QUANTITATIVE EVALUATION OF TRANSVERSE PHASE SPACE TOMOGRAPHY

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Abstract

Effects of energy distribution of objective beams on the transverse phase space tomography have been evaluated by numerical calculations. As the result, for a beam with a rectangle-shaped energy distribution, the error in the emittance derived from the reconstructed transverse phase space image is found to be as acceptably low as less than 15 % when the energy spread is less than 15 %. However, for a realistic beam by a thermionic rf gun of our interest which has a low energy tail, the reconstructed image is found severely distorted even with a extremely low energy spread of 0.5 % of the main beam component. On that beam by applying a 70 % energy cut-off with respect to the peak energy, the image is reconstructed well enough in terms of both the emittance and the phase space distribution.

INTRODUCTION

Transverse phase space tomography [1] using a quadrupole magnet and a beam profile monitor is very useful for emittance measurements especially for non-Gaussian beams, since this method directly gives transverse phase space distributions. Therefore, many measurements have been carried out by this method [2].

We have applied the method to diagnose the beam of our FEL driver Linac and thermionic rf gun [3]. However, we could not obtain reliable results due to noisy reconstructed images. The reason for that must be as follows. One can obviously see that in the transfer matrix for the quadrupole magnet in the tomographic method, the matrix parameters vary with the particle energy. Thus, the energy distribution of the objective beam distorts the reconstructed phase space distribution, since the method assumes a mono-energetic beam in reconstructing the phase space distribution. For practical application to a beam with a considerable energy spread by a thermionic rf gun of our interest, evaluation of the effect of energy distribution is indispensable. In this paper, we evaluated the effect and applicable energy distribution of this method by simulation.

In the evaluation firstly we assumed a rectangle-shaped energy distribution with Gaussian distribution in transverse phase space. Secondly we evaluated the effect of a low energy tail, i.e. much less intense low-energetic component than the main beam component, which a thermionic rf gun inherently produces.

CRITICAL ISSUE

As shown in Eq. 1, the rotation angle θ and magnification rate A are described by transfer matrix of the quadrupole magnet and free space assuming the measurement system shown in Fig.1. In Eq. 1, x_0, x_0' are transverse phase space parameters at the entrance of the quadrupole magnet, x is rotated image at the beam profile monitor, B_x is the gradient of the magnetic field in quadrupole magnet, Z is the length of the quadrupole magnet, E is the kinetic energy of the electron, and L is the distance between quadrupole magnet and beam profile monitor.

In the tomographic method, we reconstruct a phase space distribution at the entrance of quadrupole magnet by using the tomographic technique from the measured spatial distributions of beams with many different quadrupole magnet strengths at the profile monitor. However, the rotation angle θ and magnification rate A are different for different energy particle. Therefore, these differences in the θ and A cause the error in the reconstruct phase space distribution, because we can not identify the particle energy one-by-one and assume a mono-energy beam for reconstruction.

$$x = A(x_0 \cos\theta + x_0' \sin\theta) \quad , \quad k^2 = \frac{e}{m_0 (E/m_0 c^2 + 1)\beta c} B_x$$
$$\theta = \tan^{-1}\left(\frac{\frac{1}{k}\sin kZ + L\cos kZ}{\cos kZ - kL\sin kZ} \right) \tag{1}$$
$$A = \sqrt{(\cos kZ - kL\sin kZ)^2 + (\frac{1}{k}\sin kZ + L\cos kZ)^2}$$

SIMULATION

We have simulated the transverse phase space tomography procedure with our experimental setup, which consists of a quadrupole magnet and beam profile monitor as shown in Fig. 1. In this simulation, a Gaussian beam with rectangle-shaped energy distribution which is generated by a Monte Carlo simulation and a realistic beam calculated by PARMELA code [4] have been used for initial beams. The parameters of the initial particles are given at the entrance of quadrupole magnet. Then, 39 spatial projections at the profile monitor with different strength of quadrupole magnet are calculated by the transfer matrix. After that, we reconstruct the initial phase space distributions by using the Ordered Subsets – Expectation Maximization algorithm [5]. It has been already studied that the 39 projections are enough for tomographic technique [2]. In the spatial distribution calculation, space charge effects are not taken into the count to evaluate only the effect of energy distribution.

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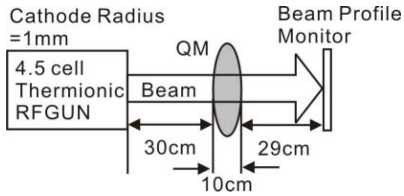


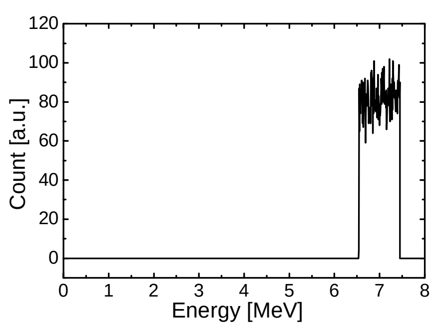
Figure 1: Geometry of our experimental setup.

