, like any oscillator system, the electron beam current needs to be higher than a threshold value, known as the start current, in order to produce coherent electromagnetic oscillation. In a recent paper [2], we have performed a calculation of start current for the case of sheet electron beam skimming over the grating surface in a SP-BWO. The attenuation of the backward wave due to finite conductivity of the grating was not taken into account in this calculation. The issue of attenuation becomes important while optimizing the parameters of SP-BWO. Recently, Andrews et al. [3] have discussed the calculation of attenuation coefficient for the backward wave supported by the grating. They have also studied the dependence of gain and attenuation in SP-BWO as a function of group velocity as the energy of the co-propagating electron beam is varied, keeping the grating parameter fixed. In this way, they have calculated the net gain. However, gain does not have a straightforward meaning in a BWO. In this paper, we therefore present a calculation of start current taking into account the effect due to attenuation. We use this calculation to study the dependence of start current on grating parameters as well as electron beam parameters.

START CURRENT CALCULATION

In a SP-BWO, the electron beam interacts with the co-propagating surface electromagnetic mode supported by the grating. The co-propagating surface mode has a group velocity in the direction opposite to the electron beam. We consider a sheet electron beam in the (y, z) plane propagating at a height b from the grating top surface along the z-axis. The grating grooves are in the y-direction and perpendicular to the grating is in the x-direction. Due to the finite conductivity of the material of the grating, the surface electromagnetic mode suffers attenuation. The attenuation coefficient can be calculated as per the prescription given by Andrews et al. in Ref. 3. Attenuation occurs in the direction in which the energy is flowing. Here, the phase velocity of the resonant surface mode is along the positive z-axis and the group velocity v_g is along the negative z-axis. Let the longitudinal component of the the electric field of the backward wave be given by $E(z, t)\exp(ik_0z - i\omega t)$. Including attenuation in the analysis, the equation for the evolution of the amplitude $E(z, t)$ of the backward wave described in Ref. 2 gets modified to

$$\frac{\partial E}{\partial z} - \frac{1}{v_g}\frac{\partial E}{\partial t} = \frac{IZ_0\chi}{2\beta\gamma\Delta y}e^{-2\Gamma_0 b}\langle e^{-i\psi}\rangle + \alpha E, \quad (1)$$

where α is the complex attenuation coefficient having positive real part. Note that we are here closely following the notations and derivations given in Ref. 2. Here, I is the electron beam current, Δy is the beam width in the y-direction assumed to be so large that two-dimensional approximation is valid, β is the electron velocity in the unit of speed of light c, γ is the electron energy in unit of rest energy, Z_0 = 377 Ω is the characteristic impedence of free space, $\Gamma_0 = k_0/\gamma$, ψ is the electron phase and χ is the residue of the singularity associated with the surface mode as defined in Ref. 2. Converting to dimensionless variables, this equation can be transformed to the following form

$$\frac{\partial \mathcal{E}}{\partial \tau} - \frac{\partial \mathcal{E}}{\partial \zeta} = -\mathcal{J}\langle e^{-i\psi}\rangle - \alpha L\mathcal{E}, \quad (2)$$

where L is the length of the grating, $\mathcal{E}$ is the dimensionless electric field, $\mathcal{J}$ is the dimensionless beam current, τ is the dimensionless time and $\zeta = z/L$ is the normalised distance along the grating [2]. The second term in the right hand side of the above equation is the contribution due to attenuation. The above equation for the evolution of surface mode electric field will be coupled to equations for the electron beam dynamics as discussed in Ref. 2. We then linearise the equations and look for the solution for the electric field growing as $e^{\nu\tau}$. For a given value of $\mathcal{J}$ and αL, we can obtain the complex growth rate ν by simultaneously solving the following set of two equations

$$\kappa^3 - (\nu + \alpha L)\kappa^2 + i\mathcal{J} = 0. \quad (3)$$

$$\kappa_1^2(\kappa_2 - \kappa_3)e^{\kappa_1} + \kappa_2^2(\kappa_3 - \kappa_1)e^{\kappa_2} + \kappa_3^2(\kappa_1 - \kappa_2)e^{\kappa_3} = 0. \quad (4)$$

Note that we have ignored the space charge parameter Q defined in Ref. 2, and assumed Q = 0. Here, κ_1, κ_2 and κ_3 are three solutions for κ in Eq. (3). For a particular

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value of αL, by solving these equations numerically, we can find out the minimum value of $\mathcal{J}$ for which the real part of ν is positive. Let us call this as dimensionless start current denoted by $\mathcal{J}_s$, which is a function of αL. For the case when there is no attenuation $(\alpha L = 0)$, we had earlier obtained that $\mathcal{J}_s$ =7.68 [2]. Here, by numerically solving Eqs. (3) and (4), we obtain the dependence of $\mathcal{J}_s$ on αL. This is shown in Fig. 1. Note that $\mathcal{J}_s$ depends only on the real part of αL.

