EMITTANCE GROWTH DUE TO SHORT-RANGE TRANSVERSE WAKEFIELDS IN THE FERMI LINAC

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Abstract

The FEL project FERMI@ELETTRA will use the existing Linac, upgraded to 1.2 GeV, to produce VUV radiation between 100-10 nm. FEL operations require a high quality beam in terms of the bunch energy spread and emittance. In this paper we present an analytical study, based on a continous model to describe the transverse motion of a single bunch. Such a study allows predicting the emittance growth under the combined influence of the short-range transverse wakefields, injection offset, initial emittance and misaligned accelerating sections. We also report a comparison between analytical and numerical (tracking code) results.

INTRODUCTION

The FERMI@ELETTRA project will use the existing ELETTRA linac. General description of the future FEL facility is in ref. [1, 2]. The machine layout has, after the second bunch compressor, five backward traveling (BTW) accelerating sections denominated LINAC 4. The main parameters for LINAC 4 are listed in table 1. FEL operations will require two lengths of the electron bunch with uniform current distribution: "short" and "long" cases, respectively, with full width (FW) bunch length of $200\mu m$ and $500\mu m$. In this paper the emittance growth under the combined influence of the short-range transverse wakefields, injection offset, initial angular divergences and misalignments of the accelerating sections in the LINAC 4 is reported.

Table 1: Main parameters for LINAC 4.

Total initial energy	$\gamma(0)$	1174	
Total final energy	$\gamma(1)$	2348	
LINAC 4 total length	L	30.375	m
Peak current	I_{max}	800	A
"Short bunch" FW length	l_b	200	μm
"Long bunch" FW length	l_b	500	μm

As indicated in ref. [3, 4], in a continous approximation the transverse motion of the relativistic beam in a misaligned accelerator under the influence of the short-range transverse wakefields can be modeled by:

$$\frac{1}{\gamma(\sigma)}\frac{\partial}{\partial\sigma}\left[\gamma(\sigma)\frac{\partial}{\partial\sigma}x(\sigma,\zeta)\right]+\kappa(\sigma)^2x(\sigma,\zeta)= \\ =\varepsilon(\sigma)\int_{-\infty}^{\zeta}w_n(\zeta-\zeta_1)F(\zeta_1)\left[x(\sigma,\zeta_1)-d_c(\sigma)\right]d\zeta_1 \tag{1}$$

where $\gamma(\sigma)$ is the energy parameter along the linac; $\sigma = s/L$ is the distance from entrance of the linac normalized to L; $\zeta = z/l_b$ is the normalized coordinate measured after the arrival of the head of the beam at location σ; $F(\zeta) = I(\zeta)/I_{max}$ is the instantaneous current divided by the maximum current of the uniform current distribution; $\kappa = kL$ is the focusing wave number k normalized to L; $w_n(\zeta)$ is normalized transverse wake function; $d_c(\sigma)$ is the lateral displacement of the accelerating sections as a function of location along the linac; $\varepsilon(\sigma) = \varepsilon_r/(\gamma(\sigma)/\gamma(0))$ is the dimensionless coupling strength between the beam and transverse wakes and ε_r, for the short-range wakes, is:

$$\varepsilon_r = \frac{4\pi\epsilon_0 W_0 I_{max} l_b L^2}{\gamma(0)I_A} \tag{2}$$

where $I_A \cong 17000A$, ϵ_0 is the dielectric constant of the vacuum and W_0 is the wake amplitude. Refs. [4, 5] give the general solution of equation of motion (1) for an accelerated beam:

$$x(\sigma,\zeta)=\frac{1}{\sqrt[4]{\psi(\sigma)}}\sum_{n=0}^{\infty}\varepsilon_r^n\Big\{x_0h_n(\zeta)j_n(\kappa_r,\varsigma) \\ +\left[x_0'g_n(\zeta)+x_0\frac{G}{4}h_n(\zeta)\right]i_n(\kappa_r,\varsigma)\Big\} \\ -\frac{1}{\sqrt[4]{\psi(\sigma)}}\sum_{n=0}^{\infty}\varepsilon_r^{n+1}f_{n+1}(\zeta)i_n(\kappa_r,\varsigma)*\delta_c(\varsigma) \tag{3}$$

where

$$\varsigma=\frac{2\sigma}{\sqrt{\psi(\sigma)}+1} \tag{4}$$

is a new variable for the longitudinal location along the linac. In these expressions x_0 and x_0' are the lateral displacement and angular divergence, respectively, of the bunch at the entrance of the linac and are time independent on end; $\psi(\sigma) = \gamma(\sigma)/\gamma(0)$ with $\gamma(\sigma) = 1 + G\sigma$ and G acceleration gradient; $\delta_c = \psi^{1/4}d_c$; $i_n(\kappa_r,\varsigma)$ and $j_n(\kappa_r,\varsigma)$

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are defined in ref. [4] in terms of Bessel functions and contain only powers and circular functions; $i_n(\kappa_r, \varsigma) * \delta_c(\varsigma)$ is the convlution of i_n and δ_c. With the assumption x_0 and $x_0^{'}$ time independent, then $f_n(\zeta) = g_n(\zeta) = h_n(\zeta)$ and it is defined by the recursion relations in ref [4].

