EFFECT OF LOSSES ON THE GAIN AND START CURRENT IN SMITH-PURCELL FREE-ELECTRON LASERS

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Abstract

The effect of dissipative losses in the grating surface of a Smith-Purcell free-electron laser is examined analytically. It is shown that losses have a significant impact on the gain of amplifiers (which operate on a convective instability) and on the growth rate of oscillators (which operate on an absolute instability), but it is not absolutely necessary for the ideal gain (without losses) to exceed the ideal attenuation (without gain) to have positive net gain or growth rate.

INTRODUCTION

Smith-Purcell free-electron lasers (SP-FELs) operate on an evanescent wave of the grating over which the electrons are passing [1]. Depending on the phase velocity of the evanescent wave, which is synchronous with the electron velocity, the group velocity may be positive or negative. If the group velocity is positive, the device operates on a convective instability as an amplifier; if the group velocity is negative, the device operates on an absolute instability and oscillates spontaneously, without external feedback, above the so-called start current [2].

As SP-FELs are designed to operate at higher frequencies, dissipative losses in the grating surface become more important. To describe the effect of these losses, we develop a dispersion relation for the evanescent wave that includes the losses, and we examine the solutions for both amplifiers and oscillators.

DISPERSION RELATION

We consider a metallic grating having a period L and wave number $K = 2\pi / L$. Such a grating supports an evanescent wave of frequency ω and wave number k that travels along the surface of the grating in the direction perpendicular to the grooves. The phase velocity is $v_\phi = c\beta_\phi = \omega / k$, where c is the speed of light. The dispersion relation $\omega(k)$ for a typical grating is shown in Figure 1. Within each Brillouin zone the curve is symmetric about the point $k/K = 1/2$, where the group velocity $v_g = c\beta_g = d\omega / dk$ vanishes. To the left of this point, called the Bragg point, the group velocity is positive, and to the right it is negative. To represent the effect of an electron beam, the region above the grating is filled with a uniform plasma moving to the right at the velocity $v = \beta c$, parallel to the top of the grating in a direction perpendicular to the grooves. The synchronous point (ω_0, k_0) is the point where the beam line of the plasma, $\omega = \beta ck$, intersects the dispersion curve as shown in Figure 1.

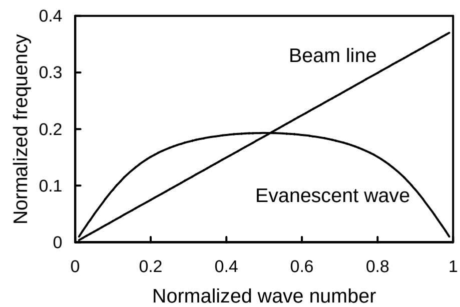

Figure 1: Dispersion relation and beam line for a typical grating.

In the neighborhood of the synchronous point (ω_0, k_0) the dispersion relation for the evanescent wave traveling along a perfect grating (no losses) in the presence of the electron beam is

$$\delta\omega - \beta_g c\delta k = \frac{\omega_p^2 S}{\gamma^3 R_\omega (\delta\omega - \beta c\delta k)^2} \tag{1}$$

where $\delta\omega$ is the complex frequency shift, δk the complex wave number shift, $\gamma = 1/\sqrt{1-\beta^2}$ the Lorentz factor, and ω_p the plasma frequency in the laboratory frame. The factors S and R_ω depend on the details of the grating profile [2]. To account for the effect of losses, we argue as follows. If we place perfect reflectors at the ends of a short section of the grating, it becomes a resonant cavity. Ignoring the effect of the beam for the moment, we can take (ω_0, k_0) as the operating point of the resonator. If we now introduce small resistive losses in the surface of the grating, the frequency shift of the resonant cavity is [3]

$$\delta\omega = -\frac{\omega_0}{2Q_c}(1+i) \tag{2}$$

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The Q of the cavity is

$$Q_c = \omega_0 \frac{\langle U \rangle}{\langle Q \rangle} \tag{3}$$

where $\langle U \rangle$ is the stored energy per unit length and $\langle Q \rangle$ the power loss per unit length. By adding this term to the left-hand side of (1) we get the complete dispersion relation, including losses

$$(\delta\omega - \beta c \delta k)^2 \left[\delta\omega - \beta_g c \delta k + \frac{\omega_0}{2Q_c}(1+i) \right] = \Delta \tag{4}$$

where

$$\Delta = \frac{\omega_p^2 S}{\gamma^3 R_\omega} \tag{5}$$

Calculations show that Δ is positive. A similar equation is obtained for gyrotrons [4].

