INFLUENCE OF AN ENERGY CHIRP ON SASE FEL OPERATION

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Abstract

Influence of a linear energy chirp in the electron beam on a SASE FEL operation is studied analytically and numerically using 1-D model. Explicit expressions for Green's functions and for output power of a SASE FEL are obtained for high-gain linear regime in the limits of small and large energy chirp parameter. Saturation length and power versus energy chirp parameter are calculated numerically. It is also shown that the effect of linear energy chirp on FEL gain is equivalent to linear undulator tapering (or linear energy variation along the undulator). A consequence of this fact is a possibility to perfectly compensate FEL gain degradation, caused by the energy chirp, by means of the undulator tapering (while keeping a frequency chirp in the radiation pulse) independently of the value of the energy chirp parameter. This opens up a possibility of a conceptual breakthrough: by a proper choice of energy chirp, undulator tapering, and bandwidth of a monochromator, installed behind the undulator, one can select a radiation pulse which is much shorter than inverse FEL bandwidth.

INTRODUCTION

Start-to-end simulations [1] of TTF FEL, Phase 1 [2], have shown a presence of a strong energy chirp (energy-time correlation) within a short high-current leading peak in electron density distribution that has driven SASE FEL process. The energy chirp was accumulated due to the longitudinal space charge after compression. According to the simulations (that reproduced well the measured FEL properties), the energy chirp had a dramatical impact on SASE FEL saturation length and output characteristics. A similar effect takes place during the operation of VUV FEL at DESY in a "femtosecond mode" [3]. Such a mode of operation might also be possible in future X-ray SASE FELs.

There also exists a concept of frequency-chirped SASE FELs[1] aiming at the shortening of radiation pulse with the help of a monochromator [4]. Energy chirp can also be used to tune the output frequency of an FEL with coherent prebunching as it was demonstrated in the experiment at the DUV FEL facility [5]. Thus, a theoretical understanding of the energy chirp effect on the FEL performance is of crucial importance.

Analytical studies on this subject were performed in [6] in one-dimensional approximation. The general form of a time-domain Green's function as an inverse Laplace transform was derived in [6]. It was then reduced to the explicit expression in the limit of small energy chirp parameter up to the first order, resulting in phase correction (and ignoring the gain correction). This explicit solution for the Green's function was used to analyze statistical properties of a chirped SASE FEL in this limit. A second order correction to the FEL gain was presented in [4] but this result is incorrect.

The goal of this paper is to study the impact of energy chirp on SASE FEL performance and to find a possible way to cure the FEL gain degradation.

GREEN'S FUNCTION

Electric field of the amplified electromagnetic wave is presented in the form[2]

$$E = \tilde{E}\exp[i\omega_0(z/c - t)] + C.C.\,,$$

where ω_0 is a reference frequency and $\tilde{E}$ is slowly-varying amplitude [7]. As it was shown in [6], for a SASE FEL, driven by an electron beam with linear energy chirp, $\tilde{E}$ can be written as follows (we use notations from [7]):

$$\tilde{E} = 2E_0 \sum_j e^{-i\hat{s}_j/\rho} e^{2i\hat{\alpha}\hat{s}_j(\hat{s}-\hat{z}/2-\hat{s}_j)} g(\hat{z}, \hat{s}-\hat{s}_j, \hat{\alpha}) \quad (1)$$

Here ρ is the efficiency parameter [7], E_0 is the saturation field amplitude [7], $\hat{z} = \Gamma z$ is a normalized position along the undulator, $\Gamma = 2k_w\rho$, $\lambda_w = 2\pi/k_w$ is the undulator period, $\hat{s} = \rho\omega_0(z/\bar{v}_{z0} - t)$ is normalized position along the electron bunch, $\bar{v}_{z0}$ is average longitudinal velocity (defined for a reference particle). Let the energy linearly depend on a particle position in the bunch (or arrival time). The energy chirp parameter

$$\hat{\alpha} = -\frac{d\gamma}{dt}\frac{1}{\gamma_0\omega_0\rho^2}$$

is defined such that, for positive sign of $\hat{\alpha}$, particles in the head of the bunch have larger energy than those in the tail. Relativistic factor γ_0 for a reference particle (placed at $\hat{s} = 0$) and reference frequency ω_0 are connected by the FEL resonance condition: $\omega_0 = 2ck_w\gamma_0^2/(1+K^2)$, K being rms undulator parameter. Note that the theory is applicable when $\rho\hat{\alpha} \ll 1$ [6]. It is also useful to define normalized detuning [7]: $\hat{C} = [k_w - \omega(1+K^2)/2c\gamma_0^2]/\Gamma$.