The expression for the start current density dI_s/dy is given by

$$\frac{dI_s}{dy} = \mathcal{J}_s(\alpha L)\frac{I_A}{2\pi\chi}\frac{\beta^4\gamma^4}{kL^3}e^{2\Gamma_0 b}, \tag{5}$$

where I_A = 17 kA is the Alfvén current. Here, k = ω/c = $2\pi/\lambda$ and λ is the free space wavelength of the surface mode. In the next section, we use the above formula to calculate and optimize the start current.

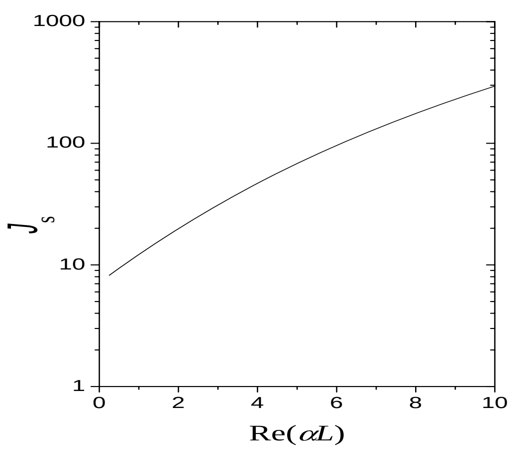

Figure 1: Plot of threshold dimensionless current $\mathcal{J}_s$ as a function of the real part of the attenuation coefficient αL.

OPTIMIZATION OF GRATING PARAMETERS

In the past, many authors [1-5] have used the parameters corresponding to the Dartmouth experiment [6] for studying the performance of SP-BWO. For the Dartmouth experiment, the grating parameters and the electron beam parameters are given in Table 1. Note that we interpret the electron beam radius in Table 1 as the parameter b in our model. Using the formula derived in the previous section, we optimize the grating parameters for this case to minimise the start current.

Table 1: Parameters for the Dartmouth experiment

Groove width (w)	62 μm
Groove depth (d)	100 μm
Period (λ_g)	173 μm
Grating length (L)	12.7 mm
Electron beam radius (b)	10 μm
Electron beam energy	35 keV

First, we calculate the attenuation coefficient for different values of groove depth d and groove width w. For w = 62 μm, we vary the groove depth d and find out the attenuation coefficient α for each groove depth. This is shown in Fig. 2(a). Note that for the calculation of the attenuation coefficient, we have chosen Aluminum as the material of the grating and used the prescription given by Andrews et al. in Ref. 3. Then, keeping w = 62 μm, we calculate χ for different values of d as shown in Fig. 2(b). For the calculation of χ, we have used the prescription given in our earlier paper [2]. Next, knowing the value of α and χ, we use Eq. (5) to obtain the start current density as a function of groove depth for w = 62 μm. As the groove depth increases from 100 μm, the χ parameter increases, which means that the start current density decreases. However, as the groove depth is increased beyond 100 μm, the attenuation also increases, which tends to reduce the start current