AMPLIFIERS

When the group velocity is positive, the device operates on a convective instability [5]. A wave incident from the upstream end of the grating travels along with the electron beam. If the wave is synchronous with the electron beam, the wave is amplified. For a steady incident wave we may take $\delta\omega = 0$ and the dispersion relation (4) simplifies to

$$(\beta c \delta k)^2 \left[\beta_g c \delta k - \frac{\omega_0}{2Q_c}(1+i) \right] + \Delta = 0 \tag{6}$$

In the absence of losses ($Q_c \to \infty$) we are left with

$$\delta k^3 = -\frac{\Delta}{\beta^2 \beta_g c^3} \tag{7}$$

This admits three roots, and the ideal gain (the gain in the absence of losses) on the fastest-growing wave is

$$\mu_\infty = -\operatorname{Im}(\delta k) = \frac{\sqrt{3}}{2} \left| \frac{\Delta}{\beta^2 \beta_g c^3} \right|^{1/3} \tag{8}$$

as found previously [2]. Since Δ is a slowly varying function of the wave number, we see that near the Bragg point ($\beta_g = 0$) the gain has the behavior $\mu_\infty \propto O\left(\beta_g^{-1/3}\right)$, so the gain is largest when the group velocity is small. In the absence of the electron beam ($\Delta = 0$), the dispersion relation (6) reduces to

$$\delta k = \frac{\omega_0}{2\beta_g c Q_c}(1+i) \tag{9}$$

The attenuation coefficient in an empty grating is therefore

$$\nu_\infty = \operatorname{Im}(\delta k) = \left| \frac{\omega_0}{2\beta_g c Q_c} \right| = \left| \frac{\langle Q \rangle}{2\beta_g c \langle U \rangle} \right| \tag{10}$$

as found previously [2]. However, while the imaginary part of (9) gives the attenuation, the real part introduces a phase shift that is not accounted for in previous work. Since $\langle Q \rangle$ and $\langle U \rangle$ are slowly varying functions of the wave number, we see that $\nu_\infty \propto O\left(\beta_g^{-1}\right)$ near the Bragg point, so the attenuation is largest when the group velocity is small. In fact, we see that

$$\nu_\infty / \mu_\infty \propto O\left(\left|\beta_g\right|^{-2/3}\right) \xrightarrow[\beta_g \to 0]{} \infty \tag{11}$$

so the attenuation in an empty grating always exceeds the ideal gain near the Bragg point.

When gain and attenuation are both present, it is convenient to write the dispersion relation in the dimensionless form

$$\Delta k^2 \left[\Delta k - (1+i) \right] + J^3 = 0 \tag{12}$$

where

$$\delta k = \frac{\omega_0}{2\beta_g c Q_c} \Delta k = \nu_\infty \Delta k \tag{13}$$

$$J = \frac{2Q_c}{\omega_0} \left| \frac{\beta_g^2 \Delta}{\beta^2} \right|^{1/3} = \frac{2\mu_\infty}{\sqrt{3}\nu_\infty} \tag{14}$$

The dispersion relation (12) admits three roots. When the electron density vanishes, one root corresponds to the structure wave (the wave that propagates along the grating in the absence of an electron beam) and it decays with $\operatorname{Im}(\Delta k) = 1$. The other two waves are the fast and slow space-charge waves, for which $\operatorname{Im}(\Delta k) = 0$ when the electron density vanishes. As the electron density increases, the slow space-charge wave becomes the growing wave and the fast space-charge wave becomes a slowly decaying wave. The structure wave becomes the strongly decaying wave. For any $J > 0$ the slow space-charge wave is always growing.

OSCILLATORS

When the synchronous point lies to the right of the Bragg point on the dispersion curve, the group velocity is negative and the device operates on an absolute instability [2,5]. The evanescent wave traveling backward bunches the plasma near the upstream end of the grating, and as the bunched beam travels toward the downstream end of the grating it excites the evanescent wave. Thus, the instability is self-excited and external feedback is not necessary. Above the start current, the device oscillates spontaneously. In an oscillator, both the frequency shift and the wave number shift are nonvanishing, so it is convenient to rewrite (4) in the dimensionless form

$$\delta^2(\delta-\kappa)+1=0 \qquad (15)$$

where the signs have been chosen for the case $\beta_g<0$, and

$$\delta_j=\left|\frac{\beta^2\beta_g c^3}{\Delta}\right|^{1/3}\left(\frac{\delta\omega}{\beta c}-\delta k_j\right) \qquad (16)$$

$$\kappa=\left|\frac{\beta^2\beta_g}{\Delta}\right|^{1/3}\left[\left(\frac{1}{\beta}-\frac{1}{\beta_g}\right)\delta\omega-\frac{\omega_0}{2\beta_g Q_c}(1+i)\right] \qquad (17)$$

in which the subscript $j=1\ldots3$ is used to identify each of the three roots. As discussed previously [2], the boundary conditions are that there is no wave incident from the downstream end of the grating, and the electron beam enters the region above the grating undisturbed. These are summarized by the equation

$$\begin{vmatrix} 1/\delta_1^2 & 1/\delta_2^2 & 1/\delta_3^2 \\ 1/\delta_1 & 1/\delta_2 & 1/\delta_3 \\ e^{-i\xi\delta_1} & e^{-i\xi\delta_2} & e^{-i\xi\delta_3} \end{vmatrix}=0 \qquad (18)$$

