QUANTUM SASE FEL WITH A LASER WIGGLER*

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Abstract

Quantum effects in high-gain FELs are ruled by the quantum FEL parameter, $\bar{\rho}=\rho mc\gamma/\hbar k_r$, which is the ratio between the momentum spread at saturation and the one photon momentum recoil. It has been shown that when $\bar{\rho}\leq 1$ the spectrum of the emitted radiation changes from the broad continuous and chaotic spectrum of the classical regime to a series of discrete and equally spaced very narrow lines, due to transitions between discrete momentum states. In this paper we show that the quantum regime can be achieved using Kilometers long magnetic wigglers or a laser wiggler. In this paper we state the scaling laws necessary to operate a Quantum SASE FEL in the Angstrom region with a laser wiggler. Specific example is given having in mind a high power Ti:Sa laser wiggler at λ= 0.8 μm, in construction at LNF-INFN Laboratories, for the SPARC/PLASMON_X project.

INTRODUCTION

It has been previously recognized that quantum effects in SASE FEL are determined by the quantum FEL parameter $\bar{\rho}$ [1], [2] defined as the usual FEL parameter times the ratio between the electron energy and the photon energy. Quantum effects become relevant when $\bar{\rho}\leq 1$. However the calculations of [1],[2] are confined to the linear regime. In [3] we have extended the theory of Quantum SASE FEL to the non linear regime and we have shown the phenomenon of quantum purification of SASE spectrum: the broad superposition of chaotic series of random spikes predicted by the classical theory changes dramatically, when $\bar{\rho}\leq 1$, to a series of discrete equally spaced very narrow lines. The question is: which are the experimental set up and the experimental parameters necessary to observe Quantum SASE in the short wavelength region, where quantum effects are expected to be relevant?

MAGNETIC VERSUS LASER WIGGLER

The Quantum FEL (QFEL) parameter is given by

$$\bar{\rho}=\rho\frac{mc\gamma}{\hbar k_r}=\gamma\rho\frac{\lambda_r}{\lambda_C} \tag{1}$$

where ρ is the classical FEL parameter [4], λ_r and $\lambda_c=h/mc\approx 0.024$ Å are the radiation wavelength and the Compton wavelength, respectively, and γ is the resonance energy in units mc^2, given by

$$\gamma=\frac{1}{b}\sqrt{\frac{\lambda}{\lambda_r}(1+a_w^2)} \tag{2}$$

where λ is the wiggler wavelength and $b=\sqrt{2}$ for the magnetic wiggler and $b=2$ for an electromagnetic (e.m.) laser wiggler. Clearly the physical meaning of $\bar{\rho}$ is the ratio between the classical momentum spread and the photon recoil $\hbar k$, so that quantum effects become important when $\bar{\rho}\leq 1$, because in this case the discreteness of momentum exchange is relevant.

Using Eqs. (1) and (2) the QFEL condition becomes

$$\rho\leq\frac{b\lambda_c}{\sqrt{\lambda_r\lambda(1+a_w^2)}} \tag{3}$$

so that, to reach the high-gain region, one needs a number of wiggler periods of the order of 1/ρ, i.e., a wiggler length L_w given by

$$L_w=N_w\lambda\approx\frac{\lambda}{\rho}\geq\frac{\sqrt{\lambda_r\lambda^3(1+a_w^2)}}{b\lambda_c} \tag{4}$$

In order to have $\lambda_r\approx 1$ Å, using Eqs.(2), (3) and (4), one has the following numbers:

Magnetic wiggler, $\lambda\approx 1$ cm, $b=\sqrt{2}$, E= 3.5 GeV, $\rho\leq 3.4\cdot 10^{-6}$, $L_w\geq 3$ Km.

Laser wiggler, $\lambda\approx 1$ μm, $b=2$, E=50 MeV, $\rho\leq 5.10^{-4}$, $L_w\geq 2$ mm!

For simplicity we have assumed $a_w \ll 1$. The opposite case would require a longer wiggler. The previous considerations clearly show that a QFEL with a magnetic

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wiggler is unpractical, whereas, using a laser wiggler it can be a table top apparatus.

Therefore, we propose a typical Compton back-scattered configuration: a low energy electron beam counter-propagating with respect to an electromagnetic wiggler (wave) provided by a high power laser.

