FREE-ELECTRON LASER WITH BESSEL BEAM CAVITY

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Abstract

The conventional cavity for a free-electron laser (FEL) oscillator usually forms an optical beam of Gaussian mode, which undergoes transverse spread along the interaction region. The transverse divergence of an optical beam will induce reduction of the FEL gain by three aspects: degenerating filling factor, causing diffraction loss and limiting the effective interaction distance. Bessel optical beam has been experimentally demonstrated diffraction-free characteristics in its propagation, which provides a possibility of improvement of FEL gain. In this paper, we present a conceptual design of a Bessel beam cavity for the free-electron laser oscillator. This cavity generates nondiffracting optical beam in the wiggler, resulting in improving the filling factor, decreasing the diffraction loss and elongating the effective interaction distance.

INTRODUCTION

Free-electron laser (FEL) oscillator is a successful device generating electromagnetic wave in the range of sub-micron to far infrared wave. The radiation is generated as a relativistic electron beam passes the periodic magnetic field, i.e., wiggler, and is amplified by a conventional cavity, usually composed of two concave reflected mirrors. As we know, the conventional cavity forms Gaussian optical beam, performing transverse divergence in its propagation. The light divergence degenerates the FEL gain from three aspects: the transverse spread of the light profile along the interaction distance leads to a bad overlap of electron beam and light beam, resulting in a small filling factor; the divergence also gives rise to diffraction loss because of the limitation of transverse size of a wiggler gap, especially for the operation of long wavelength; the effective interaction only occurs within the Rayleigh region of a Gaussian beam, usually not long, which limits the energy exchange. To overcome the divergence of the Gaussian beam, we consider designing a Bessel beam cavity for a FEL oscillator.

GENERATION OF BESSEL BEAM

The Bessel beam has been widely studied and introduced into various applications since Durnin et.al experimentally demonstrated its diffraction-free property [1-6]. A solution of the Helmholtz wave equation for an azimuthally symmetric wave of frequency ω propagating in the positive z direction gives out the Bessel beam expression

$$\psi(\bar{r},t) = J_0(k\sin(\alpha)\rho)\exp[i(k\cos(\alpha)z - \omega t)], \quad (1)$$

where $k = \omega/c$ is the wave number, c the light speed in vacuum. Clearly, the Bessel beam takes on the property of diffraction-free propagation. The integral representation of Bessel function exhibits that the Bessel beam is superposition of an infinite set of ordinary plane waves making angle α with respect to the z axis [7]. Actually, the ideal Bessel beam retaining diffraction-free property over arbitrarily large distance cannot be realized, since it requires arbitrarily large power. The physically meaning Bessel beam that can be produced is an aperture-truncated one, possessing nearly diffraction-free properties within some axial distance. The generation of aperture-truncated Bessel beam was widely studied elsewhere [8-12].

Many methods have been proposed to generate a Bessel beam. For example, one can transform a Gaussian beam to the Bessel beam through a circular slit, holographic optical element or an axicon [13]. Among them, the simplest way but with high conversion efficiency is to use an axicon, as shown in Fig. 1. The axicon is an optical element forming a line image of a small source. Based on the geometrical optics, one obtains the expressions for the extent of the line focus [13],

$$n\sin\alpha = \sin(\theta+\alpha),$$
$$l_B = R(\cot\theta - \tan\alpha) \quad (2)$$

where n is refractive index, R indicates the radius of a light beam incident on the axicon, l_B is the line focus extent, i.e., the length of Bessel beam region.

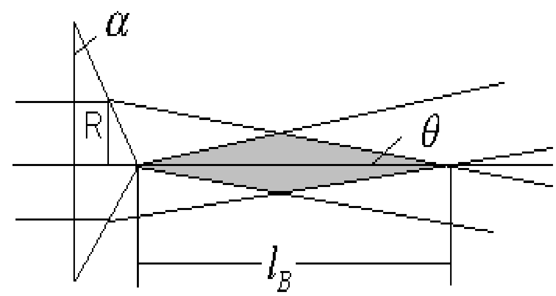

Figure 1: Bessel beam generation by axicon.

BESSEL BEAM CAVITY

The Bessel beam cavity is schematically shown in Fig.2. It is composed of two reflected mirrors (M1, M2) and two axicons (C1, C2). According to the above introduction, we know that the Bessel beam is formed between those two axicons. One of the mirrors is fully reflected, and the other is partially reflected to output the light. The electron beam is supposed to be guided in and out of the wiggler by bending magnet.

The transverse distribution of a Bessel beam is expressed by the zeroth order Bessel function. We apply the central lobe of the Bessel beam to interaction with the

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electron beam. In order to achieve a good overlap of the two beams, the radius of the central lobe should be carefully chosen according to the radius of an electron beam. In this paper, a usual electron beam radius, 0.5 mm, is assumed. Simple calculation gives the radius of central lobe of the Bessel beam 0.686 mm, which shows the same size of the electron beam at the place where the amplitude drops down to e^{-1} of the maximum. For the wave band of micrometer, there are many materials can be used to make the axicon, and the reflective index is about n=2. By using these parameters, the relation of angle α and FEL wavelength was calculated and shown in Fig.3. For today's technology, the minimum angle α with high precision that can be machined is 0.05°, which means this kind of cavity is suitable for radiation wavelength more than 2 μ m. From Fig.2 we know that the distance between the apex of the two axicons is up to the gap r_g of the wiggler. As an example, here we take 14mm for r_g, which is the same size to the electromagnetic wiggler operated on the NewSUBARU storage ring for optical klystron [14]. We leave 2 m for installation of bending magnet guiding the electron beam in and out of the interaction region, then, the maximum wiggler length is calculated and given in Fig. 3. It is seen that the maximum wiggler length drops down as the wavelength increasing, and a 5m long wiggler can be realized for 6 μ m wavelength.