The Green's function g, entering Eq. (1), is given by the inverse Laplace transform [6]:

[1]Frequency chirp of SASE FEL radiation is correlated with energy chirp in the electron beam due to the FEL resonance condition

[2]Plane wave and planar undulator are considered in this paper

$$g(\hat{z},\hat{s},\hat{\alpha}) = 2\int\limits_{\gamma'-i\infty}^{\gamma'+i\infty} \frac{dp}{2\pi i p}\exp[f(p,\hat{z},\hat{s},\hat{\alpha})] \ , \qquad (2)$$

where

$$f(p,\hat{z},\hat{s},\hat{\alpha}) = p(\hat{z}-2\hat{s}) + \frac{2i\hat{s}}{p(p+i\hat{\alpha}\hat{s})} \qquad (3)$$

We use a saddle point approximation to get an estimate of the integral (2) for large values of $\hat{z}$ [6]. The saddle point is determined from the condition $f'=0$ which leads to the 4th power equation with three parameters:

$$p^4 + 2i\hat{\alpha}\hat{s}p^3 - \hat{\alpha}^2\hat{s}^2p^2 - \frac{4i\hat{s}}{\hat{z}-2\hat{s}}\,p + \frac{2\hat{\alpha}\hat{s}^2}{\hat{z}-2\hat{s}} = 0 \quad (4)$$

Once the saddle point, p_0, is found, the Green's function can be approximated as follows:

$$g(\hat{z},\hat{s},\hat{\alpha}) = \frac{2\exp[f(p_0,\hat{z},\hat{s},\hat{\alpha})]}{p_0[2\pi f''(p_0,\hat{z},\hat{s},\hat{\alpha})]^{1/2}} \qquad (5)$$

Let us first consider the case when the energy chirp is a small perturbation, $|\hat{\alpha}|\hat{z} \ll 1$, $\hat{z} \gg 1$. A second-order expansion of the Green's function takes the following form

$$g(\hat{z},\hat{s},\hat{\alpha}) \simeq \frac{e^{-i\pi/12}}{\sqrt{\pi\hat{z}}}\exp\left[i^{1/3}\hat{z} + i^{2/3}\frac{\hat{\alpha}\hat{s}}{2}\left(1+i\frac{\hat{\alpha}\hat{z}^2}{36}\right)\right.$$
$$\left. -9i^{1/3}\left(1-\frac{\hat{\alpha}^2\hat{z}^2}{216 i^{2/3}}\right)\frac{(\hat{s}-\hat{z}/6)^2}{\hat{z}} - \frac{i}{2}\hat{\alpha}\hat{s}(\hat{z}-2\hat{s})\right] \qquad (6)$$

The leading correction term is the last term in the argument of the exponential function. It was found in [6] (note difference in definition of normalized parameters). Setting $\hat{\alpha}=0$, one gets from (6) the well-known Green's function for unchirped beam [8].

Now let us consider the case $\hat{\alpha} > 0$ and $1 \ll \hat{\alpha} \ll \hat{z}$. The Green's function for $\hat{s} \gg \hat{\alpha}^{-1}$ is approximated by:

$$g(\hat{z},\hat{s},\hat{\alpha}) \simeq \left(\frac{\hat{\alpha}}{2\pi^2\hat{z}}\right)^{1/4}\exp\left(2\sqrt{\frac{2\hat{z}}{\hat{\alpha}}} - 2\sqrt{\frac{2}{\hat{\alpha}\hat{z}}}\,\hat{s}\right) \quad (7)$$

More thorough analysis for small values of $\hat{s}$ shows that the Green's function is peaked at $\hat{s}_{\mathrm{m}} = 2^{1/3}\hat{\alpha}^{-1}$, i.e. the position of maximum is independent of $\hat{z}$ while the width of the radiation wavepacket is proportional to $\sqrt{\hat{\alpha}\hat{z}}$. The mean frequency of the radiation wavepacket corresponds to a resonant frequency at $\hat{s}=0$. Note also that the beam density excitation is concentrated near $\hat{s}=0$ within much shorter range, of the order of $\hat{\alpha}^{-7/4}\hat{z}^{-1/4}$.