THE LASER WIGGLER

Let us consider a laser wiggler with radiation propagating in the z direction opposite to an electron beam with the following specifications: W_0 is the minimum diameter of the laser beam in the focus, σ_0 is the minimum radius of the electron beam, $Z_0 = \pi W_0^2 / \lambda$ is the distance in which the radiation beam diverges (Rayleigh range), $\beta^* = 2\sigma_0^2\gamma / \varepsilon_n$ is the analogous length for the electron beam [5]. The meaning of Z_0 and β^* appears from the well known equations $W(z) = W_0\sqrt{1+(z/z_0)^2}$ and $\sigma(z) = \sigma_0\sqrt{1+(z/\beta^*)^2}$ [5]. In this section we follow [6], within factors 2. Let us define the adimensional parameters:

$$\varepsilon_1 = \frac{W_0}{2\sigma_0} \;\; ; \;\; \varepsilon_2 = \frac{\beta^*}{Z_0}. \tag{5}$$

In order to ensure a good overlapping and matching between radiation and electron beam we must impose that ε_1 and ε_2 ≥ 1. Hence, because $\varepsilon_1^2\varepsilon_2 = \gamma\lambda / 2\pi\varepsilon_n \geq 1$, our conditions guarantee that the emittance criterium is satisfied for the laser wiggler.

In [6], it has been shown that the previous relations leads to:

$$\lambda_r(A) = \lambda^3 \frac{(1+a_w^2)}{(32\pi\eta)^2} = \frac{\lambda^3(\mu m)}{\eta^2}(1+a_w^2) \tag{6}$$

where

$$\eta = \frac{\varepsilon_1^2\varepsilon_2\varepsilon_n(mm.mrad)}{8}. \tag{7}$$

The factor 8, which does not appear in [6], is due to a factor 2 in the definition of ε_1 and β^*. We remark that Eq. (6) (formally independent on the electron energy) gives a direct relation between the radiation wavelength and the wiggler wavelength in terms of two geometrical parameters and ε_n (via the η factor), and the wiggler parameter. Equation (6) can be derived using the following chain of equations

$$Z_0 = \frac{\pi W_0^2}{\lambda} = \frac{4\pi\varepsilon_1^2\sigma_0^2}{\lambda} = \frac{2\pi\varepsilon_1^2\varepsilon_n\beta^*}{\lambda\gamma} = \frac{4\pi\varepsilon_1^2\varepsilon_2\varepsilon_n}{\lambda^{3/2}}\left(\frac{\lambda_r}{1+a_w^2}\right)^{1/2} Z_0$$

Eliminating Z_0 from the first and the last equation, we obtain Eq. (6). Furthermore, using Eqs.(2) and (6) one obtain the identity

$$\gamma = \frac{16\pi\eta}{\lambda} = 50\frac{\eta}{\lambda(\mu m)} \tag{8}$$

Equation (8) fix the resonant energy only in terms of the parameter η and of the laser wiggler wavelength.

As an example, if we want to produce 1 Angstrom radiation, with $a_w \ll 1$ and $\eta = 1$, Eq.(6) and (8) gives a laser wiggler with $\lambda = 1$ µm and $\gamma = 50$.

In [6] it has been shown that the wiggler parameter is self consistently determined by the equation $a_w^2 = \frac{a_0^2}{1+a_w^2}$, where

$$a_0^2 \approx P\frac{\varepsilon_2}{\varepsilon_3}\frac{\eta}{\lambda^2}\left(\frac{\bar{\rho}^{3/2}}{\sqrt{1+\bar{\rho}}}\right) \tag{9}$$

where P is the peak laser power in TW and we have assumed a laser pulse with a gaussian transverse profile. Therefore, solving the previous equation, we obtain $a_w = a_0 / \sqrt{F(a_0)}$ where

$$F(a_0) = \frac{1+\sqrt{1+4a_0^2}}{2} = 1 + a_w^2 \tag{10}$$

Note that in the limit $4a_0^2 \ll 1$, $a_w \approx a_0$, whereas in the opposite limit $a_w \approx \sqrt{a_0}$.

Inverting Eq. (1) and using Eq.(6), one has

$$\rho = 5.10^{-4}\frac{\eta}{\lambda^2[\mu m]}\frac{\bar{\rho}}{F(a_0)} \tag{11}$$

Let us define the quantum gain length [6]

$$L_g[\mu m] = \frac{\lambda[\mu m]}{8\pi\rho\sqrt{\bar{\rho}}}\left(\sqrt{1+\bar{\rho}}\right) = 80\frac{\lambda^3[\mu m]}{\eta\bar{\rho}^{3/2}}\left(\sqrt{1+\bar{\rho}}\right)F(a_0) \tag{12}$$

where Eq. (11) has been used.

Equation (12) is not an exact equation, but is an interpolation formula which gives the correct behaviour in the quantum regime $\bar{\rho} \ll 1$ [3] and the classical expression [4] in the opposite limit. This equation can be rigorously justified in the asymptotic cases $\bar{\rho}$ very large or very small.