SINGLE BUNCH TRANSVERSE WAKE

The electromagnetic field of the charged bunch interacts with the surrounding accelerating structure generating the wakefields that acts back on the bunch itself. If the bunch travels near to the axis the transverse kick due to wakefields is dominated by dipole fields. In ref. [6] an analytical expression approximating the transverse wake function for BTW sections has been obtained (in $[V/C/m/m]$):

$$w(\zeta) = 2.831 \cdot 10^{16} \left[1 - \left(1 + \sqrt{\frac{\zeta}{\zeta_1}} \right) e^{-\sqrt{(\zeta/\zeta_1)}} \right] \quad (5)$$

with $\zeta_1 = 1.2 \cdot 10^{-4}(m)/l_b$. Let us define the normalized wake function to use in the general solution (3) as:

$$w_n = \frac{w(\zeta)}{W_0} \quad (6)$$

where $W_0 = w(1)$ is the amplitude wake.
The parameters W_0, ζ_1 and the normalized wake function $w_n(\zeta)$ depend on the total bunch length l_b; table (2) reports the values in the "short" and "long" bunch cases together with the dimensionless parameter ε_r.

Table 2: Wakes parameters.

l_b	W_0	ζ_1	ε_r
$[\mu m]$	$[V/C/m/m]$		
200	$1.048 \cdot 10^{16}$	0.600	8.620
500	$1.713 \cdot 10^{16}$	0.240	35.241

Figure 1 shows the normalized wake functions for long and short bunch cases together with their linear approximation. Figure 2 shows the functions $f_n(\zeta)$, $g_n(\zeta)$ and $h_n(\zeta)$ up to n=2. Terms of higher order are negligible; for this reason the sum in (3) converges very fast and we can consider the term up to second order (n=2). Using linear wakes instead of wake functions the relative errors in the general solution (3) is about 20%.

INITIAL LATERAL OFFSET EFFECTS

In this section we consider the effects of an initial lateral offset of the beam in an aligned accelerator; in the general solution (3) we assume $x_0^{'} = 0$ and $d_c(\sigma) = 0$. Figure 3 shows the emittance growth as a function of position along the linac for the short (a) and the long (b) bunch cases. We have used two focusing wave numbers which cover approximately the range of the optics in the LINAC 4. We can see that between the short and the long cases there is a factor 10 in emittance growth.

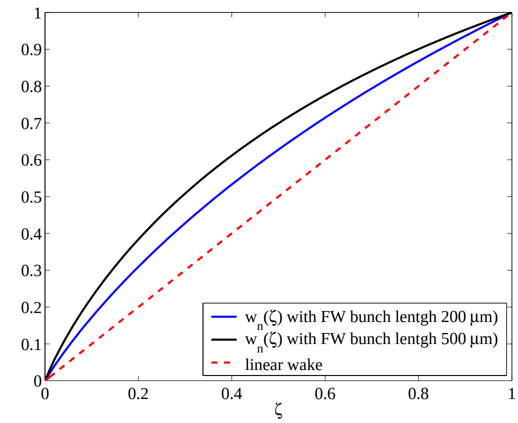


Figure 1: Comparison between wake functions (long bunch case: black line, short bunch case: blue line) and their linear approximation $w(\zeta) = \zeta$ (red dashed line).

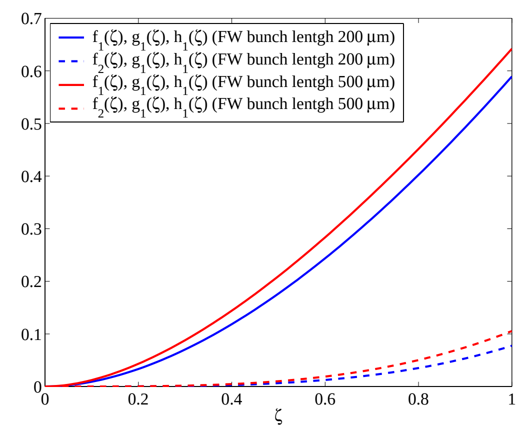
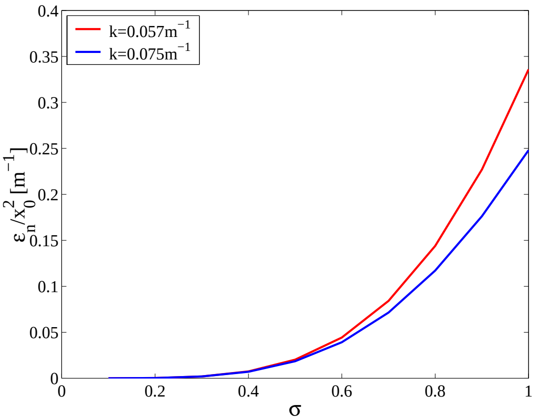


