# BUNCH COMPRESSION STABILITY DEPENDENCE ON RF PARAMETERS

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## *Abstract*

In present designs for FEL injector linacs with high electron peak currents and short bunch lengths, higher harmonic rf systems are often used to optimize the final longitudinal charge distributions [4]. This opens degrees of freedom for the choice of rf phases and amplitudes to achieve the necessary peak current with a reasonable longitudinal bunch shape. It had been found empirically that different working points result in different tolerances for phases and amplitudes.

We give an analytical expression for the sensitivity of the compression factor on phase and amplitude jitter for a bunch compression scheme involving two rf systems and two magnetic chicanes. A cancellation scheme which loosens the rf jitter tolerances is discussed and numerical results for the case of the European XFEL are presented.


## INTRODUCTION

A two-stage bunch compression system as it is used for instance in the XFEL or the LCLS is shown schematically in Fig.1. It consists of the dipole magnet chicanes BC1 and BC2, their upstream accelerating RF sections and, upstream of the first chicane, a higher harmonic rf section.

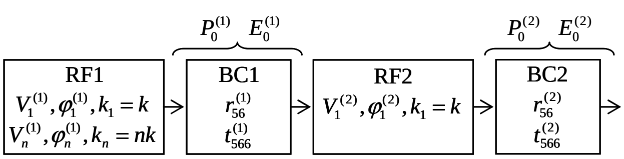


Figure 1: Two-stage bunch compression system scheme.

The basic parameters which have to be chosen are the beam energies at the chicanes BC1 and BC2 ($E^{(1)}$ and $E^{(2)}$), their compression factors $C^{(1)}$ and $C^{(2)}$, the required rf chirps ($p_0'^{(1)}$ and $p_0'^{(2)}$) and the corresponding values for the 'longitudinal dispersion' $r_{56}$. The total compression is determined by the ratio of the required peak current in the undulator (~ 5 kA for the XFEL) and the available current from the gun. The XFEL requires a total compression of $C^{(1+2)} \approx 100$.

The higher harmonic rf system (3[rd] harmonic in the XFEL case) is used to compensate higher order effects of the dispersion and the rf voltage. The amplitudes $a_{1\,\mathrm{or}\,n}$ and phases $\varphi_{1\,\mathrm{or}\,n}$ of the fundamental mode rf and the n[th] harmonic rf are related to the normalized total voltage, or, which is equivalent, to the normalized beam momentum and its derivatives by the following condition:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -k & 0 & -(nk) \\ -k^2 & 0 & -(nk)^2 & 0 \\ 0 & k^3 & 0 & (nk)^3 \end{bmatrix} \cdot \begin{bmatrix} a_1 \cos\varphi_1 \\ a_1 \sin\varphi_1 \\ a_n \cos\varphi_n \\ a_n \sin\varphi_n \end{bmatrix} = \begin{bmatrix} 1 \\ p_0'^{(1)} \\ p_0''^{(1)} \\ p_0'''^{(1)} \end{bmatrix} \quad (1)$$

Eq. 1 determines the rf settings for four 'knobs', which control the (normalized) final beam energy, the momentum chirp and the second and third derivative of the momentum deviation in the bunch center.

The first two knob settings are obviously fixed and $p_0''^{(1)}$ is needed to compensate 2nd order effects in the bunch compressor to avoid partial over-compression and thus spikes in the longitudinal charge distribution, which then cause strong wake fields [3].

The influence of higher order momentum deviations on the final bunch shape is weaker, so that $p_0'''^{(1)}$ is a relatively free parameter. In the numerical simulations shown in the last chapter, $p_0'''^{(1)}$ is scanned to find the working point with the lowest jitter sensitivities for rf phases and amplitudes.

## ERROR SENSITIVITY OF THE FIRST STAGE

We consider only the first stage and skip for this section the upper index "(1)". The energy length correlation after the first linac section and the 3[rd] harmonic section is

$$E(s_a) = E_0^{(\mathrm{gun})} + qV_1 \cos(ks_a + \varphi_1) + qV_3 \cos(3ks_a + \varphi_3).$$

$s_a$ is the relative longitudinal particle position in the bunch and $E_0^{(\mathrm{gun})}$ the beam energy at the entrance of RF1. The reference energy is $E_0 = E(0)$ and the relative momentum deviation is approximately

$$p(s_a) \approx E(s_a)/E_0 - 1.$$

The longitudinal position after compression is

$$s_b = s_a - \left(r_{56} p + t_{566} p^2 + O_3(p)\right),$$

and the bunch is compressed by the ratio

$$C = \left[\frac{\partial s_b}{\partial s_a}\right]^{-1} = \left[1 - \left(r_{56} + 2t_{566} p + O_2(p)\right)\frac{\partial p}{\partial s_a}\right]^{-1}.$$

