COLLECTIVE EFFECTS IN THE THOMSON BACK-SCATTERING BETWEEN A LASER PULSE AND A RELATIVISTIC ELECTRON BEAM

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Abstract

The interaction between high-brilliance electron beams and counter-propagating laser pulses produces X rays via Thomson back-scattering. If the laser source is long and intense enough, the electrons of the beam can bunch and a regime of collective effects can be established. In the case of dominating collective effects, the FEL instability can develop and the system behaves like a free-electron laser based on an optical undulator. Coherent X-rays can be irradiated, with a bandwidth very much thinner than that of the corresponding incoherent emission. The emittance of the electron beam and the distribution of the laser energy are the main quantities that limit the growth of the X-ray signal. In this work we analyse with a 3-D code the transverse effects in the emission produced by a relativistic electron beam when it is under the action of an optical laser pulse and the X-ray spectra obtained. The scalings typical of the optical wiggler, characterized by very short gain lengths and the overall time durations of the process make possible considerable emission also in violation of the Pellegrini criterion for static wigglers. A generalized form of this criterion is validated on the basis of the numerical evidence.

INTRODUCTION

A Thomson back-scattering set-up can be considered in principle as a source of intense X-ray pulses which is at the same time easily tunable and highly monochromatic. Due to recent technological developments in the production of high brilliance electron beams and high power CPA laser pulses, it is now even conceivable to make steps toward their practical realisation.

The radiation generated in the Thomson back-scattering is usually considered incoherent and calculated by summing at the collector the intensities of the fields produced in single processes by each electron. If the laser pulse is long enough, however, collective effects can establish and become dominant. The system in this range of parameters behaves therefore like a free-electron laser, where the static wiggler is substituted by the optical laser pulse.

From the point of view of the theoretical description of the process, it is convenient to start with the same set of one-dimensional equations that are used in the theory of high-gain free-electron laser amplifier [2]. To take into account the many aspects of the process connected with the finite transverse geometry of the electron beam and of the laser and radiation pulses it is necessary to consider 3D equations.

The procedure we have followed to write a system of 3D equations will be described in the next section of this preliminary report. A first set of numerical results and a short discussion of their importance will be given at the end of the paper.

3D EQUATIONS

We start from the Maxwell-Lorentz equations that describe both laser and collective electromagnetic fields and from the relativistic equations of motion for the electrons of the beam. The laser and collective fields are given in terms of the corresponding scalar and vector potentials in the Coulomb gauge.

We assume that the laser is circularly polarised with the following form of the vector potential $\mathbf{A}_L$ (the laser pulse propagates along the z-axis in the negative direction):

$$\mathbf{A}_L(\mathbf{r},t)=\frac{a_{L0}}{\sqrt{2}}(g(\mathbf{r},t)e^{-i(k_L z+\omega_L t)}\hat{\mathbf{e}}+cc)+O(\frac{\lambda_L}{w_0}) \qquad (1)$$

where $\lambda_L = 2\pi/k_L$ is the laser wavelength, w_0 the laser spot size, $\omega_L=ck_L$ the angular frequency and $\hat{\mathbf{e}}=(\mathbf{e}_x+i\mathbf{e}_y)/\sqrt{2}$. The envelope g(**r**,t)s considered to be a slowly varying function of all variables xyz and t and is defined as a complex number with $|g(\mathbf{r},t)|\le 1$. In the case, for instance, of a laser pulse with a Gaussian transverse shape, the envelope has the form [1]

$$g(\underline{r},t)=\Phi(z+ct)\frac{1+i\dfrac{z}{z_0}}{1+\dfrac{z^2}{z_0^2}}\exp\left[-4\frac{x^2+y^2}{w_0^2(1+\dfrac{z^2}{z_0^2})}-4i\frac{x^2+y^2}{w_0^2(\dfrac{z}{z_0}+\dfrac{z_0}{z})}\right] \qquad (2)$$

where $z_0 = \pi w_0^2/4\lambda_L$ is the Rayleigh length and the form of the (real) function Φ (with $0\le\Phi(z)\le 1$), depends on the shape of the pulse along the z-axis. Notice that $\mathbf{A}_L$ is perpendicular to the z-axis up to terms of the order of λ_L/w_0, which is consistent with the gauge requirement $\nabla\cdot\mathbf{A}_L=0$.

