# MULTI-OBJECTIVE OPTIMIZATION FOR PURE PERMANENT-MAGNET UNDULATOR MAGNETS ORDERING USING MODIFIED SIMULATED ANNEALING \*

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#### Abstract

Undulator filed errors influence the electron beam trajectories and lower the radiation quality. Angular deflection of electron beam is determined by first field integral, orbital displacement of electron beam is determined by second field integral and radiation quality can be evaluated by rms field error or phase error. Appropriate ordering of magnets can greatly reduce the errors. We apply a modified simulated annealing algorithm to this multi-objective optimization problem, taking first field integral, second field integral and rms field error as objective functions. Undulator with small field errors can be designed by this method within a reasonable calculation time even for the case of hundreds of magnets (first field integral reduced to 10<sup>-6</sup>T·m, second integral to  $10^{-6}$ T·m<sup>2</sup> and rms field error to 0.01%). Thus, the field correction after assembling of undulator will be greatly simplified. This paper gives the optimizing process in detail and puts forward a new method to quickly calculate the rms field error and field integrals.

# **INTRODUCTION**

Undulators and wigglers are the main components of third-generation synchrotron radiation sources and freeelectron lasers [1]. Undulators are mostly built of permanent magnets with iron poles (hybrid undulators) or without iron poles (Pure Permanent-Magnet undulators or PPM undulators). The unavoidable remanence inhomogeneities of these magnets and construction errors cause the undulator magnetic field errors [2], which affect the trajectories of the electron beam and lower the radiation quality of undulator or FEL [3]. Angular deflection of electron beam is determined by first field integral along the beam axis, orbital displacement of electron beam is determined by second field integral along the beam axis, and radiation quality can be evaluated by rms field error or phase error [4]. To reduce the undulator magnetic field errors, we need a sufficiently precise mechanical construction and a certain method to overcome the influence of remanence inhomogeneities. That is, we should not only provide a precise method to measure the individual permanent magnets and assemble them to form undulator [5], but also provide an appropriate ordering of undulator magnets before assembly. And the sorting procedure of ordering is mainly based on "Simulated Annealing" [6] although "genetic algorithms" have also been used [7] and it is widely \*This paper is supported by Foundation of National Key Program for

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applied to the PPM undulator where linear superposition can be used.

We apply a modified simulated annealing algorithm to this multi-objective optimization problem, taking first field integral, second field integral and rms field error as objective functions. And we put forward a new quick method to directly calculate the rms field error and field integrals according to remanence and positions of individual magnets. Illustrations are drawn from the reconstruction of the first undulator (modulator of optical klystron) of CHG-FEL for National Synchrotron Radiation Laboratory.

## **OBJECTIVE FUNCTIONS**

Figure 1 shows the structure of undulator. The undulator consists of three configuration magnets, which are horizontal ("H") magnets with the main remanence in the horizontal direction, vertical ("V") and terminal ("T") magnets with the main remanence in the vertical direction. "T" magnets differ from "V" magnets only in width and their width is half those of "V" magnets. *X*, *Y*, *Z* are length, height, width of the magnet. And *g* is the gap of undulator magnetic field.



Figure 1: Structure of undulator.

For purposes of discussion, we assume a rectangular coordinate system with the electron propagation axis in the *z* direction, the principal component of the magnetic field  $B_y$  in the vertical *y* direction, and the *x*-*z* plane being thus the plane of the electron sinusoidal motion.

## Measurement and Calculation of Magnets

To measure all the magnets equivalently, we define a reference position with the main component in the upward vertical y direction. Therefore, for a magnet, the

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remanence components  $B_{r1}$ ,  $B_{r2}$ ,  $B_{r3}$  are in the *x*, *y*, *z* direction respectively. For ideal magnets,  $B_{r1}=B_{r3}=0$ . For real magnets, in general,  $|B_{r1}|:|B_{r2}|:|B_{r3}|$  approximately equals 1:1000:10.