In the case $\hat{\alpha} < 0$ and $1 \ll |\hat{\alpha}| \ll \hat{z}$ the Green's function is given by:

$$g(\hat{z},\hat{s},\hat{\alpha}) \simeq \frac{2^{1/4}e^{-i\pi/2}}{\pi^{1/2}|\hat{\alpha}|^{5/4}\hat{z}^{3/4}\hat{s}}\exp\left(2\sqrt{\frac{2\hat{z}}{|\hat{\alpha}|}}\right.$$
$$\left. + i|\hat{\alpha}|\hat{z}\hat{s} + \frac{2i}{|\hat{\alpha}|^2\hat{s}} - \frac{2\sqrt{2}}{|\hat{\alpha}|^{7/2}\hat{z}^{1/2}\hat{s}^2} - 2\sqrt{\frac{2}{|\hat{\alpha}|\hat{z}}}\,\hat{s}\right) \qquad (8)$$

The width of the radiation wavepacket (and of beam density excitation as well) is of the order of $|\hat{\alpha}|^{-7/4}\hat{z}^{-1/4}$. The maximum of the wavepacket is positioned at $\hat{s}_{\mathrm{m}} = 2^{5/4}|\hat{\alpha}|^{-7/4}\hat{z}^{-1/4}$, i.e. the wavepacket is shrinking and back-propagating (with respect to the electron beam) with increasing $\hat{z}$. The mean frequency of the wavepacket is blue-shifted with respect to resonant frequency at $\hat{s}=0$. In normalized form this shift is $\Delta\hat{C} = -|\hat{\alpha}|\hat{z}/2$.

LINEAR REGIME OF SASE FEL

The normalized radiation power (normalized efficiency), $<\hat{\eta}> = P_{\mathrm{SASE}}/\rho P_{\mathrm{beam}}$, can be expressed as follows [7]:

$$<\hat{\eta}> = \frac{<|\tilde{E}|^2>}{4E_0^2} \ , \qquad (9)$$

where $<\ldots>$ means ensemble average. One can easily get from (1):

$$<\hat{\eta}(\hat{z},\hat{\alpha})> = \frac{1}{N_{\mathrm{c}}}\int_0^\infty d\hat{s}|g(\hat{z},\hat{s},\hat{\alpha})|^2 \ . \qquad (10)$$

Here $N_{\mathrm{c}} = N_\lambda/(2\pi\rho)$ is a number of cooperating electrons (populating $\Delta\hat{s}=1$), N_λ is a number of electrons per wavelength. The local power growth rate [9] can be computed as follows:

$$G(\hat{z},\hat{\alpha}) = \frac{d}{d\hat{z}}\ln <\hat{\eta}(\hat{z},\hat{\alpha})> \ . \qquad (11)$$

Applying Eqs. (10), (11) to the asymptotical cases, considered in the previous Section, we get the following results. For the case $|\hat{\alpha}|\hat{z} \ll 1$, $\hat{z} \gg 1$ the FEL power is given by

$$<\hat{\eta}> \simeq \frac{\exp\left\{\sqrt{3}\hat{z}\left[1-(\hat{\alpha}\hat{z}/12)^2/3\right] + \hat{\alpha}\hat{z}/12\right\}}{3^{5/4}\sqrt{\pi\hat{z}}N_{\mathrm{c}}} \quad (12)$$

and the local power growth rate is

$$G(\hat{z},\hat{\alpha}) \simeq \sqrt{3}\left[1-\left(\frac{\hat{\alpha}\hat{z}}{12}\right)^2\right] - \frac{1}{2\hat{z}} + \frac{\hat{\alpha}}{12} \ . \qquad (13)$$