We must also impose that the electron beam characteristic length β^* is larger than the gain length, i.e.,

$$\varepsilon_3 = \beta^* / L_g \geq 1 . \quad (13)$$

It can be easily shown that

$$\sigma_0^2[\mu m] = \frac{\varepsilon_n \beta^*}{2\gamma} = \frac{\varepsilon_n \varepsilon_3 L_g}{2\gamma} \approx \varepsilon_n \varepsilon_3 \frac{\lambda^4[\mu m]}{\eta^2 \bar{\rho}^{3/2}} \left(\sqrt{1+\bar{\rho}}\right) F(a_0) . \quad (14)$$

Furthermore, the relative energy spread, in the quantum regime, is subjected to the limitation [3]

$$\frac{\Delta\gamma}{\gamma} \leq 4\rho\sqrt{\bar{\rho}} = \frac{2\cdot 10^{-3}}{F(a_0)} \frac{\eta}{\lambda^2[\mu m]} \bar{\rho}^{3/2} \quad (15)$$

where Eqs. (11) has been used.

In [6] it has been shown that the current, I, necessary to achieve a given value of the QFEL parameter, $\bar{\rho}$, is given by

$$I(A) = 3\cdot 10^2 \frac{1}{P\lambda^5} \left(\frac{\varepsilon_n \varepsilon_3^2 \eta^3}{\varepsilon_2 F(a_0)} \right) (1+\bar{\rho}) \quad (16)$$

where the wiggler laser power, P, is given in TW.

ε_n (mm mrad)	0.5	1	0.5	1
ε_1	1	1	1	1
ε_2	12	6	10	5
ε_3	20	30	50	40
Laser power (TW)	20	100	50	100
Current (A)	230	353	440	531
λ_r (Å)	1.5	1.8	2.0	2.1
E_b (MeV)	23.4	23.4	19.5	19.5
$\Delta\gamma/\gamma$ (10^{-4})	2.5	2.2	2.4	2.2
τ (psec)	84	148	229	195
d(μm) = 2 σ_0	7.3	13.7	13.2	17.2
L_{gain} (10^3 μm)	1.1	1.3	1.2	1.3
N of photons (10^{11})	1.2	3.3	6.3	6.5
P(MW)	1.8	2.4	2.7	3.0

Table 1. Example of the various parameters for $\bar{\rho}$=0.2 and λ = 0.8 μm (laser wiggler).

We remark that in the quantum limit $\bar{\rho}$ <<1 the current is independent on $\bar{\rho}$, whereas in the classical limit, $\bar{\rho}$ >>1, the current increases linearly with $\bar{\rho}$, making more problematic the use of an e.m. wiggler to see classical SASE.

The minimum laser time duration required is given by:

$$\tau(p\sec) = \beta^* / c = 3.3\cdot 10^{-3} \varepsilon_3 L_g [\mu m] \quad (17)$$

where L_g is given by Eq. (12).

Furthermore, using Eq. (10), Eq. (6) can be rewritten as:

$$\lambda_r = \frac{\lambda^3}{\eta^2} F(a_0) . \quad (18)$$

The units are: λ in μm, λ_r in Å and ε_n in mm mrad and P in TW. Explicit examples for a laser wiggler at 0.8 μm are given in Table 1. Furthermore, keeping in mind that in QFEL one can have maximum of the order of one photon per electron, the order of magnitude of the number of photons, N, and the radiated power, P_r, are given by $N \approx \tau I / e$ and $P_r \approx \hbar\omega I / e$.

EMITTANCE LIMITATIONS

In this paragraph we will assume to be in the quantum regime, i.e., $\bar{\rho}$ <<1.

The emittance criterium is satisfied for the laser wavelength imposing that ε_1, ε_2 of Eq.(5) are larger than one. Roughly speaking, this is equivalent to require that the electron beam is contained in the laser wiggler beam, otherwise the electron would not be interacting with the wiggler. If one defines the quantity ε_1, ε_2, Z_0 and W_0 referred to the radiation beam at the wavelength λ_r, then, in order to guarantee that the radiated field is contained in the radiating electron beam, we should impose that ε_1 and ε_2 are less than unity, because the field outside the electron beam cannot be amplified. Therefore, the emittance criterium, i.e. $2\pi\varepsilon_n / \gamma\lambda_r \leq 1$, should be reversed regarding the radiation field. However, the emittance cannot be arbitrarily large because the transverse emittance implies a spread on the resonant wavelength, which must be small enough not to degrade the FEL action. In fact, the resonance condition with off-axis propagation reads:

$$\lambda_r = \frac{\lambda(1 + a_w^2 + \gamma^2\theta^2)}{4\gamma^2} \quad (19)$$