(19)

where the parameter

$$\xi=\left|\frac{\Delta}{\beta^2\beta_g c^3}\right|^{1/3} Z=\frac{2}{\sqrt{3}}\mu_\infty Z$$

is proportional to the ideal gain and the grating length. The numerical solutions are the same as those obtained with losses neglected [6]. Some results are shown in Figure 2.

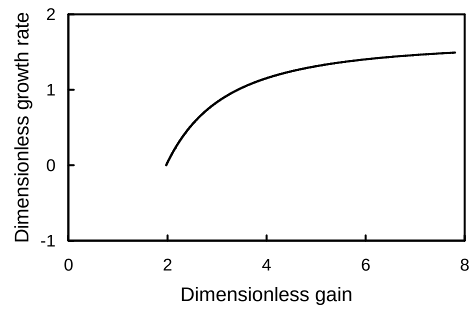

Figure 2: Dimensionless growth rate $\mathrm{Im}(\kappa)$ as a function of the dimensionless gain ξ.

In terms of the ideal gain μ_∞ and empty-grating attenuation ν_∞, we see from (17) that the growth rate of the oscillation is

$$\mathrm{Im}(\delta\omega)=\frac{2}{\sqrt{3}}\frac{\beta\beta_g c\mu_\infty}{\beta_g-\beta}\left[\mathrm{Im}(\kappa)-\frac{\sqrt{3}\nu_\infty}{2\mu_\infty}\right] \qquad (20)$$

As a test of these predictions, we can compare our results to the simulations reported recently by Donohue and Gardelle [7]. As shown in Figure 3, the agreement is remarkably good.

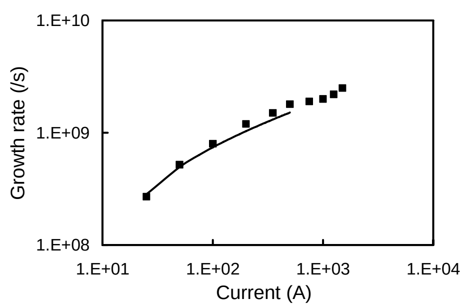

Figure 3: Computed growth rate(curve)compared with the simulations of Donohue and Gardelle (squares)[7].

For the mode to grow with time, it is necessary that $\mathrm{Im}(\delta\omega)>0$, so the start current, above which oscillations grow, corresponds to the condition

$$\mathrm{Im}(\kappa)>\left|\frac{\beta^2}{\beta_g^2\Delta}\right|^{1/3}\frac{\omega_0}{2Q_c}=\frac{\sqrt{3}\nu_\infty}{2\mu_\infty} \qquad (21)$$

In the simulations of Donohue and Gardelle, losses are absent, so the start condition is $\mathrm{Im}(\kappa)>0$. From Figure 2 we see that this corresponds to $\xi>\xi_0\approx 1.973$. In the simulations this occurs at a current of 8.5 A, just off the left edge of Figure 3, and below this current the oscillations vanish.

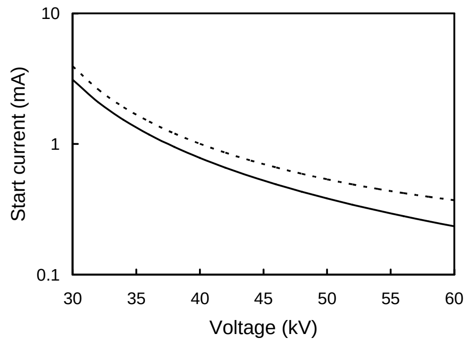

Figure 4: Predicted start current in the SP-FEL experiments at Dartmouth [8] including losses (solid curve) and ignoring losses (dotted curve).

The only available experimental data are from the experiments at Dartmouth [8,9]. In these experiments losses were significant but not large. The predicted start current with and without losses is shown in Figure 4. At 40 kV, the difference is about 25 percent. In those experiments, the start current at 40 kV was observed to be on the order of 1 mA, in reasonable agreement with the predictions. Losses are expected to become more important at higher frequencies.

CONCLUSIONS

From the above analysis we see that dissipative losses in the grating of a SP-FEL have an impact on the gain and growth rate. In an amplifier, there is always one wave (the slow space-charge wave) for which the net gain is positive, no matter how large the attenuation. However, the impact of losses is more important in an oscillator.

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