We suppose that $\varphi(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$ have a slow dependence on x and y, i.e., that they vary on a transverse scale L_T much greater than the radiation wavelength

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λ=2π/k and write, accordingly to the single-mode hypothesis frequently used in 1D treatments

$$\begin{aligned}\mathbf{A}(\mathbf{r},t) &= M(\mathbf{r},t)\hat{\mathbf{e}} + cc + O(\lambda / L_T) \\ &= A(\mathbf{r},t)e^{i(kz-\omega t)}\hat{\mathbf{e}} + cc + O(\lambda / L_T)\end{aligned} \tag{3}$$

where $M(\mathbf{r},t) = A(\mathbf{r},t)e^{i(kz-\omega t)}$ and ω=ck is the radiation angular frequency. We write the relativistic equation of motion of each electron in the form

$$\frac{\mathrm{d}\boldsymbol{\Pi}_{\mathrm{j}}(\mathrm{t})}{\mathrm{dt}} = \mathrm{e}\left[\nabla(\varphi - \boldsymbol{\beta}_{\mathrm{j}}(\mathrm{t})\cdot(\mathbf{A}_{\mathrm{L}}+\mathbf{A}))\right]_{\mathbf{x}=\mathbf{r}_{\mathrm{j}}(\mathrm{t})} \tag{4}$$

with $\mathbf{r}_j(t)$ the instantaneous position of the electron, $\boldsymbol{\Pi}_j(t)=\boldsymbol{\pi}_j(t)-(e/c)(\mathbf{A}_L+\mathbf{A})_{\mathbf{x}=\mathbf{r}_j(t)}$ the generalized momentum and $\boldsymbol{\pi}_j(t)=mc\mathbf{p}_j(t)$ the mechanical momentum in which we use the definition $\mathbf{p}_j(t)=\gamma_j(t)\boldsymbol{\beta}_j(t)$.

By projecting equation (4) on the plane transverse to the z-axis, we see that $\boldsymbol{\Pi}_{\mathrm{j}\perp}$ is a constant of the motion to dominant order in the small parameters and, as is usually done in 1D treatments, we shall even assume that $\boldsymbol{\Pi}_{\mathrm{j}\perp} = 0$ always to dominant order, which leads to the equation

$$\boldsymbol{\beta}_{\mathrm{j}\perp}(\mathrm{t}) = \frac{\mathrm{e}}{\mathrm{mc}^2\gamma_{\mathrm{j}}(\mathrm{t})}(\mathbf{A}_{\mathrm{L}}+\mathbf{A})_{\mathbf{x}=\mathbf{r}_{\mathrm{j}}(\mathrm{t})} + \ldots. \tag{5}$$

By inserting (5) into (4) and neglecting space charge effects, one has

$$\frac{\mathrm{d}\boldsymbol{\Pi}_{\mathrm{j}}(\mathrm{t})}{\mathrm{dt}} = -\frac{\mathrm{e}^2}{2\mathrm{mc}^2\gamma_{\mathrm{j}}(\mathrm{t})}\left[\nabla(\mathbf{A}_{\mathrm{L}}+\mathbf{A})^2\right]_{\mathbf{x}=\mathbf{r}_{\mathrm{j}}(\mathrm{t})} + \ldots.. \tag{6}$$

where

$$(\mathbf{A}_L+\mathbf{A})^2 = a_{L0}^2\,|g|^2 + 2|A|^2 + \sqrt{8}a_{L0}\,\mathrm{Re}(g^* A e^{i\theta}) + .. \tag{7}$$

$\theta=(k+k_L)z+c(k_L-k)t$ and the term $2|A|^2$ in the r.h.s. of (7) is usually omitted.