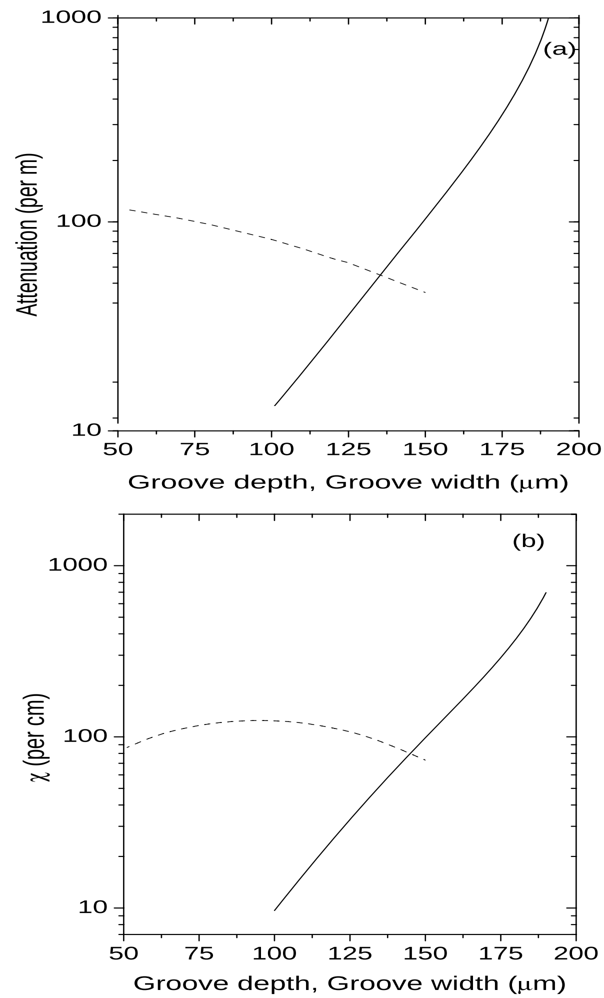

Figure 2: Plot of the real part of attenuation coefficient α (a), and the χ parameter (b) as a function of groove depth (solid curves) for groove width = 62 μm, and as a function of groove width (dashed curves) for groove depth = 150 μm.

density. Consequently, there is an optimum groove depth at which dI_s/dy is minimum. As we find in Fig. 3, the start current density is minimum for d = 150 mm. The value of dI_s/dy reduces from 40 mA/mm to 7.7 mA/mm when the groove depth is changed from 100 μm to 150 μm.

After optimizing the groove depth, we next optimize the groove width. Keeping d = 150 μm, we vary w from 50 μm to 150 μm and obtain the attenuation coefficient and the χ-parameter as shown in Fig. 2. Then, using Eq. (5), we obtain the value of dI_s/dy for different values of groove width, keeping groove depth fixed at 150 μm. This is shown in by the dotted curve in Fig. 3. We find that the optimum value of groove width is 110 μm for which the start current density is 5.6 mA/mm.

Hence, we find that the parameters for the Dartmouth experiment are not optimized for the minimum value of the start current. The optimized value of the groove depth and the groove width are 150 μm and 110 μm respectively. For these parameters, the start current density is 5.6 mA/mm, which is a substantial reduction compared to start current density of 40 mA/mm corresponding to the parameters given in Table 1.

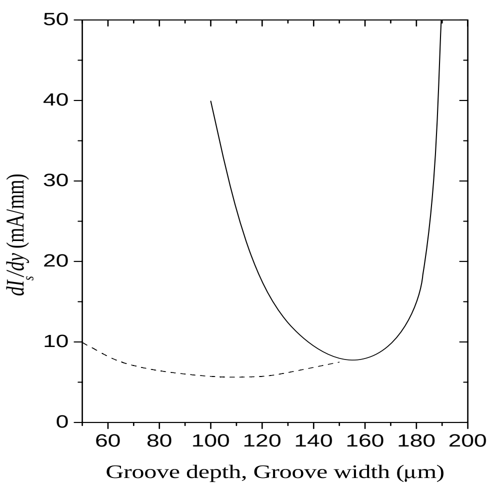

Figure 3: Plot of the start current density dI_s/dy with groove depth (solid line) as well as groove width (dashed line). For the solid line, we have kept groove width fixed at 62 μm. For the dashed line, the groove depth is fixed at 150 μm.

In the above calculations, when we change the groove width and the groove depth, the resonant wavelength λ of the surface mode also changes. The value of λ can be obtained by finding out the location of the singularity of the reflection matrix associated with the surface mode, as discussed in Ref. 2. Fig. 4 shows the variation of the resonant wavelength. We find that for w = 110 μm and d = 150 μm, for which the start current is minimum, the resonant wavelength is 819 μm. Note that for the calculation of dI_s/dy, one needs to put the value of the free-space resonant wavelength λ of the surface mode in Eq. (5), which has been taken from Fig. 4.

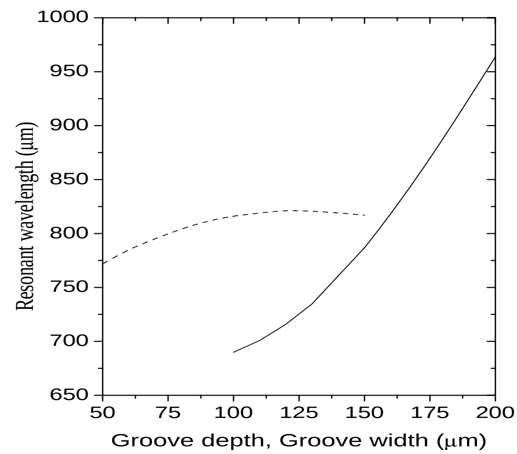

Figure 4: Plot of the free-space resonant wavelength λ with groove depth (solid line) as well as groove width (dashed line). For the solid line, we have kept grove width = 62 μm. For the dashed line, groove depth = 150 μm.