It reaches maximum $G_{\mathrm{m}} = \sqrt{3}\left[1-(|\hat{\alpha}|/16)^{2/3}\right] + \hat{\alpha}/12$ at the position $\hat{z}_{\mathrm{m}} = 3^{1/2}2^{2/3}/|\hat{\alpha}|^{2/3}$. Although the condition $|\hat{\alpha}|\hat{z} \ll 1$ was used to derive Eqs. (12), (13), they are pretty accurate up to the values $|\hat{\alpha}|\hat{z}$ of the order of unity as it was seen from comparison with numerical simulations.

For the case $\hat{\alpha} > 0$ and $1 \ll \hat{\alpha} \ll \hat{z}$ we get rather simple expressions:

$$<\hat{\eta}(\hat{z},\hat{\alpha})> \simeq \frac{\hat{\alpha}}{8\pi N_{\mathrm{c}}}\exp\left(4\sqrt{\frac{2\hat{z}}{\hat{\alpha}}}\right) , \qquad (14)$$

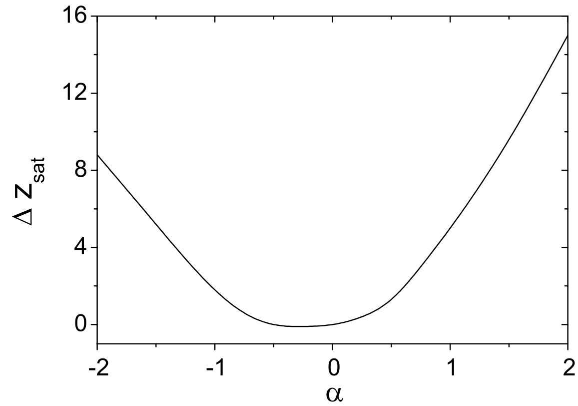

Figure 1: Increase of saturation length $\Delta\hat{z}_{\rm sat} = \hat{z}_{\rm sat}(\hat{\alpha}) - \hat{z}_{\rm sat}(0)$ versus parameter $\hat{\alpha}$. Here $\hat{z}_{\rm sat}(0) = 13$.

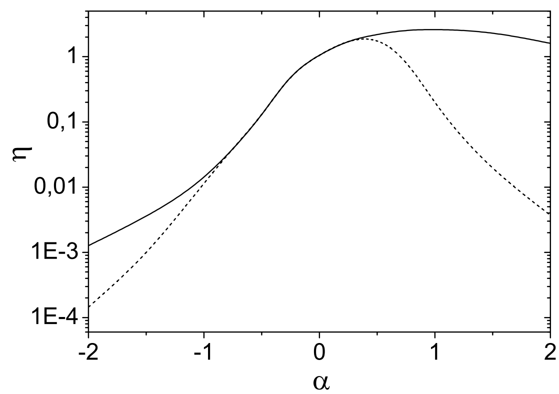

Figure 2: Normalized output power versus parameter $\hat{\alpha}$. Solid: $\hat{z} = \hat{z}_{\rm sat}(\hat{\alpha})$ (see Fig. 1); dash: $\hat{z} = \hat{z}_{\rm sat}(0) = 13$.

$$G(\hat{z},\hat{\alpha}) \simeq 2\sqrt{\frac{2}{\hat{\alpha}\hat{z}}}\ . \qquad (15)$$

For large negative values of $\hat{\alpha}$ we obtain:

$$<\hat{\eta}> \simeq \frac{1}{2^{7/4}\pi^{1/2}|\hat{\alpha}|^{3/4}\hat{z}^{5/4}N_{\rm c}} \exp\left(4\sqrt{\frac{2\hat{z}}{|\hat{\alpha}|}}\right), \qquad (16)$$

$$G(\hat{z},\hat{\alpha}) \simeq 2\sqrt{\frac{2}{|\hat{\alpha}|\hat{z}}} - \frac{5}{4\hat{z}}\ . \qquad (17)$$