The axial motion of the single particle is obtained by projecting Eq.(6) on the z-axis and using (4) and(7), which gives

$$\frac{dp_{jz}(t)}{dt} = -\frac{e^2}{2m^2c^3\gamma_j(t)}\left\{ a_{L0}^2\left[\frac{\partial}{\partial z}|g|^2\right]_{\mathbf{x}=\mathbf{r}_j(t)} - 2\sqrt{2}(k+k_L)a_{L0}\,\mathrm{Im}\left[(g^*A)_{\mathbf{x}=\mathbf{r}_j(t)}e^{i\theta_j(t)}\right]\right\} \tag{8}$$

where the phase angles $\theta_j(t)$ of the particles in the combined laser plus collective field are given by

$$\theta_j(t)=(k_L+k)z_j(t)+c(k_L-k)t \tag{9}$$

The first term in the r.h.s. of Eq.(8) gives the ponderomotive force exerted on the electron as it enters or leaves the laser pulse, while the second term is due to the action of all other electrons of the beam and is therefore responsible for collective effects and, in particular, the free-electron laser (FEL) instability.

The transverse motion is likewise obtained by projecting Eq.(6) on the x,y plane. By omitting rapidly varying terms simply by taking the mean value of both sides of this equation over the laser period, one obtains finally

$$\frac{d\mathbf{p}_{j\perp}(t)}{dt} = -\frac{e^2}{2m^2c^3\gamma_j(t)}\left\{ a_{L0}^2(\nabla_\perp|g|^2)_{\mathbf{x}=\mathbf{r}_j(t)} + 2\sqrt{2}a_{L0}\,\mathrm{Re}\left[(\nabla_\perp(g^*A))_{\mathbf{x}=\mathbf{r}_j(t)}e^{i\theta_j(t)}\right]\right\} \tag{10}$$

As in Eq.(8), the first term to the right of (10) gives the ponderomotive focussing or defocussing actions due to the laser transverse gradients while the second term takes into account collective contributions to the transverse motion. Notice also that, due to the assumed slow dependence on both x and y, this last term is smaller than the corresponding term in the r.h.s. of Eq.(8).

The last point consists in the derivation of an approximate equation for the collective vector potential $\mathbf{A}(\mathbf{r},t)$ directly from the Maxwell equation:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\mathbf{A}(\mathbf{r},t) = 4\pi c\mathbf{J}_b - c\frac{\partial}{\partial t}\nabla\varphi$$

where the beam current density is taken in its microscopic form, i.e., $\mathbf{J}_{\mathrm{b}} = -\mathrm{ec}\sum_{\mathrm{j}}\boldsymbol{\beta}_{\mathrm{j}}(\mathrm{t})\delta(\mathbf{x}-\mathbf{r}_{\mathrm{j}}(\mathrm{t}))$, the integer j running from 1 to N the total number of electrons of the beam. By dropping, as we have already said, the contribution from the scalar potential φ, remembering that $\mathbf{A}$ is to dominant order perpendicular to the z-axis and using again Eq.(5), we write

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\mathbf{A}(\mathbf{r},t) = -\frac{4\pi e^2}{m}\sum_j \frac{(\mathbf{A}_L+\mathbf{A})}{\gamma_j(t)}\delta(\mathbf{x}-\mathbf{r}_j(t)).$$

We neglect $\mathbf{A}$ with respect to $\mathbf{A}_L$ in the driving term of this equation and take definitions (1) and (3) into account to obtain the form

$$\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2}\right)M(\underline{r},t) = c^2\nabla_\perp^2 M - \frac{\omega_b^2 a_{L0}}{\sqrt{2}}\frac{V_b}{N}\sum_j \frac{g(\underline{r},t)}{\gamma_j(t)}e^{-i(k_L z + ck_L t)}\delta(\mathbf{x}-\mathbf{r}_j(t)) + \ldots.$$

where $\omega_b^2 = 4\pi e^2 n_b/m$, $n_b = N/V_b$ being the average value of the beam volume density (V_b is the volume of the beam).