After measurement of individual magnets, we can obtain the values of  $B_{r1,n}$ ,  $B_{r2,n}$ ,  $B_{r3,n}$ , and here *n* is the serial number of magnets from 1 to  $N_{total}$  (the total number of all the magnets). We choose *N* magnets among the  $N_{total}$  according to  $B_{r2,n}$  and arrange them to assemble the undulator.

If the magnet is used in the "V" or "T" configuration,  $B_{r1}$  corresponds to  $B_{rx}$ ,  $B_{r2}$  to  $B_{ry}$  and  $B_{r3}$  to  $B_{rz}$ . If the magnet is placed in the "H" position,  $B_{r1}$  corresponds to  $B_{rx}$ ,  $B_{r2}$  to  $B_{rz}$  and  $B_{r3}$  to  $B_{ry}$ .

Take the centre of the magnet as the origin of the coordinate, and the magnetic field in *y* direction  $B_y(x,y,z)$ , generated by this system outside the magnet, is given by the following expression:

$$B_{y}(x, y, z) = b(X, Y, Z) - b(-X, Y, Z) - b(X, -Y, Z) + b(-X, -Y, Z) - b(X, Y, -Z) + b(-X, Y, -Z)$$
(1)  
+ b(X, -Y, -Z) - b(-X, -Y, -Z)

For ideal "V" and "T" magnets,

$$b(X,Y,Z) = \frac{B_r}{4\pi} \operatorname{tg}^{-1} \left[ \frac{\left(x - \frac{X}{2}\right) \left(z - \frac{Z}{2}\right) / \left(y - \frac{Y}{2}\right)}{\sqrt{\left(x - \frac{X}{2}\right)^2 + \left(y - \frac{Y}{2}\right)^2 + \left(z - \frac{Z}{2}\right)^2}} \right]$$
(2)

For ideal "H" magnets,

$$b(X,Y,Z) = \frac{-B_r}{4\pi} \ln \left[ \frac{X}{2} - x + \sqrt{\left(x - \frac{X}{2}\right)^2 + \left(y - \frac{Y}{2}\right)^2 + \left(z - \frac{Z}{2}\right)^2} \right]$$
(3)

 $B_y$  of individual magnets can be calculated using Equations (1), (2) and (3). For calculating rms field error of undulator, we should record the values in the positions whose distances from the centre of the magnet are integer times of *Z*. "V" magnets are recorded as  $B_{V0}$ ,  $B_{V1}$ ,  $B_{V2}$ ,  $B_{V3}$ .... And for "H" magnets, they are  $B_{H0}$ ,  $B_{H1}$ ,  $B_{H2}$ ,  $B_{H3}$ ....



Figure 2: Distribution of magnetic field  $B_{y_1}$  1<sup>st</sup> integral and 2<sup>nd</sup> integral of individual magnet (remanence  $B_r$ =1T).

Figure 2 shows the distribution of magnetic field  $B_y$  (a), first integral (b) and second integral (c) of individual magnets (remanence  $B_r$ =1T).

To get the first integral of individual magnet  $I_1$ , we integrate  $B_y$  along the *z*-axis from  $-\Delta z$  to  $\Delta z$ , where  $\Delta z$  is the distance between the centre of the magnet and the measured position. Integrating  $B_y$  twice, we can get the second integral  $I_2$ . For magnets with unit remanence (1T), "V" magnets:  $I_1 = K_1$ ,  $I_2 = K_1 \Delta z$ ; "T" magnets:  $I_1 = K_1/2$  and  $I_2 = K_1 \Delta z/2$ ; "H" magnets:  $I_1 = 0$  and  $I_2 = K_2$ . When  $\Delta z$  is big enough,  $K_1$  and  $K_2$  are constants, decided by X, Y, Z, x, y. In our illustration, X=100mm, Y=23mm, Z=23mm, x=0, y=Y/2+g/2,  $\Delta z=600$ mm,  $K_1=2.4143$ mm,  $K_2=170.78$ mm<sup>2</sup>,  $B_{v0}=0.0904$ T,  $B_{v1}=0.0246$ T,  $B_{v2}=-0.0034$ T,  $B_{v3}=-0.0043$ T;  $B_{H0}=0$ T,  $B_{H1}=0.0505$ T,  $B_{H2}=0.0106$ T,  $B_{H3}=-0.0012$ T.