DISCUSSIONS

We would now like to discuss the dependence of the start current on the length of the grating. It is important to include three-dimensional effects for such an analysis [7]. A fully three-dimensional analysis of SP-BWO does not exist yet. However, we will attempt an approximate analysis here. The surface mode in the (y, z) plane is expected to diffract freely as an optical packet of wavelength $\beta\lambda$ since the grating is open in y-direction. The minimum average rms beam size of the optical beam over the length L due to diffraction effects is given by $\sqrt{\beta\lambda L/4\pi}$. In order to maximize the overlap between the electron beam and the optical beam, the rms electron beam size σ_y in the y-direction should be chosen equal to this. Putting the electron beam size in the y-direction in Eq. (5), we obtain that the start current I_s should be proportional to $\mathcal{J}_s(\alpha L)/L^{5/2}$. As we increase the length, the start current increases due to increase in $\mathcal{J}_s$. This is however counterbalanced by the $L^{5/2}$ factor in the denominator. For $\alpha L < 1$, $\mathcal{J}_s$ can be assumed to be slowly varying. Hence, the start current should decrease as $1/L^{5/2}$. For the larger values of αL, the attenuation effects will be dominating since $\mathcal{J}_s$ becomes exponential as shown in Fig. 1. In this case, the start current increases with grating length.

One of the important requirements for the operation of SP-BWO is that the electron beam should be sufficiently close to the grating surface. As the electron beam size in the x-direction increases, the start current increases exponentially due to the $\exp(2\Gamma_0 b)$ factor in Eq. (5). Hence, the rms electron beam size σ_x in the x-direction should be around $1/4\Gamma_0$. In order that the electron beam size in the

x–direction is maintained around this value over a length L, we need that the normalised electron beam emittance ϵ_x is less than $\beta\gamma/L(4\Gamma_0)^2$. We therefore notice that although the start current reduces by increasing the grating length, the requirement on electron beam emittance in the x-direction becomes more stringent.

One should also confirm that the space charge effect does not blow up the emittance. For this to be valid, the space charge term in the envelope equation should be less than the emittance term. This leads to the following condition in the x-direction [7]

$$\frac{4}{\beta\gamma}\frac{I}{I_A}\frac{\sigma_x^3}{(\sigma_x+\sigma_y)\epsilon_x^2} < 1. \qquad (6)$$

It is important to point out here that for the Dartmouth parameters given in Table 1, the above inequality is not satisfied. After optimizing the grating parameters, the start current is reduced and then the above inequality is satisfied.

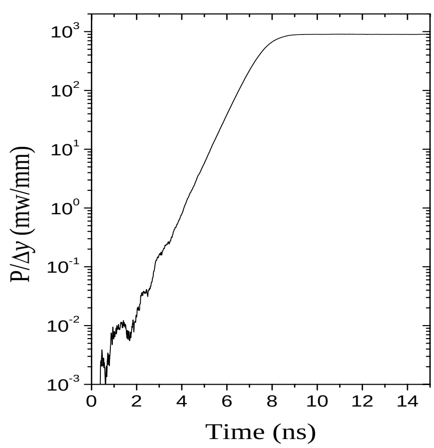

Figure 5: Plot of power per unit beam width in the surface mode as a function of time at the entrance of the grating for the optimized grating prameters.

We would like to point out that for the optimized grating parameters that we discussed, although the start current is reduced significantly compared to the grating parameters in Table 1, the attenuation is increased as seen in Fig. 2. This means that the heat loss in the grating will be more for the optimized parameters, which implies that the power conversion efficiency of SP-BWO will be reduced. For example, for the optimized grating parameters that we discussed, $I_s/\Delta y$ = 5.6 mA/mm. For this case, if we take $I/\Delta y$ = 7 mA/mm, we obtain $P/\Delta y$ = 900 mW/mm at saturation as shown in Fig. 5. This translates to a power conversion efficiency of 0.37%. Note that this calculation is performed by numerically solving the coupled Maxwell-Lorentz equations as discussed in Ref. 8. On the other hand, for the Dartmouth parameters, we have $I_s/\Delta y$ = 40 mA/mm. For this case, if we take $I/\Delta y$ = 50 mA/mm, we obtain $P/\Delta y$ = 14.3 W/mm at saturation. This translates to a power conversion efficiency of 0.56%.

CONCLUSIONS

We have derived a simple formula for the start current density in a SP-BWO taking the attenuation due to finite conductivity of the grating material into account. This formula has been used to optimize the parameters of the grating. We find that for the parameters corresponding to the Dartmouth experiment, the start current can be reduced by six times by optimizing the grating parameters.

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