Secular terms in the perturbation treatment of the preceding equation can be avoided by imposing that the amplitude $A(\mathbf{r},t)$ is a solution of the following equation where a continuous average of both sides over a specified volume V_m has been done in order to eliminate the delta functions and change from a microscopic to a macroscopic collective field

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)A - i\frac{c}{2k}\nabla_\perp^2 A = -i\frac{\omega_b^2 a_{L0}}{2\sqrt{2}ck}\frac{1}{N_s}\sum_s \frac{g(\mathbf{r}_s(t),t)}{\gamma_s(t)}e^{-i\theta_s(t)} + \ldots. \tag{11}$$

This equation shows the typical driving ("bunching") factor

$$b = \frac{1}{N_s} \sum_s \frac{g(\mathbf{r}_s(t),t)}{\gamma_s(t)} e^{-i\theta_s(t)} \qquad (12)$$

which is similar to that appearing in the 1D version of the theory. The phase angles $\theta_s(t)$ have been defined in (9), while the integer s in (11) and (12) runs over all values of j for which, at time t and position **x**, $r_j = \sqrt{(x-x_j)^2+(y-y_j)^2+(z-z_j)^2} < R_m$ if we choose V_m as a sphere with radius R_m. $N_s(\mathbf{r},t)$ is the number of electrons that satisfy the preceding inequality.

Equations (8), (10) and (11) are our basic equations. Once restated using the non dimensional variables $\bar{t} = 2\rho a_L t$ and $\bar{\mathbf{x}} = 2\rho k_L \mathbf{x}$, they may be summarized as follows:

$$\frac{d}{d\bar{t}}\bar{\mathbf{r}}_j(\bar{t}) = \rho \frac{\mathbf{P}_j(\bar{t})}{\bar{\gamma}_j(\bar{t})} \qquad (13)$$

$$\frac{d}{d\bar{t}} P_{jz}(\bar{t}) = -\frac{\bar{a}_{L0}^2}{2\rho\gamma_0^2}\frac{1}{\bar{\gamma}_j}\left[\frac{\partial}{\partial \bar{z}}|g|^2\right]_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j} - \frac{2}{\bar{\gamma}_j}\mathrm{Re}\,al\left[(g^*\bar{A})_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j} e^{i\theta_j(\bar{t})}\right] + ... \qquad (14)$$

$$\frac{d}{d\bar{t}}\mathbf{P}_{j\perp}(\bar{t}) = -\frac{\bar{a}_{L0}^2}{2\rho\gamma_0^2}\frac{1}{\bar{\gamma}_j}\left[\bar{\nabla}_\perp |g|^2\right]_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j} - \frac{4\eta}{1+\frac{k_L}{k}}\frac{1}{\bar{\gamma}_j}\mathrm{Im}\left[(\nabla_\perp(g^*\bar{A}))_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j} e^{i\theta_j(\bar{t})}\right] + ... \qquad (15)$$

$$\left(\frac{\partial}{\partial \bar{t}} + \frac{\partial}{\partial \bar{z}}\right)\bar{A}(\bar{\mathbf{x}},\bar{t}) - i\eta\bar{\nabla}_\perp^2\bar{A} = b \qquad (16)$$

with the phase angles written in non dimensional form as

$$\theta_j(\bar{t}) = \frac{k}{2\rho k_L}\left(\left(1+\frac{k_L}{k}\right)\bar{z}_j(\bar{t}) + \left(\frac{k_L}{k}-1\right)\bar{t}\right) \qquad (17)$$

and

$$\gamma_j^2 = 1 + \gamma_0^2\rho^2 P_{jz}^2 + \bar{a}_{L0}^2(|g|^2)_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j(\bar{t})} + ... \qquad (18)$$

In the preceding equations γ_0 is the average value of γ over all electrons of the beam at t=0, $\bar{\gamma}_j = \gamma_j/\gamma_0$, $\mathbf{P}_j = \mathbf{p}_j/\gamma_0\rho$, where $\bar{a}_{L0} = \frac{e}{mc^2}a_{L0}$ is the laser parameter, $\eta = \frac{k_L}{k}\rho$, $\bar{A} = -i\left(\frac{\omega_b^2 a_{L0}}{4\sqrt{2}\omega\omega_L\gamma_0\rho}\right)\frac{eA}{mc^2}$, and the FEL parameter $\rho = \frac{1}{\gamma_0}\left(\frac{\omega_b^2 a_{L0}^2}{16\omega_L^2}\right)^{\frac{1}{3}}$.