NONLINEAR REGIME

We studied nonlinear regime of a chirped SASE FEL operation with 1-D version of the code FAST [7, 10]. Analytical results of two previous Sections were used to check how well we simulate energy chirp effect. Green's function was modelled by exciting density modulation on a short scale, $\Delta\hat{s} \ll 1$. SASE FEL initial conditions were simulated in a standard way [7]. The results of numerical simulations in all cases were in a good agreement with analytical results presented above. The main results of simulation of nonlinear regime are presented in Figs. 1,2. Saturation length and power are functions of two parameters, $\hat{\alpha}$ and $N_{\rm c}$. For our simulations we have chosen $N_{\rm c} = 3 \times 10^7$ - a typical value for VUV SASE FELs. Note, however, that the results, presented in Figs. 1,2, very weakly depend on $N_{\rm c}$. Fig. 1 shows increase of saturation length with respect to unchirped beam case. In Fig. 2 the output power is plotted versus chirp parameter for two cases: when undulator length is equal to a saturation length for a given $\hat{\alpha}$ and when it is equal to the saturation length for the unchirped beam case. One can see sharp reduction of power for negative $\hat{\alpha}$ while a mild positive chirp ($\hat{\alpha} < 0.5$) is beneficial for SASE.

ENERGY CHIRP AND UNDULATOR TAPERING

Let us consider now the case when there is no energy chirp ($\hat{\alpha} = 0$) and the detuning parameter changes linearly along the undulator [7]: $\hat{C}(\hat{z}) = \hat{b}_1\hat{z}$. This change can be due to variation of undulator parameters ($K(\hat{z})$ and/or $k_{\rm w}(\hat{z})$) or due to an energy change $\gamma_0(\hat{z})$. We have found from numerical simulations that in such case the effect on FEL gain is exactly the same as in the case of energy chirp and no taper if $\hat{\alpha} = 2\hat{b}_1$ for any value of $\hat{\alpha}$ (Fig. 3 shows an example). Therefore, all the results of two previous Sections can be also used for the case of linear variation of energy or undulator parameters with the substitution $\hat{\alpha} \rightarrow 2\hat{b}_1$. The amplitudes of Green's functions are also the same while the phases are obviously different. In case of $\hat{b}_1 = 0$, $\hat{\alpha} \neq 0$ there is a frequency chirp along the bunch while in the case $\hat{b}_1 = 0$, $\hat{\alpha} \neq 0$ the frequency is changing along the undulator.

An effect of undulator tapering (or energy change along the undulator) on FEL gain was studied in [9] in the limit $\hat{b}_1 \ll 1$. Comparing our Eq. (12) (with the substitution $\hat{\alpha} \rightarrow 2\hat{b}_1$) and Eq. (45) of Ref. [9], we can see that quadratic correction term in the argument of the exponential function is the same but the linear term is two times larger in [9]. The reason for discrepancy is that the frequency dependence of the pre-exponential factor in Eq. (42) of Ref. [9] is neglected.

A symmetry between two considered effects (energy chirp and undulator tapering) can be understood as follows. If we look at the radiation field acting on some test electron from an electron behind it, this field was emitted at a retarded time. In first case a radiating electron has a detuning due to an energy offset, in the second case it has the same detuning because undulator parameters were different at a retarded time. The question arises: can these two effects compensate each other? We give a positive answer based

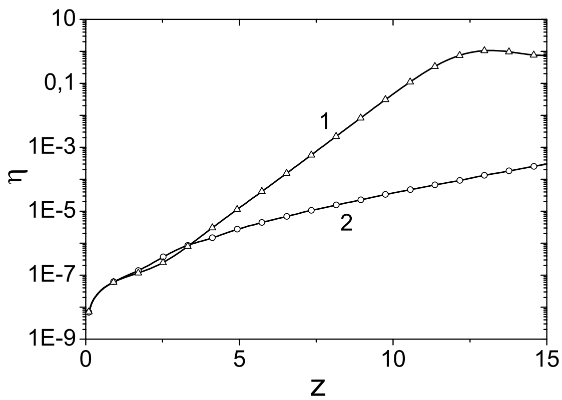

Figure 3: Normalized power versus undulator length. Solid line 1: $\hat{\alpha} = 0$, $\hat{b}_1 = 0$; triangles: $\hat{\alpha} = 4$, $\hat{b}_1 = -2$; solid line 2: $\hat{\alpha} = 4$, $\hat{b}_1 = 0$; circles: $\hat{\alpha} = 0$, $\hat{b}_1 = 2$.