The nominal compression at design working point ($p_0 = 0$ and $p_0' = p'(0)$) is $C_0 = \left(1 - r_{56} p_0'\right)^{-1}$, with $p' = \partial p / \partial s_a$. To estimate the error sensitivity with respect to the variation of an rf parameter (e.g., $\varphi = \varphi_{\mathrm{design}} + x$ for the phase or $A = A_{\mathrm{design}} + x$ for the amplitude), we write the relative momentum deviation

$p = p(s_a, x)$ explicitly as function of the error parameter $x$ and the longitudinal bunch position $s_a$. The relative error sensitivity of the compression factor is:

$$\frac{1}{C_0}\frac{\partial C}{\partial x} = A\frac{\partial p}{\partial x} + B\frac{\partial p'}{\partial x}, \qquad (2)$$

with $A = -2(C_0 - 1)t_{566}/r_{56}$ and $B = -C_0 r_{56}$. It depends on the sensitivity of the momentum ($\partial p/\partial x$) and that of the chirp ($\partial p'/\partial x = \partial^2 p/\partial x \partial s_a$).

To calculate the effect of phase jitter of the fundamental harmonic rf we write for the momentum:

$$p(s_a, x) = a_1 \cos(k\, s_a + \varphi_1 + x) + a_n \cos(nk\, s_a - \varphi_n),$$

with $p(0,0) = 0$ and $p'(0,0) = (1 - C_0^{-1}) \cdot r_{56}^{-1}$.

The phase sensitivity of the compression factor with respect to fundamental harmonic rf-jitter then follows as

$$\frac{1}{C_0}\frac{\partial C}{\partial x} = -a_1 (A \sin\varphi_i + Bk_i \cos\varphi_i). \qquad (3)$$

This expression is derived for a single stage compression system, but as we show in the next chapter, the presence of the second compression stage causes only minor changes for the case of the XFEL parameters.

For the special case of one rf system on the fundamental mode and a simple chicane consisting of four dipole magnets, where $r_{56} < 0$ and $t_{566} \approx -1.5 r_{56}$, the phase sensitivity is given by (if $E_0^{(\mathrm{gun})} << E_0$):

$$\frac{1}{C_0}\frac{\partial C}{\partial x} \approx (1 - C_0) \cdot \left( 3\tan\varphi_1 + \frac{1}{\tan\varphi_1} \right).$$

In the case of high compression ($C_0 >> 1$) and acceleration not too far off crest ($|\varphi_1| << 1$), the relative compression error in such a system is approximately

$$\Delta C / C_0 \approx -C_0\, \Delta\varphi_1 / \varphi_1 .$$

The tolerances for the phase stability and absolute phase are by the bunch compression factor tighter than the tolerance on the final peak current. In the case of the XFEL, phase stability requirements become as demanding as a few hundredth of a degree.

## ERROR SENSITIVITY OF A TWO- STAGE COMPRESSION SYSTEM

In the following, we extend our sensitivity analysis to a two-stage compression system. In this case, the energy length correlation after RF2 is

$$E^{(2)}(s_b(s_a)) = E^{(1)}(s_a) + qV_1^{(2)} \cos(ks_b(s_a) + \varphi_1^{(2)}).$$

The reference energy is $E_0^{(2)} = E^{(2)}(0)$ and the relative momentum deviation is given by:

$$p^{(2)}(s_b) \approx E^{(2)}(s_b)/E_0^{(2)} - 1$$

$$p^{(2)}(s_b) \approx up^{(1)}(s_a) + p^{(2r)}(s_b)$$

with $u = E_0^{(1)}/E_0^{(2)}$ and

$$p^{(2r)}(s_b) = V_1^{(2)} \cos(ks_b + \varphi_1^{(2)})/E_0^{(2)} - 1$$

is the chirp created by the second fundamental mode rf system RF2 (see Fig. 1). The longitudinal position after the 2nd compressor follows as

$$s_c = s_b - (r_{56}^{(2)} p^{(2)} + O_2(p^{(2)})).$$

The total compression factor of the two stages is

$$C^{(1+2)} = \frac{1}{(C^{(1)}\tilde{C}^{(2)})^{-1} - (r_{56}^{(2)} + O(p^{(2)})) \cdot u \dfrac{\partial p^{(1)}}{\partial s_a}},$$