# Field Integrals Calculation of Undulator

For nth real magnets, field integral is given as: If it is "H" magnet (n=3, 4, 4i-1, 4i...),

$$\begin{cases} I_{1,n} = (-1)^{|\mathrm{NT}\left(\frac{n-1}{4}\right)} (-1)^n K_1 B_{r_{3,n}} \\ I_{2,n} = (-1)^{|\mathrm{NT}\left(\frac{n-1}{4}\right)} \left[ (-1)^n K_1 B_{r_{3,n}} \Delta z_n + K_2 B_{r_{2,n}} \right] \end{cases}$$
(4)

If it is "V" magnet (*n*=5, 6, 4*i*+1, 4*i*+2...),

$$\begin{cases} I_{1,n} = (-1)^{|\mathrm{INT}\left(\frac{n-1}{4}\right)} K_1 B_{r_{1,n}} \\ I_{2,n} = (-1)^{|\mathrm{INT}\left(\frac{n-1}{4}\right)} \left[ K_1 B_{r_{1,n}} \Delta z_n + (-1)^{n-1} K_1 B_{r_{1,n}} \right] \end{cases}$$
(5)

If it is "T" magnet (n=1, 2, N-1 and N),

$$\begin{cases} I_{1,n} = (-1)^{\mathrm{INT}\left(\frac{n}{4}\right)} (-1)^n K_1 B_{r_{3,n}} \\ I_{2,n} = (-1)^{\mathrm{INT}\left(\frac{n}{4}\right)} \left[ (-1)^n \frac{K_1}{2} B_{r_{3,n}} \Delta z_n + \frac{K_2}{2} B_{r_{2,n}} \right] \end{cases}$$
(6)

For undulator, the first field integral  $I_1$  and second field integral  $I_2$  are given as:

$$I_1 = \sum_{n=1}^{N} I_{1,n} \qquad I_2 = \sum_{n=2}^{N} I_{2,n}$$
(7)

# Rms Field Error Calculation of Undulator

The peak values of magnetic field appear at the centre of "V" magnets. So the total number of peak values in the undulator is INT(N/4)-1. The peak value is given as:

$$\begin{split} \left|B_{i}\right| &= B_{V0} \left[B_{r2,(4i+1)} + B_{r2,(4i+2)}\right] \\ &+ B_{V1} \left[B_{r3,(4i-1)} - B_{r3,4i} - B_{r3,(4i+3)} + B_{r3,(4i+4)}\right] \\ &+ B_{V2} \left[-B_{r3,(4i-3)} - B_{r2,(4i-2)} - B_{r2,(4i+5)} - B_{r2,(4i+6)}\right] \\ &+ B_{V3} \left[-B_{r3,(4i-5)} + B_{r3,(4i-4)} + B_{r3,(4i+7)} - B_{r3,(4i+8)}\right] \quad (8) \\ &+ B_{H1} \left[B_{r2,(4i-1)} + B_{r2,4i} + B_{r2,(4i+3)} + B_{r2,(4i+4)}\right] \\ &+ B_{H2} \left[B_{r3,(4i-3)} - B_{r3,(4i-2)} - B_{r3,(4i+5)} + B_{r3,(4i+6)}\right] \\ &+ B_{H3} \left[-B_{r2,(4i-5)} - B_{r2,(4i-4)} - B_{r2,(4i+7)} - B_{r2,(4i+8)}\right] \end{split}$$

