SPATIAL COHERENCE EFFECTS IN THE TRANSITION RADIATION SPECTRUM FOR RELATIVISTIC CHARGED BEAMS: THEORETICAL RESULTS AND BEAM DIAGNOSTICS IMPLICATIONS

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Abstract

In the electromagnetic radiative phenomena originated by relativistic charged beams, angular distortions as well as variations of the photon flux are commonly observed as a function of the ratio between the beam transverse size and the observed wavelength, even at a wavelength shorter than the longitudinal bunch length. In the framework of a single particle theory of the transition radiation, diffractive alterations of the spectrum due, for instance, to the finite size of the radiator screen are already known. For relativistic three-dimensional charged beams, it could be interesting to check if the transition radiation emission undergoes modifications depending on the finite value of the beam transverse size with respect to the observed wavelength. Taking into account the beam diagnostics applications of the transition radiation in a linear accelerator, such an experimental check can offer promising perspectives. The theoretical background and physical basis of the spatial coherence effects affecting the spectral distribution of the transition radiation intensity in conditions of temporal incoherence will be presented. The main beam diagnostics applications will be also contoured.

INTRODUCTION

A relativistic charged beam passing through a vacuum-metal interface can originate an electromagnetic radiation emission. The steeply local variation of the dielectric properties of the medium imposes a boundary condition to the charge electric field at the interface. Thanks to such a constraint, the *virtual quanta* field of the charge can materialize at the vacuum-metal interface and propagate as a radiation field, the so called transition radiation field [1, 2]. Covering a broad wavelength band extending up to the X-rays region, the spectral distribution of the transition radiation suffers a low frequency diffractive cut-off occuring as the transverse extension of the harmonic component of the *pseudo-photon* field becomes comparable with the finite size of the radiator surface [3]. This manifests in either a modification of the angular distribution or an intensity variation of the emitted radiation. At a high frequency, a suppression of the single electron spectrum of the radiation intensity is instead observable when the metallic screen behaves, under the action of the rapidly varying electromagnetic field, as a real conductor with vacuum-like dielectric properties [2]. In a linear accelerator, transition radiation finds a large and consolidate application as a tool of beam diagnosis thanks to the instantaneous and highly directional emission of the radiation, the dependence of its angular distribution on the beam energy and the possibility to minimize the radiator intercepting consequences on the beam thanks to the use of thin metallic coated dielectric films. Concerning the possible beam diagnostics applications of the transition radiation originated by relativistic three-dimensional charged beams, it could be interesting to study if and how the angular distribution of the radiation spectral intensity is sensitive to a variation of the transverse size (σ) of the charged beam with respect to the observed wavelength (λ) belonging, for instance, to the optical region. In other words, it could be interesting to check if, similarly to what can be commonly observed in other electromagnetic radiative phenomena - such as the synchrotron radiation or the single photon bremsstrahlung - also in the transition radiation a diffractive distortion of the angular distribution along with an intensity variation occurs as a function of the ratio σ/λ. According to [4], the angular distribution of the transition radiation should be sensitive to a variation of the ratio σ/λ even though this is observed in a condition of temporal incoherence, i.e., at a wavelength much shorter than the longitudinal length of the three-dimensional charged beam. In the following, the transition radiation spectral intensity will be derived and possible beam diagnostics applications of the source-size (σ/λ) effects will be contoured.

PSEUDO-PHOTON AND TRANSITION RADIATION FIELDS

In the laboratory reference frame, a relativistic three-dimensional distribution of N electrons in a rectilinear and uniform motion hits a metallic planar surface S located in the plane $z = 0$ of the reference frame and surrounded by a non-dispersive medium [4]. In the visible optical region, *ideal conductor* properties can be reliably attributed to the metallic screen. At the time t=0, the center of mass of the N electrons, moving with a velocity $\vec{w}$ along the z-axis, collides with the metallic surface S. For a strict and essential description of the radiation emission mechanism, the spatial distribution of the charged beam $\rho(\vec{r}, t)$ can be assumed time-invariant and referred to the particle configuration corresponding to the collision time ($t = 0$):

$$\rho(\vec{r}, t) = \sum_{j=1}^{N} \delta(\vec{r} - \vec{r}_j) = \sum_{j=1}^{N} \delta(\vec{r}_0 - \vec{r}_{0j}). \qquad (1)$$