$C^{(1)}$ being the compression of the 1st stage and

$$\tilde{C}^{(2)} = \left[ 1 - (r_{56}^{(2)} + O(p^{(2)})) \frac{\partial p^{(2r)}}{\partial s_b} \right]^{-1}$$

the compression of the 2nd stage without the chirp of the 1st stage. With the help of the nominal parameters

$$\tilde{C}_0^{(2)} = [1 - r_{56}^{(2)} p_0'^{(2r)}]^{-1},$$

$$C_0^{(2)} = [1 - r_{56}^{(2)} p_0'^{(2)}]^{-1},$$

$$C_0^{(1+2)} = C_0^{(1)} C_0^{(2)} = \left[ (C_0^{(1)} \tilde{C}_0^{(2)})^{-1} - r_{56}^{(2)} u p_0'^{(1)} \right]^{-1}$$

where

$$p_0'^{(2)} = (\partial p^{(2)}/\partial s_b)_{s_b = 0}$$

and

$$p_0'^{(2r)} = p_0'^{(2)} - C_0^{(1)} u p_0'^{(1)},$$

we can write the sensitivity of the total compression factor to errors of parameters of the first rf system as

$$\frac{\partial C^{(1+2)}}{C_0^{(1+2)} \partial x} = \frac{C_0^{(1+2)}}{C_0^{(1)} \tilde{C}_0^{(2)}} \left\{ \frac{\partial C^{(1)}}{C_0^{(1)} \partial x} \right\} - C_0^{(1+2)} r_{56}^{(2)} u \frac{\partial p'^{(1)}}{\partial x},$$

neglecting second order effects downstream of the first magnet chicane.

For the parameter range of the XFEL bunch compression system the second term is small, so that the error sensitivity behaves similar to that of the 1st stage (multiplied with $C_0^{(1+2)}/C_0^{(1)}\tilde{C}_0^{(2)}$). For the case that the beam is on crest in RF2, the two-stage system has the same sensitivity as the single stage compressor (for the same total compression factor). Adding chirp in the second linac by operating the beam off-crest reduces the sensitivity somewhat (in principle of up to the compression factor of the 2nd stage), but for reasonable penalty in rf power the gain for XFEL parameters is small (see last chapter).

## PHASE JITTER COMPENSATION

A possible way to loosen the extremely tight phase tolerances is suggested by Eq. 2. For the four-magnet chicane, the constants $A$ and $B$ have the same sign. Since $r_{56}$ is negative and a negative chirp is needed for compression, a shift in phase which increases the beam momentum must be accompanied with an increased chirp (more negative!) to reduce the sensitivity. The effectively reduced longitudinal dispersion of the magnet chicane due to the higher beam energy is then compensated by the stronger chirp.

An rf system with a single frequency cannot provide this compensation, as for instance Eq. 3 shows: As, again, the constants $A$ and $B$ have the same sign, the right hand terms can compensate each other only if $\cos\varphi_i > 0$ and $\sin\varphi_i < 0$ (acceleration, but with positive (de-compressing) chirp) or if $\cos\varphi_i < 0$ and $\sin\varphi_i > 0$ (negative (compressing) chirp, but de-acceleration).

In the presence of a higher harmonic rf system, it is possible to find a working point where the terms of Eq. 2 compensate each other to a significant degree for both systems.

Such a working point for the case of the XFEL is sketched in Fig. 2. The sum of the fundamental and 3rd harmonic field is shown in red. The total field accelerates and has a negative slope $p' = \left(1 - C_0^{-1}\right) \cdot r_{56}^{-1}$ as it is required for compression. The fundamental mode rf provides the acceleration and the higher harmonic system the chirp.

The compensation is shown in the lower graphs of Fig. 2: Increasing amplitude due to phase offsets in both rf systems is always accompanied by a stronger chirp.

To determine the optimal rf parameters which provide the required beam energy, rf chirp and 2nd momentum derivative (see Eq. 1) as well as the minimum phase sensitivity for both systems, we varied the third derivative of the momentum $p_0'''^{(1)}$. The results of this scan are shown in the next chapter.