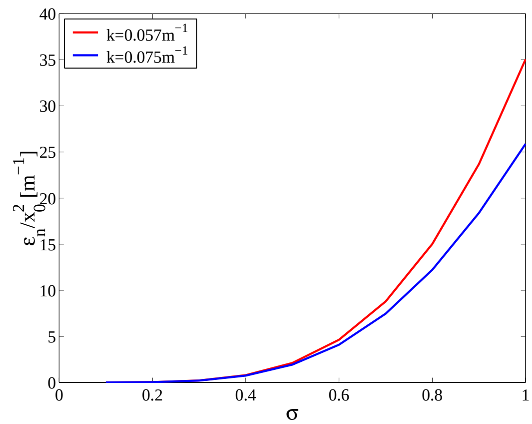
Figure 2: Functions $f_n(\zeta)$, $g_n(\zeta)$ and $h_n(\zeta)$ up to second order for long and short bunch.

Comparisons with tracking

In order to confirm the analytical results they have been compared to results from the Elegant [7] tracking code. Figure 4 shows the lateral displacement (a) and the angular divergence (b) as a function of position inside the bunch at the end of LINAC 4 for an initial offset of $200 \mu m$. We can see excellent agreement between analytical and numerical results. We have repeated the comparison for different initial offsets and the results are listed in table 3. The comparisons have been performed even for the long bunch case, confirming again the good agreement. The results will be report in a following paper. There are two discrepancies between the tracking and the analytical solution: (a) in Elegant we have imposed a focusing strength that does not change with the linac energy, while the analytical case foresees a focusing strength that decreases with the linac energy, i.e. $k(\sigma) = k_r/\sqrt{\psi(\sigma)}$. For this reason we need small corrections on the focusing strength in the analytical



(a)



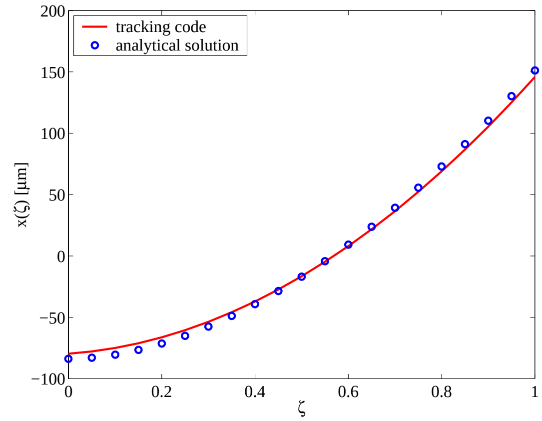
(b)

Figure 3: Normalized emittance growth along the LINAC 4 in the short (a) and long (b) bunch cases.

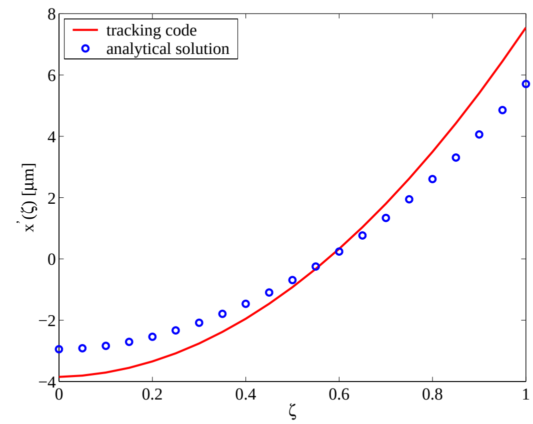
model respect to tracking; (b) we have found slight differencies in the centroid offset at the linac end; nevertheless it has not influence on the calculation of the normalized projected emittance.

Table 3: Comparison between analytical solution and ELEGANT tracking with bunch length $200\mu m$ and different initial offset.