Because $0 \leq \theta \leq \varepsilon_r / 2\sigma$ the maximum spread is given by $\frac{\Delta\lambda}{\lambda} = \frac{\varepsilon_n^2}{4\sigma^2(1+a_w^2)} \approx 2\frac{\Delta\gamma}{\gamma}$. Imposing condition (15), one has

$$\frac{\Delta\gamma}{\gamma} = \frac{\varepsilon_n^2}{8\sigma^2(1+a_w^2)} < 4\rho\sqrt{\bar{\rho}} = \frac{2\cdot 10^{-3}}{F(a_0)} \frac{\eta}{\lambda^2[\mu m]} \bar{\rho}^{3/2} . \quad (20)$$

Note that the emittance criterium can be written as

$$X = 2\pi\varepsilon_n / \gamma\lambda_r < 1 \quad (21)$$

whereas condition (20), using Eq.(6,8,14) with $\bar{\rho} << 1$, can be written as

$$X \leq 20\varepsilon_3 \quad (22)$$

where ε_3 is given by Eq.(13). Hence, if $\varepsilon_3 >> 1$ the emittance criterium can be relaxed by orders of magnitude. Note that, for $\varepsilon_1 = 1$ one has $X = Z_r / \beta^*$, where Z_r is the Rayleigh length of the radiation field. In this case, criterium (22) can be written as

$$X \leq 4\sqrt{Z_r / L_g} \quad (23)$$

Finally, using Eq.(14), with $\bar{\rho} << 1$, Eq. (20) can be written as

$$\varepsilon_n \leq \frac{1.6 \cdot 10^{-2}\, \varepsilon_3 \lambda^2}{\eta} F(a_0)\,. \quad (24)$$

Eq.(24), using Eq.(7), is equivalent to

$$\varepsilon_n \leq 0.4\lambda \sqrt{\frac{\varepsilon_3 F(a_0)}{\varepsilon_1^2 \varepsilon_2}}\ . \quad (25)$$

Eqs.(22), (24) and (25) give the emittance limitation for a QFEL with a laser wiggler.

QFEL EXPERIMENTAL STUDIES

The high brightness SPARC photo-injector [7] and the High Intensity Laser Laboratory (HILL) [8] under development at LNF, will provide an excellent facility to test the Quantum SASE FEL (QFEL) theoretical predictions [3]. The main component of this test facility is a 800 nm, 100 TW-class Ti:Sa laser system synchronized to the SPARC photo-injector. Eventually an additional beam line will be built in the SPARC bunker in order to transport the SPARC electron beam at an interaction point with the incident laser beam. A final focus system, already foreseen for the Thomson backscattering source [8], will allow conducting experiments of generation of X-ray radiation via QFEL interaction of electron bunches in a laser wiggler, as schematically shown in Figure 1.

Parametric studies based on the QFEL scaling laws discussed in this paper, 3D simulations of the QFEL interaction and electron beam dynamics studies are under way in order to match the SPARC photo-injector performances to the requirement of a realistic QFEL experiment.

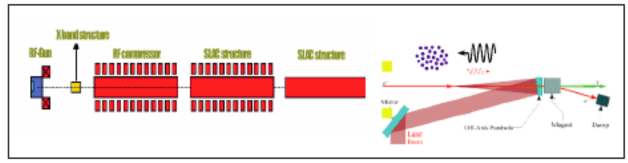

Figure 1: Schematic experimental setup for a QFEL interaction of electron bunches in a laser wiggler.

CONCLUSIONS

In conclusion, we have shown that the construction of a Quantum SASE FEL with a magnetic wiggler is very problematic because it would require a wiggler length larger than 1 Km, with electron energy in the GeV region, whereas with a laser wiggler of 1 μm wavelength, the apparatus can be table top with the wiggler length of the order of mm. Furthermore, we have shown that the criterium for emittance can be relaxed by orders of magnitude.

We have given the expression of the relevant quantities for the design of a Quantum SASE FEL with a laser wiggler in terms of the quantum FEL parameter $\bar{\rho}$, showing that the required current increases linearly with $\bar{\rho}$ in the classical regime, when $\bar{\rho} >> 1$, whereas is independent of $\bar{\rho}$ in the quantum regime, when $\bar{\rho} \leq 1$. This fact makes more difficult the use of the laser wiggler in the classical regime than in the quantum regime. If a quantum SASE FEL can be built one would have X-ray coherent FEL that can be a table-top object and the technological problem would go from high-energy accelerator plus long magnetic wigglers to the widely used high-power laser technology. We have given an example assuming a Ti:Sa laser wiggler at 0.8 μm.

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