Here, i=1,2,..., INT(N/4)-1, is the serial number. For undulator, the rms field error is given as:

$$\sigma_{B} / \overline{|B|} = \frac{1}{\overline{|B|}} \sqrt{\frac{1}{\mathrm{INT}(N/4) - 4}} \sum_{i=2}^{\mathrm{INT}(N/4) - 2} \left( \left|B_{i}\right| - \overline{|B|} \right)^{2}$$
(9)

# MULTI-OPTIMIZATION MODIFIED SA

Simulated annealing (SA) is a technique used to find good approximate solutions for combination optimization problem. We apply a multi-optimization modified SA algorithm (with heating procedure and the best move strategy based on standard SA) to optimize the problem of PPM undulator magnets ordering [8]-[9].

The multi-objective functions are (s is a solution):

 $f_1(s)$ : the rms field error of undulator;

 $f_2(s)$ : the absolute value of first field integral;

 $f_3(s)$ : the absolute value of second field integral.

## Preliminaries

• A weighting function *S*(*s*) is chosen:

$$S(s) = w_1 \frac{f_1(s)}{f_1(s_0)} + w_2 \frac{f_2(s)}{f_2(s_0)} + w_3 \frac{f_3(s)}{f_3(s_0)}$$
(10)

Here,  $w_1$ ,  $w_2$  and  $w_3$  are weighting factors, and  $w_1+w_2+w_3=1$ .

- Parameters of the heating procedure are initialized as: h (the heating factor) and  $H_{step}$  (the heating length).
- Parameters of the cooling procedure are initialized as:  $T_0$  (initial temperature, calculated in modified SA, but given in standard SA),  $\alpha$  (the cooling factor <1) and  $N_{step}$  (the cooling length of temperature step).
- Two stopping criteria are fixed: *T<sub>stop</sub>* (the final temperature) and *N<sub>stop</sub>* (the maximum number of iterations without improvement).
- A neighborhood *N*(*s*) of feasible solution in the vicinity of s is defined.

## Procedures

- Initialization. Initialize  $w_1$ ,  $w_2$ ,  $w_3$ , h,  $H_{step}$ , a,  $N_{step}$ ,  $T_{stop}$  and  $N_{stop}$ , set  $m=n=H_{count}=N_{count}=0$ ; Draw at random an initial solution  $s_0$ , and evaluate  $f_1(s_0)$ ,  $f_2(s_0)$ ,  $f_3(s_0)$  and  $S(s_0)$ ; Set a list of potentially efficient solutions PE.
- Heating procedure. Draw at random a solution s'  $N(s_n)$ , and evaluate  $f_1(s')$ ,  $f_2(s')$  and  $f_3(s')$  and S(s'); If  $S(s')>S(s_n)$ , we accept the new solution:  $s' \rightarrow s_{n+1}$ ,  $H_{count} = H_{count}+1$ , else,  $s_n \rightarrow s_{n+1}$ . Update the list PE with the solution s', and set n=n+1. If  $n=H_{step}$ , then break and set  $T_0=h \times H_{count}$ , else, iterate.
- Cooling procedure. Randomly draw K solutions from the N(s<sub>n</sub>), and evaluate f<sub>1</sub>, f<sub>2</sub>, f<sub>3</sub> and S of them. Let the best solution among the generated K solutions be s'; Replace s<sub>n</sub> by s' with probability:

$$p = \min\left[1, \exp\left(\frac{S(s_n) - S(s')}{T_m}\right)\right]$$
(11)

If s' is accepted  $s' \rightarrow s_{n+k}$ ,  $N_{count}=0$ , else,  $s_n \rightarrow s_{n+k}$ ,  $N_{count}=N_{count}+K$ . Update the list PE with the solution s'; Set n=n+K. m=m+K. If  $m/N_{step}$  is an integer, then

 $T_m = \alpha T_{m-k}$ , else  $T_m = T_{m-k}$ ; If  $T_m < T_{stop}$  or  $N_{count} = N_{stop}$ , then stop, else, iterate.