	ELEGANT	**Analytical**
X_0	$\Delta\varepsilon_{n,rms}$	$\Delta\varepsilon_{n,rms}$
$[\mu m]$	$[\mu rad]$	$[\mu rad]$
50	$0.08 \cdot 10^{-2}$	$0.08 \cdot 10^{-2}$
100	$0.33 \cdot 10^{-2}$	$0.34 \cdot 10^{-2}$
150	$0.75 \cdot 10^{-2}$	$0.76 \cdot 10^{-2}$
200	$1.32 \cdot 10^{-2}$	$1.34 \cdot 10^{-2}$
250	$2.07 \cdot 10^{-2}$	$2.10 \cdot 10^{-2}$
300	$2.97 \cdot 10^{-2}$	$3.02 \cdot 10^{-2}$
500	$8.24 \cdot 10^{-2}$	$8.40 \cdot 10^{-2}$



(a)



(b)

Figure 4: Comparison between analytical (blue circles) and tracking (red line) results for the lateral displacement (a) and angular divergence (b) at the LINAC 4 end due to initial offset of $200\mu m$ in the short bunch case.

MISALIGNED BTW SECTIONS EFFECTS

In this section we consider the effects of the misaligned BTW accelerating structures without lateral offset and angular divergence of the beam at the entrance of the linac. We assume in the general solution (3) $x_0 = 0$ and $x_0^{'} = 0$. A bad case where the accelerating sections are all misaligned by d_c is here considered. Figure 5 shows the normalized lateral displacements as a function of position inside the bunch at linac end in short and long bunch cases. We can see that the displacement of the bunch tail at linac end increase by a factor about 5 from short to long bunch case. Table 4 lists the normalized emittance growth for different misalignments d_c of the BTW accelerating sections for both bunch length cases. In the short bunch case the emittance growth is limited while in the long bunch case the emittance growth becomes important if the misalignment of the cavities is more than $100\mu m$. At the moment we have not a comparison with Elegant tracking; it will be carried out in future.

Using Elegant we have seen that for the short bunch case the emittance growth caused by the transverse wakefields, when including initial offset in position and divergence and randomly misaligned cavities can be completely neutralized using a two steps algorithm: first, a trajectory correction is performed; then, a local bump performed with few correctors permits to find a trajectory for which the growth of the projected emittance is minimal at the linac end. The results are reported in [8]. In future we will apply this algorithm to the long bunch case.

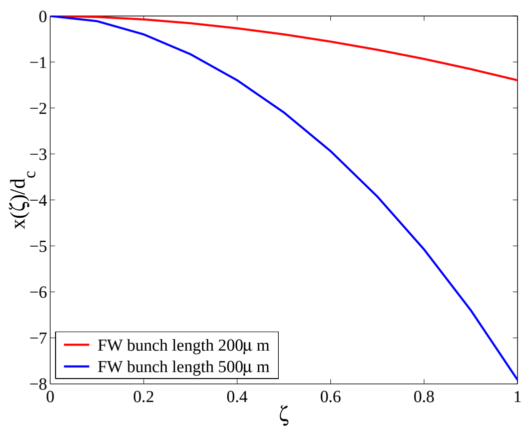


Figure 5: Normalized lateral displacements as a function of position inside the bunch at the LINAC 4 end in the short (red line) and long (blue line) bunch cases.

Table 4: Normalized emittance growth for different misalignments d_c of the BTW accelerating sections.

	Short short	**Long bunch**
d_c	$\Delta\varepsilon_{n,rms}$	$\Delta\varepsilon_{n,rms}$
$[\mu m]$	$[\mu rad]$	$[\mu rad]$
50	$0.10 \cdot 10^{-2}$	0.10
100	$0.38 \cdot 10^{-2}$	0.40
150	$0.86 \cdot 10^{-2}$	0.90
200	$1.53 \cdot 10^{-2}$	1.60
250	$2.39 \cdot 10^{-2}$	2.49
300	$3.44 \cdot 10^{-2}$	3.59
500	$9.55 \cdot 10^{-2}$	10.67

CONCLUSION

An estimate of the emittance growth under the combined influence of transverse wakefields, initial offset and misaligned accelerating sections has been carried out using a formalism presented in [4, 5] that allows direct calculation, at any position inside the bunch and along the linac, of the transverse displacement of the beams. Tracking simulations have also been performed to support the consistency of the analytical solution. An excellent agreement between numerical and analytical results has been obtained.

The analytical results have been applied to short and long single bunch operations of the FERMI@ELETTRA FEL project. The analysis has highlighted a worsening in the emittance growth along the linac mainly in the long bunch case. Previous tracking simulations have pointed out that, at least in the short bunch case, the emittance growth can be completely neutralized.

ACKNOWLEDGEMENT

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