RESULTS AND EVALUATION

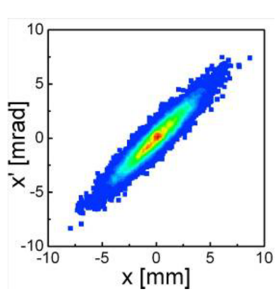
Gaussian beams with rectangle-shaped energy distribution

Gaussian beams with rectangle-shaped energy distributions (Fig. 2(a)) with different energy spreads up to 15% were generated by a Monte Carlo method.. The unnormalized emittance of the initial phase space distribution is 1.5π mm mrad (Fig. 2(b)).

In Fig. 3, the reconstructed phase space distributions of each energy spread are shown. The reconstructed images are the almost same as the initial one. The calculated unnormalized emittances of the reconstructed phase space distributions also consistent with the unnormalized emittance of initial phase space distribution. The ratios of the unnormalized emittances of the reconstructed images to that of the initial image are plotted in Fig. 4 and fitted with a parabolic function of the energy spread. As the result, the energy spread of the initial beams distorts the reconstructed images. However, the errors of emittances are acceptably low as less than 15 % when the energy spread is less than 15 % (see Fig. 4).



(a)



(b)

Figure 2: Gaussian beam with rectangle-shaped energy distribution, (a) Example of energy spectrum (FWHM = 10 %), (b) Phase space distribution (Gaussian, ε = 1.5π mm mrad)

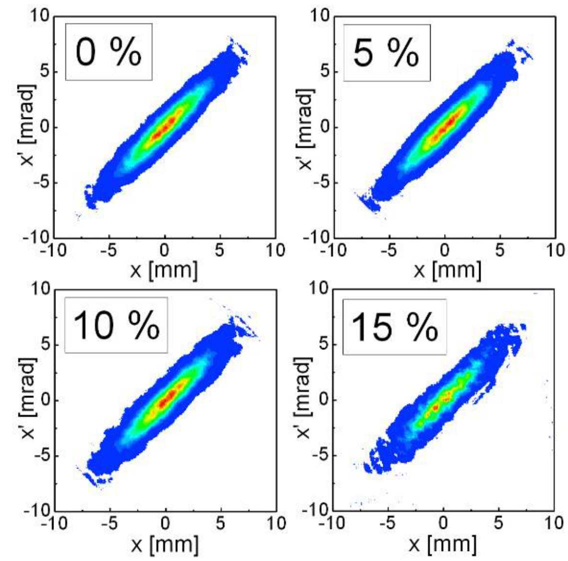


Figure 3: Reconstructed phase space distributions of Gaussian beam with rectangle-shaped energy distribution for different energy spread, upper left: mono energy, upper right: 5 %, lower left: 10 %, lower right: 15 %.

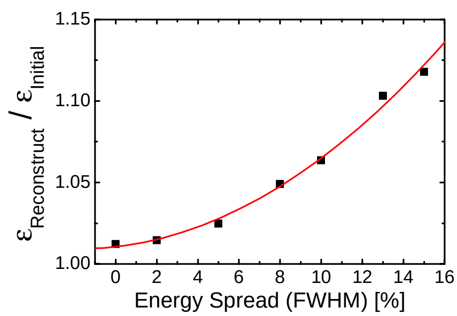
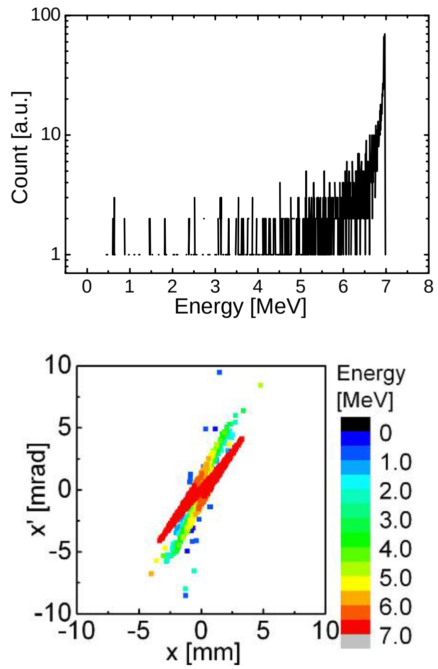


Figure 4: The ratios of the unnormalized emittances from the reconstructed phase space of Gaussian beams with rectangle-shaped energy distributions to that from the initial one. The red line is the fitted curve by parabolic function.