on numerical simulations (see Fig. 3 as an example): by setting $\hat{b}_1 = -\hat{\alpha}/2$ we get rid of gain degradation, and FEL power at any point along the undulator is the same as in the case of unchirped beam and untapered undulator. This holds for any value of $\hat{\alpha}$. For instance, if one linearly changes magnetic field H_w of the undulator, the compensation condition can be written as follows (nominal values of parameters are marked with subscript '0'):

$$\frac{1}{H_{\mathrm{w}0}} \frac{dH_{\mathrm{w}}}{dz} = -\frac{1}{2} \frac{(1+K_0^2)^2}{K_0^2} \frac{1}{\gamma_0^3} \frac{d\gamma}{cdt} \qquad (18)$$

Of course, in such a case we get frequency chirped SASE pulse. Since compensation of gain degradation is possible also for large values of $\hat{\alpha}$ (there is no theoretical limit on the value of chirp parameter, except for above mentioned condition $\rho\hat{\alpha} \ll 1$), one can, in principle, organize a regime when a frequency chirp within an intensity spike is much larger than the natural FEL bandwidth (given by $\rho\omega_0$).

GENERATION OF ATTOSECOND PULSES

Many schemes for generation of femto- and attosecond pulses from X-ray SASE FELs are proposed. Here we mention the schemes considered in [11, 12] making use of energy modulation of a short slice in the electron bunch by a high-power few-cycle optical pulse in a two-period undulator. Due to energy modulation the frequency of SASE radiation in X-ray undulator (resonant to, say, 0.1 nm [11]) is correlated to the longitudinal position within the few-cycle-driven slice of the electron beam. The largest frequency offset corresponds to a single-spike pulse in time domain (about 300 as in [11]) . The selection of single-spike pulses is achieved by using a crystal monochromator after the X-ray undulator [11].

Using the compensation effect, described in the previous Section, one can modify this scheme such that a monochromator is not required. Indeed, there is a strong energy chirp around zero-crossing of energy modulation (for specific parameters of Ref. [11] the chirp parameter is $\hat{\alpha} \simeq 2$). If one uses undulator tapering with $\hat{b}_1 \simeq -1$ then only a short slice around zero-crossing produces powerful FEL pulse. The main part of the bunch is unmodulated and is, therefore, suffered from strong negative undulator tapering (from Fig. 1 one can estimate a suppression factor of 10^4). Therefore, a high-contrast attosecond pulse is directly produced in the undulator.

The fact that a SASE FEL can operate with a strong chirp parameter (in combination with undulator tapering) without gain degradation, opens up a possibility of a conceptual breakthrough: one can get from SASE FEL a radiation pulse which is much shorter than inverse FEL bandwidth. Indeed, in the case of $\hat{\alpha} \gg 1$, the idea of Ref. [4] can be generalized to a time scales that are much shorter than a duration of intensity spike. In this case the frequency chirp inside an intensity spike (its duration is given by inverse FEL bandwidth) is much larger than FEL bandwidth. By appropriate choice of a monochromator bandwidth [6] one can select an X-ray pulse that is shorter by a factor $\sqrt{2\hat{\alpha}}$ than inverse FEL bandwidth.

To illustrate a possible technical realization of this idea, we can suppose that the energy modulation by a few-cycle optical pulse in the scheme of Ref. [11] is increased by a factor 3 so that $\hat{\alpha} \simeq 6$. In combination with undulator tapering and a monochromator, this will allow to obtain intense X-ray pulses that are shorter than 100 as. Finally, without discussing technical limits, we should stress that a "fundamental" limit on pulse duration $(\rho\omega_0)^{-1}$ can be overcome. That is the most important result of this paper.

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