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The corresponding Fourier transform [4] $\rho(\vec{k},\omega)$

$$\rho(\vec{r},t)=\frac{1}{(2\pi)^4}\int d\vec{k}\,d\omega\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}\rho(\vec{k},\omega) \qquad (2)$$

reads

$$\rho(\vec{k},\omega)=2\pi\left(\sum_{j=1}^{N}e^{-i\vec{k}\cdot\vec{r}_{0j}}\right)\delta(\omega-\vec{w}\cdot\vec{k}). \qquad (3)$$

In the Fourier space, according to the gauge of Lorentz, the propagation equations of the 4-potential $(\vec{A},\Phi)$ are:

$$\begin{cases} (-\frac{\omega^2}{v^2}+k^2)\vec{A}(\vec{k},\omega)=\frac{4\pi\,e}{c}\vec{w}\rho(\vec{k},\omega) \\ (-\frac{\omega^2}{v^2}+k^2)\Phi(\vec{k},\omega)=\frac{4\pi\,e}{\epsilon}\rho(\vec{k},\omega) \end{cases}$$

Therefore, the charge electric field reads

$$\vec{E}(\vec{k},\omega)=-i\vec{k}\Phi(\vec{k},\omega)+\frac{i\omega}{c}\vec{A}(\vec{k},\omega)= \qquad (4)$$
$$=-\frac{i8\pi^2 e}{\epsilon}\frac{(\vec{k}-\omega\vec{w}/v^2)(\sum_{j=1}^{N}e^{-i\vec{k}\cdot\vec{r}_{0j}})}{k^2-(\omega/v)^2}\delta(\omega-\vec{w}\cdot\vec{k})$$

and the harmonic component of the charge electric field in the transverse plane can be finally obtained:

$$E_{x,y}(\vec{r},\omega)=\frac{-ie}{\pi\epsilon}\times \qquad (5)$$
$$\times\int d\vec{k}e^{i\vec{k}\cdot\vec{r}}\frac{k_{x,y}(\sum_{j=1}^{N}e^{-i\vec{k}\cdot\vec{r}_{0j}})}{k^2-(\omega/v)^2}\delta(\omega-\vec{k}\cdot\vec{w}).$$

At the vacuum-metal interface ($z=0$), the transverse component of the electric field resulting from the relativistic charge and the induced charge satisfies the following continuity condition

$$E_{x,y}(x,y,z=0,\omega)+E^{ps}_{x,y}(x,y,z=0,\omega)=0, \qquad (6)$$

which allows the harmonic component of the *pseudo-photon* (*virtual quanta*) field of the relativistic charge in the vacuum ($\epsilon=1$) to be finally derived:

$$E^{ps}_{x,y}(x,y,z=0,\omega)=\sum_{j=1}^{N}e^{-i(\omega/w)z_{0j}}K_{x,y}(x,y,\vec{\rho}_{0j},\omega) \qquad (7)$$

with

$$K_{x,y}(x,y,\vec{\rho}_{0j},\omega)=\frac{i\,e}{w\pi}\int d\vec{\tau}\,e^{i\vec{\tau}\cdot\vec{\rho}}\,\frac{\tau_{x,y}e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2+\alpha^2}, \qquad (8)$$

where $\vec{\rho}_{0j}=(x_{0j},y_{0j})$ with $j=1,..,N$ are the electron coordinates in the transverse plane and $\vec{\tau}=(\tau_x,\tau_y)$.