Realistic beam from thermionic rf gun

For a realistic beam, we use the PARMELA calculation which simulates the 4.5-cell thermionic RF gun as the initial beam distribution. The beam parameters at the entrance of the quadrupole magnet given by PARMELA code are shown in Fig. 5. The energy distribution of the beam consists of the main component and low energy tail (Fig 5(a)), and the phase space distribution is strongly correlated with energy distribution (Fig. 5(b)).

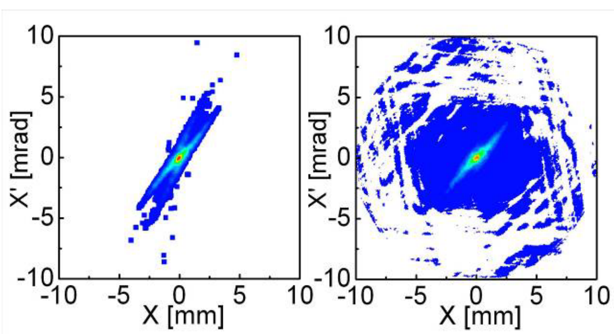
Initial phase space distribution and reconstructed phase space distribution by tomographic method are shown in Fig. 6. The unnormalized emittance calculated from the initial phase space distribution is 1.0π mm mrad and that from the reconstructed phase space distribution is 2.7π mm mrad. Due to the noisy image, the emittance of the reconstructed phase space distribution is calculated about three times as large as that of initial one.



(a)

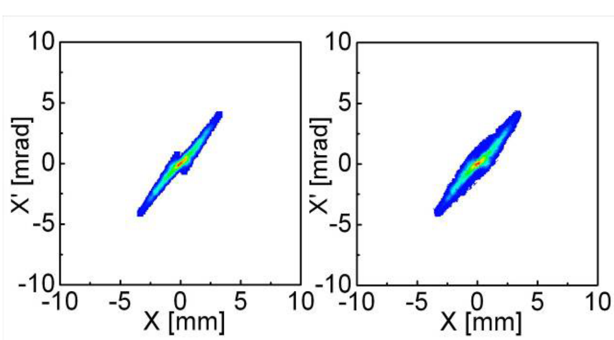
(b)

Figure 5: Realistic distribution from PARMELA at the entrance of the quadrupole magnet, (a) energy spectrum (FWHM = 0.5 % with tail), (b) phase space distribution (ε = 1.0π mm mrad), strongly correlated with energy distribution (the colors indicate the energy level).



(a) (b)

Figure 6: Phase space distributions of the realistic beam, (a) initial (ε = 1.0π mm mrad), (b) reconstructed (ε = 2.7π mm mrad)



(a) (b)

Figure 7: Phase space distributions of the realistic beam in the case of $E_{cut\text{-}off}$ / E_{peak} = 90 %, (a) initial (ε = 0.43π mm mrad), (b) reconstructed (ε = 0.52π mm mrad).

In this case, reconstructed image is very noisy, even though the energy spread of beam is very small (about 0.5% in FWHM). As shown in previous section, the error from main component should not be large if the energy spread is 0.5 %. Therefore, the noise in the reconstructed image is originated from low energy particles.

To evaluate the contribution of low energy particles to noise on reconstructed image we introduced an energy filtering in the following procedure:

1) Select the particles whose energy is higher than cut-off energy ($E_{cut\text{-}off}$).
2) Reconstruct the phase space image by using only selected particles.

In the case of $E_{cut\text{-}off}$ / E_{peak} = 90 %, the initial and reconstructed phase space distributions are shown in Fig. 7. For the equivalent comparison, the energy filtering also applied to the initial distribution. As is shown in Fig.7(b), the noises in the reconstructed image are dramatically reduced by the energy filtering. By the energy filtering, the unnormalized emittance of the initial beam is 0.43π mm mrad and that of reconstructed image is 0.52π mm mrad, which consistent to emittance of initial phase space distribution. For practical use, we have to cut the low energy tail of the objective beam by using a energy filtering section, such as a bending and a slit.

Evaluation of the tolerable cut-off energy

To evaluate the tolerable cut-off energy of the tomographic method, the beams of different cut-off energies are also simulated.

The results of emittance calculation from the reconstructed phase space distributions are the function of cut-off energy and plotted in Fig. 8. As shown in Fig. 8, this method can be validated for the electron beams whose cut-off energies are higher than 50 %. We found, however, that the reconstructed image is still noisy with the 50 % cut-off energy (see Fig. 9).