The resonant frequency is at $\omega \approx \frac{4\gamma_0^2\omega_L}{1+a_{L0}^2}$.

NUMERICAL RESULTS AND DISCUSSION

We have solved equations (13)-(16) in the following case: the laser has a wavelength λ_L=0,8 micron and the parameter a_{L0} = 0.8. The diameter of the laser focal spot w_0 has been varied from 1000 down to a few tens of microns in order to analyze the dependence of the process on the shape of the transverse distribution of the laser energy. The bunch of electrons has been chosen with an average value of γ, $\langle\gamma\rangle$=30, corresponding to an energy of 15 MeV. This value of $\langle\gamma\rangle$ leads to a resonant wavelength λ=2.22 Angstrom. The quantum parameter $\bar{\rho} = \rho\frac{mc\langle\gamma\rangle}{\hbar k}$ = 2 [3,4] and the classical equations (13)-(16) are expected to be fully valid. The collective effects appear and saturate after 10-15 gain lengths which in our case correspond to times of the order of 5-7 ps (each gain length corresponds to Lg=110 micron), i.e., of the same order of the duration of the laser pulse.

The electron beam we have considered has a mean radius σ_0=10 micron , a total charge of 1 nC and a length L_b= 120 micron, so that the Pierce parameter is ρ=710^{-4}. Its energy spread $\Delta\gamma/\gamma$ ranges from 0 to 1.5 10^{-4} and the initial normalized transverse emittance ε_n has been varied from 0 up to 3.

Fig 1 shows the typical growth of the collective potential amplitude in time, as well as the bunching factor. The amplitude of the vector potential $|A|^2$ has been calculated in the middle of the electron bunch at the position z_m =<z> and averaged on the transverse plane.

In this case w_0= 500 micron, ε_n=1,11 mm mrad and the signal saturates at t=5 psec.

Figures 2 gives the level curves of the potential amplitude $|A|^2$ in a transverse plane x,y in the electron frame of reference and in the middle of the electron beam *vs* time. The case shown has been obtained with ε_n=1,85 mm mrad and w_0=500 micron. The transverse increase of the radiation spot is substantially due to the divergence of the electron beam, because the Rayleigh length Z_R of the

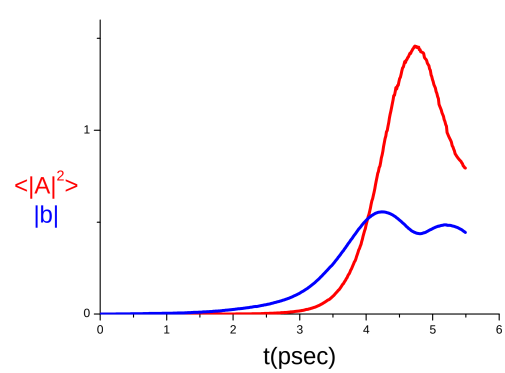

Figure 1: |A|2(zm) averaged on the transverse section vs t in psec and |b| for w0= 1000 m, n=1,11 mm mrad.

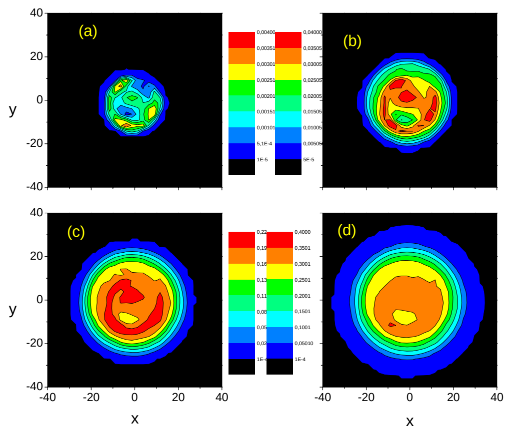

Figure 2: (a)number of time steps 200, (b)400, (c)600, (d) 800.