Owing to the passage of the relativistic charge, the vacuum-metal interface behaves like the scattering surface of the *pseudo-photon* field ($E^{ps}_{x,y}$). According to the Huygens-Fresnel principle, this propagates as a radiation field ($E^{tr}_{x,y}$) from the metallic screen in the backward and forward half-spaces. At a distance R from the metallic screen, $E^{tr}_{x,y}$ can be expressed by means of the Helmholtz-Kirchhoff integral theorem, which, in the *far field approximation* (i.e., wavelength $\lambda\ll R$), can be represented as the Fourier transform of $E^{ps}_{x,y}$ with respect to the coordinates $\vec{\rho}=(x,y)$ of the planar surface S [1]:

$$E^{tr}_{x,y}(\vec{\kappa},\omega)=\frac{k}{2\pi R}\int_S d\vec{\rho}\,E^{ps}_{x,y}(\vec{\rho},\omega)e^{-i\vec{\kappa}\cdot\vec{\rho}} \qquad (9)$$

with $\vec{\kappa}=(k_x,k_y)$. From the previous equation and from Eqs.(7,8) the radiation field can be finally derived:

$$E^{tr}_{x,y}(\vec{\kappa},\omega)=\sum_{j=1}^{N}e^{-i(\omega/w)z_{0j}}H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j}) \qquad (10)$$

with

$$H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})=\frac{i\,e\,k}{2\pi^2 Rw}\int_S d\vec{\rho}\,d\vec{\tau}\,\frac{\tau_{x,y}\,e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}}{\tau^2+\alpha^2}\,e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}}. \qquad (11)$$

It is worth to be noted [4] that the longitudinal ($e^{-i(\omega/w)z_{0j}}$) and transverse ($e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}$) components of the particle phase factor in Eq.(3) play a different role in the definition of the transition radiation field. The transverse component of the particle phase factor contributes - according to the Huygens-Fresnel principle - to define the amplitude term $H_{x,y}(\vec{\kappa},\omega,\vec{\rho}_{0j})$ of the radiation field. The longitudinal one instead, reflecting the temporal sequence of the particle collisions on the metallic screen, determines the relative phase of the field amplitude terms and takes part in the definition of the causality correlation between them.

TRANSITION RADIATION SPECTRUM

The angular distribution of the transition radiation spectral intensity - i.e., the radiated energy per solid angle and frequency units - is proportional to the square module of the Poynting vector of the radiation field:

$$\frac{d^2I}{d\Omega d\omega}=\frac{cR^2}{4\pi^2}\left(\left|E^{tr}_x(k_x,k_y,\omega)\right|^2+\left|E^{tr}_y(k_x,k_y,\omega)\right|^2\right). \qquad (12)$$

With reference to Eqs.(10,11), the following expression for the spectral density of the transition radiation intensity for a bunch of N relativistic electrons can be obtained:

$$\frac{d^2I}{d\Omega d\omega}=\frac{cR^2}{4\pi^2}\sum_{\mu=x,y}\Big(\sum_{j=1}^{N}|H_{\mu,j}|^2+$$
$$+\sum_{j,l(j\neq l)=1}^{N}e^{-i(\omega/w)z_{0j}}e^{i(\omega/w)z_{0l}}H_{\mu,j}H^*_{\mu,l}\Big) \qquad (13)$$

with $H_{\mu,j}=H_\mu(\vec{\kappa},\omega,\vec{\rho}_{0j})$ and $\mu=x,y$. By averaging the previous expression with respect to the continuous distribution functions [$\rho_x(x_{0j})$, $\rho_y(y_{0j})$ and $\rho_z(z_{0j})$] of the particle coordinates (x_{0j}, y_{0j}, z_{0j}) [4], the transition radiation spectrum

$$\langle\frac{d^2I}{d\Omega d\omega}\rangle_{av} = \frac{cR^2}{4\pi^2}\sum_{\mu=x,y}(N\langle|H_{\mu,j}|^2\rangle_{av} + +N(N-1)\langle e^{-i(\omega/w)z_{0j}}e^{i(\omega/w)z_{0l}}H_{\mu,j}H^*_{\mu,l}\rangle_{av}) \quad (14)$$

assumes the following form [4]:

$$\langle\frac{d^2I}{d\Omega d\omega}\rangle_{av} = \frac{d^2I^S}{d\Omega d\omega}\left[N + N(N-1)F(\omega)\right], \quad (15)$$