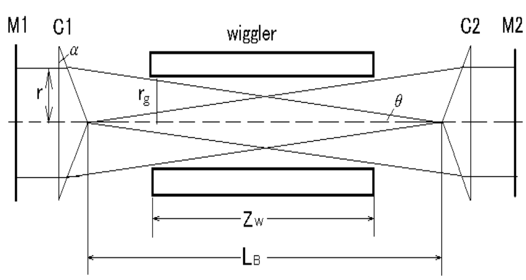

Figure 2: Conceptual design of a Bessel beam cavity for free-electron laser.

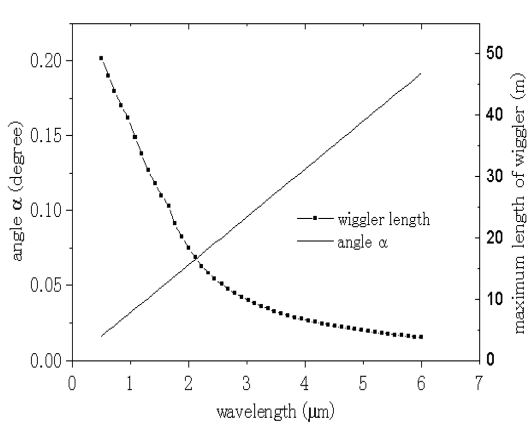

Figure 3: Angle α and maximum wiggler length wtih respect to the radiation wavelength.

IMPROVEMENT OF GAIN

From Madey's theorem it is known that the FEL gain is proportional to the derivative of the spontaneous emission spectrum. Any effect causing inhomogeneous broadening of this spectrum induces reduction in gain. An ideal gain should be multiplied by correction factors to reach the real FEL gain. There are many correction factors for such as energy spread, finite emittance, longitudinal slippage, cavity loss and filling factor. Filling factor, which accounts for the overlap between the electron and optical beam, appears to be the most significant of any of the corrections.

For the Bessel beam, the filling factor is close to 1 since we have chosen the proper transverse size of the optical beam to match that of the electron beam. For the Gaussian beam, the filling factor can be calculated by [15]

$$F = \frac{A_e}{A},$$

$$A = 2\pi\left(\frac{\overline{w}^2}{4} + \sigma_x^2\right)^{1/2}\left(\frac{\overline{w}^2}{4} + \sigma_y^2\right)^{1/2},$$

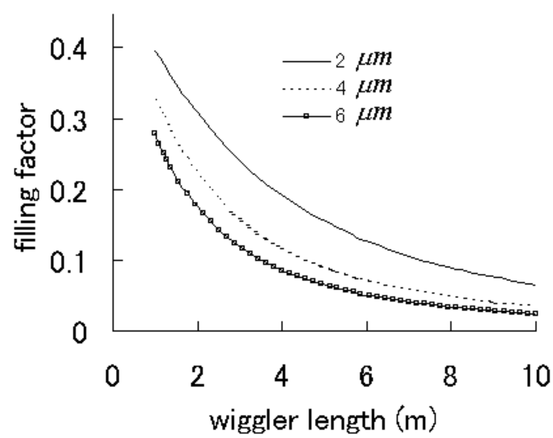

Figure 4: Filling factor with respect to the wiggler length for different wavelength.

$$\overline{w} = \frac{w_0}{L_u}\int_{-L_u/2}^{L_u/2}\sqrt{1+\left(\frac{\lambda z}{\pi w_0^2}\right)}\,dz,$$

where A_e is the transverse area of electron beam, $\sigma_{x(y)}$ the electron beam transverse size, w_0 the Gaussian beam waist, L_u the wiggler length and λ is the radiation wavelength. The divergence of the optical beam along the wiggler length has been taken into account in Eq. (3). For an optimum optical beam with the waist at the centre of the wiggler and a Rayleigh length of $1/3$ of the wiggler length, we give the calculation results as shown in Fig.4. Obviously, the filling factor decreases as the wiggler length growing. For the case of 6 μ m, the filling factor is lower than 0.1 at a wiggler length of about 5 m. If we use the Bessel beam cavity, the filling factor is close to 1, which means an improvement by about ten times.

CONCLUSION

We try to extend the application of nondiffracting Bessel optical beam to FEL oscillator in order to enhance the interaction of electron beam. A novel cavity of generating Bessel beam is conceptually designed, with employing two axicons. Calculations show that the improvement of filling factor can be achieved, resulting in increase of FEL gain.

Besides the improvement of filling factor, the Bessel beam is able to elongate the effective interaction distance, which will benefits the energy extraction. Certainly, the diffraction-free property of the Bessel beam can avoid the diffraction loss, which also results in promotion of FEL gain.

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