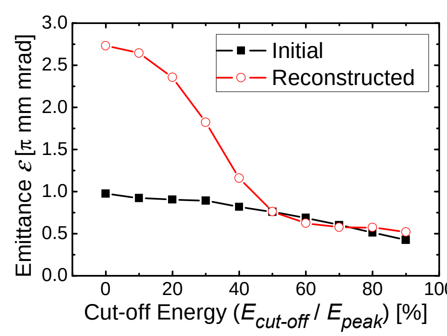


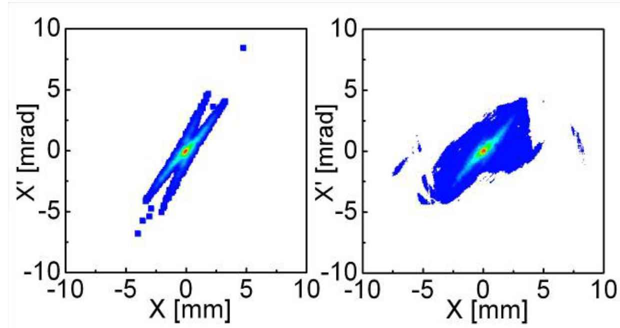
Figure 8: Calculated emittances as the function of cut-off energy ($E_{cut\text{-}off}$ / E_{peak} ≥ 50 % looks enough for emittance measurement).

Since that, we cannot evaluate the accuracy of phase space measurement by comparison of emittance alone. In the medical computed tomography field, the normalized mean square error (NMSE ξ) defined as Eq. 2 is commonly used as the criterion of the accuracy of the reconstructed image [6]. Therefore, we introduce this criterion to evaluate accuracy of this method.

$$\xi = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}(y_{ij}-s_{ij})^2}{\sum_{i=1}^{M}\sum_{j=1}^{N}s_{ij}^2} \quad (2),$$

where y_{ij} are signals on the reconstructed phase space, s_{ij} are signals on the initial phase space, and M, N is pixel numbers.

Calculated results of ξ of the beams with different cut-off energies are shown in Fig. 10. As is shown in Fig. 10, this method seems to be validated to the cases at $E_{cut\text{-}off}$ / E_{peak} ≥ 70 %, since the ξ is constant at $E_{cut\text{-}off}$ / E_{peak} ≥ 70 % and about 6 % (ξ is about 8 % in the case of the Gaussian beam with rectangle-shaped energy distribution whose energy spread is 15 %). As shown in Fig. 11, initial phase space distribution and the reconstructed phase space distribution are almost the same, as the ξ indicates. The emittances are almost the same, too.



(a) (b)

Figure 9: Phase space distributions of the realistic beam in the case of $E_{cut\text{-}off}$ / E_{peak} = 50 %, (a) initial (ε = 0.76π mm mrad), (b) reconstructed (ε = 0.76π mm mrad).

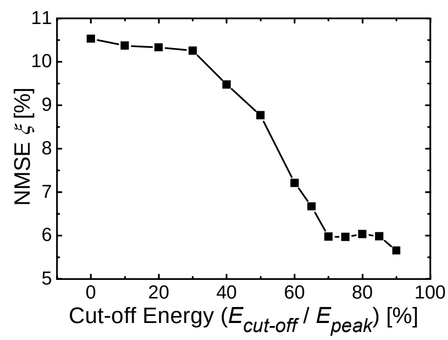
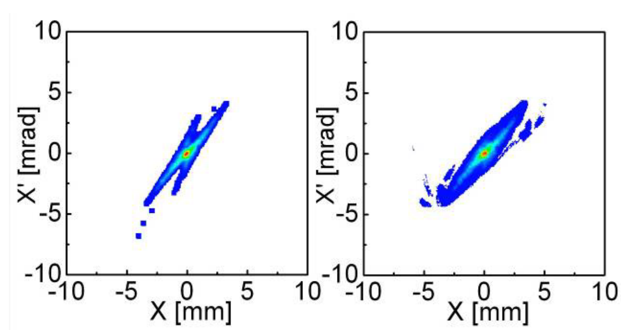


Figure 10: Normalized mean square errors of the realistic beam as the function of cut-off energy ($E_{cut\text{-}off}$ / E_{peak} ≥ 70 % looks enough).



(a) (b)

Figure 11: Phase space distributions of the realistic beam in the case of $E_{cut\text{-}off}$ / E_{peak} = 70 %, (a) initial (ε = 0.60π mm mrad), (b) reconstructed (ε = 0.58π mm mrad).

CONCLUSION

The effect of energy distribution has been evaluated on the validity of the transverse phase space tomography by particle simulations. For a Gaussian beam with rectangle-shaped energy distribution the reconstructed image was distorted by the energy spread of the beam. However the error in the calculated emittance is less than 15 % when the energy spread is less than 15 %. For a realistic beam which has a low energy tail the reconstructed image was severely distorted. Therefore, the low energy tail mainly distorts the reconstructed image and we need an energy cut-off process. By using 70 % energy cut-off, the reconstructed image is good enough in terms of emittance and phase space distribution. In practical use, the phase space distribution of the beam should be measured just after the energy filtering section, such as a bending magnet and a slit.

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