where

$$F(\omega) = \left|\int dz\,\rho_z(z)e^{-i(\omega/w)z}\right|^2 \quad (16)$$

is the charge *longitudinal form factor*, defined as the square module of the Fourier transform of the particle distribution function along the longitudinal axis, and

$$\frac{d^2I^S}{d\Omega d\omega} = \frac{cR^2}{4\pi^2}\sum_{\mu=x,y}\langle|H_{\mu,j}|^2\rangle_{av} = \frac{c(ek)^2}{16\pi^6w^2}\times \times\sum_{\mu=x,y}\left|\int_S d\vec{\rho}\,d\vec{\tau}\,\frac{\tau_\mu\,\rho_x(\tau_x)\rho_y(\tau_y)}{\tau^2+\alpha^2}\,e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}}\right|^2 \quad (17)$$

is the spectral distribution function of the radiation intensity in the case of a metallic surface S with a generic shape and size. With respect to the case of radiation emitted by a single electron, Eq.(17) is a function of the charge *transverse form factor*, which, depending on the Fourier transforms $\rho_x(k_x)$ and $\rho_y(k_y)$ of the charge in the transverse plane, in the case of gaussian beams reads:

$$\rho_x(k_x)\rho_y(k_y) = e^{-\frac{k^2sin^2\theta}{2}(\sigma_x^2cos^2\phi+\sigma_y^2sin^2\phi)}. \quad (18)$$

Details of the average operation are below described [4]:

$$\langle|H_{\mu,j}|^2\rangle_{av} = \langle H_{\mu,j}H^*_{\mu,j}\rangle_{av} = \langle H_{\mu,j}\rangle_{av}\langle H^*_{\mu,j}\rangle_{av} \quad (19)$$

where $\langle H_{\mu,j}\rangle_{av}$ with $\mu = (x, y)$ is given by

$$\langle H_{\mu,j}\rangle_{av} = \frac{i\,e\,k}{2\pi^2Rw}\int_S d\vec{\rho}\,d\vec{\tau}\,\frac{\tau_\mu\,\langle e^{-i\vec{\tau}\cdot\vec{\rho}_{0j}}\rangle_{av}}{\tau^2+\alpha^2}\,e^{i(\vec{\tau}-\vec{\kappa})\cdot\vec{\rho}}. \quad (20)$$

with the average of the particle phase factor given by

$$\langle e^{-i(\tau_xx_{0j}+\tau_yy_{0j})}\rangle_{av} = \langle e^{-i\tau_xx_{0j}}\rangle_{av}\langle e^{-i\tau_yy_{0j}}\rangle_{av} = = \rho_x(\tau_x)\rho_y(\tau_y) \quad (21)$$

where the relation between $\rho_x(\tau_x)$ and $\rho_y(\tau_y)$ and the average of the particle phase factors $e^{-i\tau_xx_{0j}}$ and $e^{-i\tau_yy_{0j}}$ is:

$$\rho_x(\tau_x) = \langle e^{i\tau_xx_{0j}}\rangle_{av} = \int dx_{0j}\rho_x(x_{0j})e^{-i\tau_xx_{0j}} \quad (22)$$

and analogously for $\rho_y(\tau_y)$. Concerning the temporal coherent part of the spectrum, the average operation reads:

$$\langle e^{-i(\omega/w)z_{0j}}e^{i(\omega/w)z_{0l}}H_{\mu,j}H^*_{\mu,l}\rangle_{av} = \langle e^{-i(\omega/w)z_{0j}}\rangle_{av}\langle e^{i(\omega/w)z_{0l}}\rangle_{av}\langle H_{\mu,j}\rangle_{av}\langle H^*_{\mu,l}\rangle_{av} = = F(\omega)\langle|H_\mu(\vec{\kappa},\omega,\vec{\rho}_{0j})|^2\rangle_{av} \quad (23)$$

where $\langle|H_\mu(\vec{\kappa},\omega,\vec{\rho}_{0j})|^2\rangle_{av}$ is given by Eqs.(20, 21) and

$$F(\omega) = \langle e^{-i(\omega/w)z_{0j}}\rangle_{av}\langle e^{i(\omega/w)z_{0l}}\rangle_{av} = = \int dz_{0j}\rho_z(z_{0j})e^{-i(\omega/w)z_{0j}}\int dz_{0l}\rho_z(z_{0l})e^{i(\omega/w)z_{0l}} \quad (24)$$

is the *longitudinal form factor* in Eq.(16).