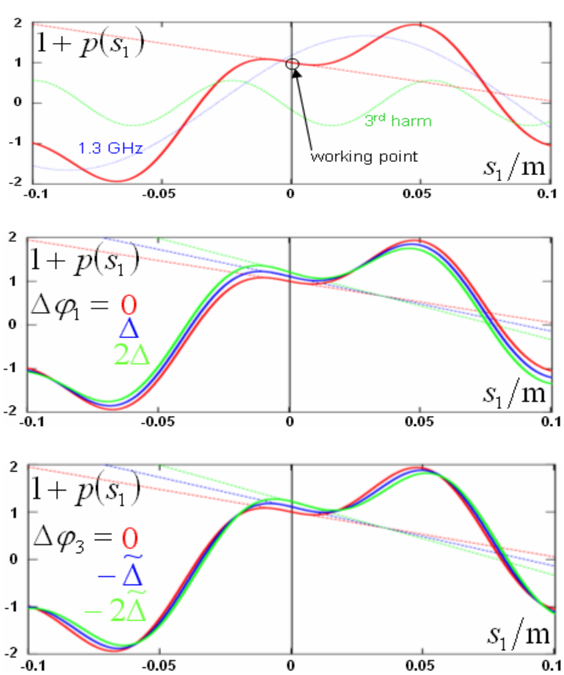


Figure 2: On top: Voltages of fundamental and 3rd harmonic rf and the sum voltage. The lower graphs show the sum voltage and its slope at the working point for different phase offsets for the fundamental mode rf system (middle) and for the 3rd harmonic system (bottom).

# OPTIMAL WORKING POINT FOR THE XFEL BUNCH COMPRESSION SYSTEM

In [1] it has been shown that growth of the projected emittance due to CSR effects is minimal for $C^{(1)} \approx 20$.

Space charge effects give a lower limit for the beam energy for the compression stages: Simulation calculations [2] for the XFEL suggest that $I/\gamma^2 > \sim 1$ mA is a good criterion to avoid emittance growth, requiring $E_0^{(1)} > \sim 0.5$ GeV and $E_0^{(2)} > \sim 1$ GeV for design peak currents. Actually, the design beam energy for stage 2 has been chosen to be $E_0^{(2)} = 2\mathrm{GeV}$ to provide head room for higher peak currents.

To compensate the energy offsets caused by the wake fields of the accelerating structures of the main linac, we choose the chirp for the 1st stage to be about 10 MeV per RMS bunch length. In the linac section upstream of the second magnet chicane, the bunch travels on crest.

To find the working point for the rf systems upstream of the first chicane, we scanned different settings of the third momentum derivative $p_0'''^{(1)}$, while keeping the accelerating voltage, the chirp and the second derivative constant.

The sensitivities of the total compression factor with respect to amplitude and phase jitter of these rf systems were calculated with a longitudinal tracking code which includes the non-linearities of rf systems and compressor chicanes, wake field effects and the longitudinal space charge forces.

The resulting necessary voltage for the fundamental harmonic system (Fig. 3) and the phase and amplitude sensitivities of both rf systems (Fig. 4-7) turn out to be mostly smooth functions of the 3rd harmonic rf amplitude, which is plotted on the horizontal axis.

The phase and amplitude offsets which are plotted on the vertical scale cause a change of the final peak current of 10%.

The different curves refer to different settings of chicane beam energies and compression factors (see Table 1). For all cases, the total compression factor is $C^{(1+2)} = 100$.

Table 1: Parameters for different compression scenarios.

| | 1 | 2 | 3 | 4 | **5** | **6** |
|---|---|---|---|---|---|---|
| $E_0^{(1)}/\mathrm{MeV}$ | 500 | 500 | 500 | 500 | **400** | **400** |
| $C^{(1)}$ | 20 | 14 | 20 | 14 | **14** | **14** |
| $r_{56}^{(1)}/\mathrm{mm}$ | 84.4 | 82.3 | 101.4 | 109.3 | **89.1** | **68.4** |
| $\varphi_1^{(2)}/\mathrm{deg}$ | 0 | 0 | 20 | 20 | **20** | **20** |
| $r_{56}^{(2)}/\mathrm{mm}$ | 19.2 | 29.0 | 19.0 | 29.3 | **29.3** | **23.5** |

Case 1 represents the present design values. For phase jitter compensation, the integrated voltage of the 3rd harmonic rf has to be around 250 MV (see Fig. 4 and 5). Perfect compensation of the terms in Eq. 3 for the fundamental mode rf occurs at a setting of 250 MV for the 3rd harmonic system. At 240 MV, the 3rd harmonic system itself is perfectly compensated. For a voltage a little less than 250 MV, both systems have phase tolerances looser than one degree. The fundamental mode rf needs close to 800 MV to reach the 500 MeV beam energy at the exit of the 1st compression stage (see Fig. 2).

In the cases two to four, bunch compression factors are varied and the linac upstream of BC2 is run 20 degrees off-crest (3 and 4).