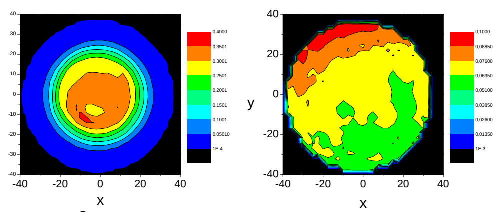

Figure 3: $|A|^2$ and the bunching |b| after 900 time steps.

radiation is very much larger than the gain length L_g so that the radiation diffraction can be neglected. This can also be seen from the scaled equations (15) and (16), where the very small parameter η multiplies respectively the collective term in the transverse momentum equation (15) and the diffraction term in the wave equation (16). From these figures one can also see that the initial jagged shape of the intensity changes in time, smoothing itself and tending to assume a central peaked form.

In Fig. 4 the dependence of the maximum of $<|A|^2>$ on the transverse normalized emittance is shown, while in Fig. 5 the typical spectrum of the radiation is presented. It has to be noted that the spectrum bandwidth is considerably larger than in the 1D case as is shown in Fig. 5, where the curve with ε_n=1,11 presents $<\delta\omega/\omega>$ =4 10^{-3} ,a factor of 10 larger than ρ. We must note that we have considerable emission also in violation of the Pellegrini criterion for a static wiggler. In fact, the emittances considered exceed largely the value $\gamma\lambda/4\pi$, which in this case is 5,5 10^{-4} μm. On the other hand, on the fact that Z_R/Lg=5,6 10^4, the criterion of Pellegrini can be rewritten in a generalized form for both static and optical undulators as $\varepsilon_n \le \alpha\sqrt{Z_R/L_g}\lambda_R\gamma/4\pi$ [5] where $\alpha=\sqrt{d\omega/(\omega\rho)} \approx \sqrt{10}$, giving the more relaxed limit $\varepsilon_n<0{,}43\mu$.

The shape of the spectrum which is similar to a function $\mathrm{sinc}^2x=(\sin x/x)^2$ is due to the sharp z-dependence of the electron distribution. The last figure 6 shows the most critical effect, i.e., the dependence of the growth of the signal on the transverse energy distribution of the laser pulse. In fact a spot size with a diameter smaller than 100 micron, for instance, does not seem to lead to any FEL like instability. The collective signal in this condition, therefore, does not grow. A possible remedy could be the development and use of laser beams with flat transverse energy distributions.

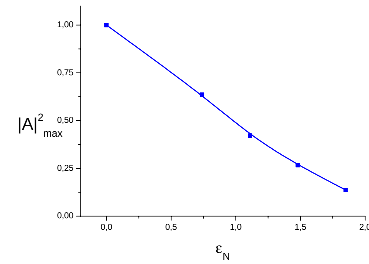

Figure 4: $|A|^2_{max}$ versus ε_n for w_0=500.

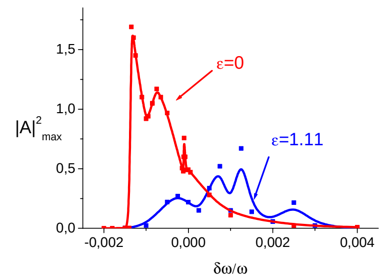

Figure 5: $|A|^2_{max}$ *vs* δω/ω for ε_n=0 and ε_n =1.11 (w_0=500).

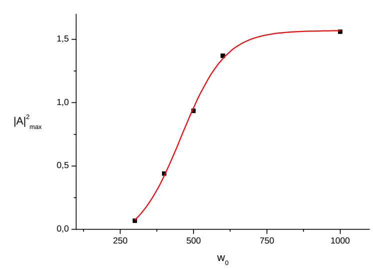

Figure 6: $|A|^2_{max}$ versus w_0 for ε_n =0.

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