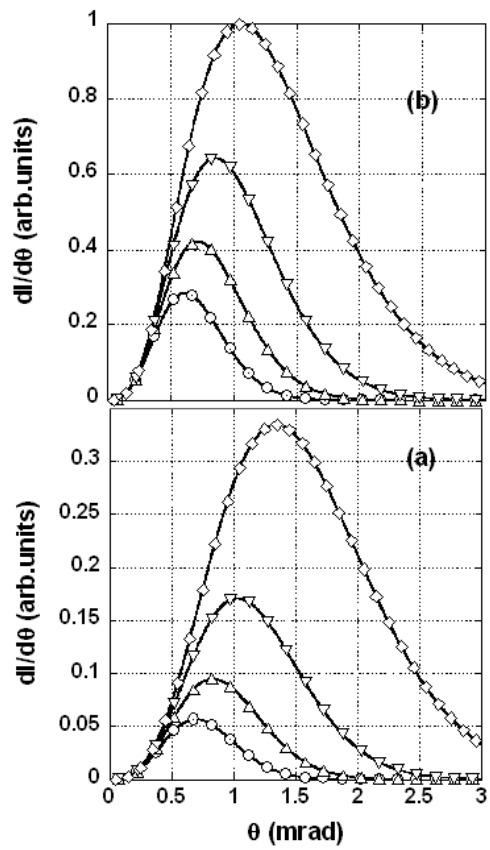

Figure 1: For an observed wavelength $\lambda = 0.5\ \mu m$ and a beam energy of 300 (a) and 500 (b) MeV, the polar angle distribution of the transition radiation intensity $dI(\theta)/d\theta$ is here reported as a function of the transverse size σ of the charged beam: 125 ($\odot$), 100 ($\triangle$), 75 ($\bigtriangledown$) and 50 μm ($\diamond$).

SPATIAL COHERENCE AND BEAM DIAGNOSTICS IMPLICATIONS

In a linear accelerator, transition radiation is observed in a condition of temporal incoherence ($F(\omega) = 0$) for most part of the beam diagnostics applications. For a measurement of the beam energy and transverse size, the radiation is mainly detected in the visible optical region by means of a quite standard experimental set-up [5]. By focusing, for example, the backward emitted transition radiation on the pixel matrix of a charge-couple-device (CCD) camera, the transverse size of the charged beam can be estimated from the measured image of the light spot. An estimation of the beam energy can be obtained from the angular distribution of the radiation measured by a CCD camera placed at the focus of a convergent lens or at a given distance from a lens with the focus on the metallic screen. For such observation conditions, even in the case of ultra-relativistic charged beam, an infinite surface ($S = \infty$) can be in practice assumed for the metallic screen. In such a case, for a gaussian beam with a cylindrical symmetry, the transition radiation spectrum reads:

$$\frac{d^2 I^{\infty}}{d\Omega d\omega} = \frac{(e\beta)^2}{\pi^2 c} \frac{sin^2\theta \, e^{-(k\sigma sin\theta)^2}}{(1-\beta^2 cos^2\theta)^2}, \qquad (25)$$