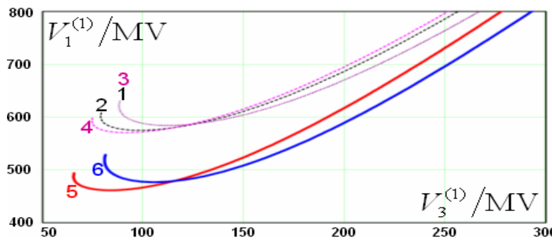


Figure 3: Amplitude of fundamental harmonic rf system vs. 3rd harmonic rf amplitude. The varied parameter is $p_0'''^{(1)}$.

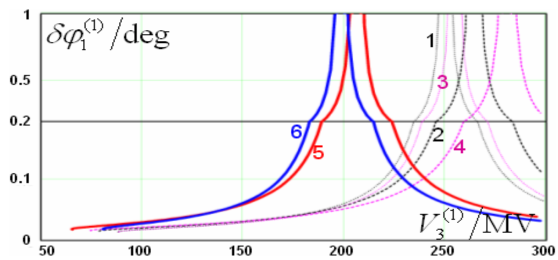


Figure 4: Phase sensitivity of the fundamental harmonic rf system vs. 3rd harmonic rf amplitude

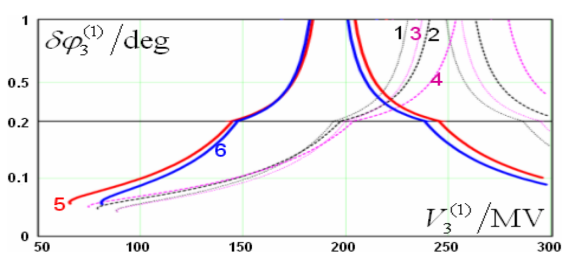


Figure 5: Phase sensitivity of the 3rd harmonic rf system vs. 3rd harmonic rf amplitude.

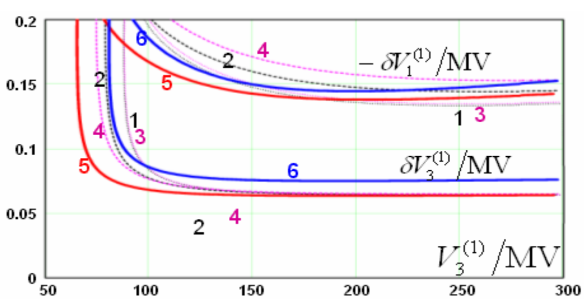


Figure 6: Amplitude sensitivity of both rf systems vs. 3rd harmonic rf amplitude

The cases 5 and 6 have reduced beam energy after BC1 and a consequently reduced voltage requirement for the 3rd harmonic rf system (200 kV). For the cases 1 to 5, chirp and cavity wakes (of the next 100 linac rf modules) approximately compensate each other; in case 6 we drop that requirement.

The amplitude sensitivities in Fig. 6 are voltage offsets in units of MV. Over the most part of the scan, the relative amplitude sensitivity is constant at a level of $1.5 \cdot 10^{-4}$ for the 3rd harmonic rf and $3 \cdot 10^{-4}$ for the fundamental mode. In the parameter regime where the amplitude jitter sensitivity would be reduced, the phase tolerances become extremely tight.

Fig. 7 shows the variation in the charge profile during the scan for the example of case 5 at three points.

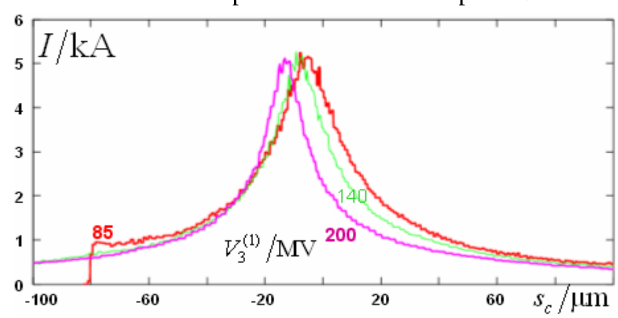


Figure 7: Final longitudinal beam distribution for three different third harmonic rf voltages (case 5)

## CONCLUSION

The phase jitter sensitivity of a bunch compression system can be reduced by more than an order of magnitude if the amplitudes and phases of the fundamental mode rf and the higher harmonic rf system are correctly chosen to provide phase jitter compensation.

For the case of the European XFEL, the 3rd harmonic system has to be operated with an amplitude of 250 MV, more than twice the minimum value necessary to compensate the non-linearities of the fundamental mode rf and the magnet chicanes. At that working point, phase jitter tolerances are of the order of a degree for both rf systems, compared to a few hundredth of a degree in the previous design. Amplitude jitter tolerances are $1.5 \cdot 10^{-4}$ for the 3rd harmonic rf and $3 \cdot 10^{-4}$ for the fundamental mode rf.

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