# **OPTIMIZATION RESULTS**

We use the following parameters:  $w_1=0.6$ ,  $w_2=0.2$ ,  $w_3=0.2$ , h=1.0,  $H_{step}=100$ ,  $\alpha=0.9$ ,  $N_{step}=200$ ,  $T_{stop}=0.0001$ ,  $N_{stop}=500$ , K=10.

We initialize 10 orderings of magnets, and optimize each ordering by five methods, including modified SA, modified SA only with heating procedure (K=1), standard SA, Local Search (T→0) and Exhaustion approach. It takes approximately 100 seconds for each process (CUP 2.0G).

Table 1 shows the optimization results of modified SA and some other methods after 10 calculations of each. We can see that the ordering of magnets provided by SA can reduce the undulator field errors much more greatly compared with the results of other methods listed in the table. And modified SA is better than other SA Algorithms with smaller mean value and rms divergence.

Table 1: Optimization results of Modified SA and other methods

Algorithm	Rms field error (10 <sup>-4</sup> )	1 <sup>st</sup> integral (10 <sup>-6</sup> T⋅m)	$2^{nd}$ integral ( $10^{-6}$ T·m <sup>2</sup> )
Initial solution	58.6±9.7	295±239	312±251
Modified SA	1.71±1.12	1.18±1.91	0.71±0.87
Modified SA (only with heating procedure)	1.82±1.14	1.55±1.40	3.22±4.67
Standard SA	2.13±1.28	1.13±1.59	0.42±0.47
Local Search	3.15±1.87	3.89±7.09	0.64±1.54
Exhaustion approach	31.9±3.8	26.1±23.5	17.4±17.9

In order to demonstrate the precision of the quick method in calculation of undulator field errors, we design a very precise three-dimensional program in which we first calculate undulator field according to remanence and positions of individual magnets, and then analyze the undulator. From Table 2, we can see the two results are very close (can be ignored considering the construction errors), but actually the time used by our quick method is over 1000 times less than the latter.

 

 Table 2: Calculation results of objective functions by quick method and three-dimensional program (in parentheses)

	Before optimization	After optimization
Rms field error $\sigma_{B}/B$	0.653%(0.634%)	0.103‰(0.099‰)
1 <sup>st</sup> integral (10 <sup>-6</sup> T·m)	-463.4 (-464.9)	-1.33 (-2.01)
$2^{nd}$ integral $(10^{-6}T \cdot m^2)$	-566.5 (-568.3)	-0.79 (0.23)
Rms phase error $\sigma_\phi$	/ (13.11)	/ (0.78)

Figure 3 shows the trajectory of electron beam before and after optimization. Angular deflection  $\Delta x$  and orbital displacement  $\Delta x$  of electron beam are given as:

$$\Delta x' = \frac{300I_1}{E[MeV]} \qquad \Delta x = \frac{300I_2}{E[MeV]} \tag{12}$$



Figure 3: Electron trajectory before and after optimization

Figure 4 shows the spectrum of spontaneous emission before and after optimization.



Figure 4: Spontaneous emission spectrum before and optimization

## CONCLUSION

The optimization results exposed in this paper show that the undulator field errors can be greatly reduced (rms field error reduced to 0.01%, first field integral to  $10^{-6}$ T·m, second integral to  $10^{-6}$ T·m<sup>2</sup> and rms phase error to 1 degree). The field after assembly will inevitably differ from predictions from the optimization results due to the construction errors, and because of this reason, we don't consider the multipole field errors and set the weighting factors 0.6 for rms field error, 0.2 for first integral, 0.2 for second integral [5].

Once the magnets have been constructed and measured, the remaining errors can be overcome by swapping magnets, or more commonly by "shimming" [10]. Optimization of the magnets ordering can also save the time for field measurement and correction after assembling.

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