with $\beta = w/c$ [see Eqs.(17,18)]. For a constant value of the observed wavelength $\lambda = 0.5\ \mu m$ and in correspondence of a beam energy of 300 and $500\ MeV$, the angular distribution of the radiation intensity has been calculated on the basis of Eq.(25) for different values of the beam transverse size $\sigma = 125, 100, 75$ and $50\ \mu m$, as it is reported in Fig(1). As the transverse size σ of the charged beam is reduced with respect to the observed wavelength λ, the radiation intensity increases as well as the angular distribution of the radiation undergoes a broadening, tending as a limit to the single electron shape. Such a behavior is quite reasonable if the radiation emission, being originated in the end by the interaction of a relativistic charge with the conduction electrons of the metallic screen, is compared with other similar electromagnetic radiative phenomena involving relativistic charged beams, such as the synchrotron radiation or the single photon bremsstrahlung. Moreover, if the scattering nature of the transition radiation, resulting from the diffusion of the *virtual quanta* field on the metallic surface, is considered, then such a behavior finds a natural explanation. Since the radiation field wave front results from the interference at the observation point of the spherical wave fronts, which, according to the Huygens-Fresnel principle, are originated by all the points of the metallic screen that are enveloped by the *virtual quanta* field, the transition radiation is expected to be sensitive to the transverse distribution of the electrons. As the observed wavelength is small compared with the beam transverse size, the radiation field bears a phase information about the charge distribution in the transverse plane and, consequently, a diffractive distortion of the angular distribution and intensity of the emitted photons should be observed. As the wavelength becomes comparable with the beam transverse size, the particle phase information is lost and the radiation field behaves like a field generated by a single macro-particle. Consequently, the radiation intensity reaches a coherent threshold with the typical angular distribution of a single electron hitting the metallic screen. According to [4], even for a condition of temporal incoherence $F(\omega) = 0$ and provided that the observed wavelength is suitable to resolve the beam transverse size ($\lambda < \sigma$), the spectral intensity and the angular distribution of the transition radiation depend not only on the energy but also on the transverse size of the charged beam. Such a result, illustrated here in the case of an infinite metallic screen ($S = \infty$), can be interpreted as an intrinsic diffractive effect due to the finite transverse extension of the charged beam with respect to the observed wavelength. In the framework of a single particle theory of the transition radiation [3], similar diffractive effects of the radiation angular distribution and intensity are expected as the finite transverse size of the metallic screen S becomes comparable with the extension of the *pseudo-photon* field. From the point of view of the possible beam diagnostics applications, a dependence on σ/λ of the angular distribution of the transition radiation means that a beam energy estimation cannot be disjoined from a measurement of the beam transverse size. Moreover, in the case of a non-symmetric charged beam ($\sigma_x \neq \sigma_y$), the angular distribution of the radiation spectral intensity is expected to show an azimuthal angular dependence deriving from the *transverse form factor* [see Eqs.(17,18)]. The observation of the transition radiation with the aim to retrieve information about the energy and the size of the charged beam implies a careful spectral analysis of the angular and the intensity distribution of the radiation, which requires the use of either CCD cameras to detect the angular distribution or photomultipliers to measure the intensity of the transition radiation.

CONCLUSIONS

Diffractive source-size effects affecting the spectral distribution of the transition radiation intensity are here illustrated. Depending on the ratio between the transverse size of the charged beam and the observed wavelength, they can modify the radiation angular distribution and intensity with respect to the ideal single particle values. Such effects can be useful diagnosis tools of the energy and the transverse size of a relativistic charged beam.

REFERENCES

[1] M.L. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media*, Wiley, New York (1972).

[2] V.L. Ginzburg, V.N. Tsytovich, *Transition Radiation and Transition Scattering*, Adam Hilger, Bristol (1990).

[3] W. Barry, 7^{th} Beam Instrumentation Workshop, AIP Conference Proccedings, Argonne, **390** (1996) 173; N.F. Shul'ga, S.N. Dobrovol'sky, JETP Lett. **65** (1997) 611; A.P. Potylitsyn, Nucl. Instr. Meth. B **145** (1998) 169; M. Castellano et al., Nucl. Instr. Meth. A **435** (1999) 297.

[4] G.L. Orlandi, Opt. Commun. **211**, (2002) 109; G.L. Orlandi, submitted to Optics Communications.

[5] L. Wartski et al., J. Appl. Phys. **46**, 8 (1975); D.W. Rule, Nucl. Instr. Meth. B, **24/25** (1987) 901; A.H. Lumpkin et al., Nucl. Instr. Meth. A **285** (1989) 343; M. Castellano et al., Part. Acc. Conf. Proc., **3** (1999) 2196.