SIMPLIFIED METHOD FOR EXPERIMENTAL SPECTRAL RATIO CALCULATION OF CHG-FEL*

Nian Chen#, Pengfei Zhang, Duohui He, Ge Li, Shancai Zhang, Yuhui Li
National Synchrotron Radiation Laboratory, University of Science and Technology of China, Hefei, Anhui 230029, China

Abstract

Coherent harmonic generation free-electron laser (CHG-FEL) acts as a harmonic amplifier of the seed laser. The spectral ratio, which is defined as the ratio of coherent radiation intensity and incoherent radiation intensity in infinitesimal bandwidth and solid angle aperture, can evaluate the performance of CHG-FEL. In the experiment, we can only get the experimental integral ratio integrated over the actual bandwidth and solid angle aperture of the radiation measurement system. Because the coherent radiation and incoherent radiation are very different in the spatial and spectral structure, the experimental integral ratio is greatly influenced by the measurement system and can not be directly used to evaluate the performance of CHG-FEL. So we must calculate the experimental spectral ratio according to the experimental integral ratio, the bandwidth and solid angle aperture of the measurement system and the parameters of CHG-FEL. And our work is to give a simplified method for the calculation of experimental spectral ratio.

INTRODUCTION

Coherent harmonic generation free-electron laser (CHG-FEL) is one way of storage ring FEL [1-3]. The main component of CHG-FEL is optical klystron (OK), which consists of three sections - two undulators separated by a dispersive section. In the first undulator (modulator), an energy modulation is imposed on the electron beam by interaction with a seed laser. The energy modulation is converted to a coherent spatial density modulation as the electron beam traverses the dispersion section which is a single-period wiggler with strong magnetic field. The second undulator (radiator), tuned to a higher harmonic n_0 (n_0=1, 2, 3…) of the seed laser wavelength λ_s, causes the micro-bunched electron beam to emit coherent radiation whose fundamental wavelength is λ_s/n_0. CHG-FEL acts as a harmonic amplifier of the seed laser. It becomes high-gain harmonic generation free-electron laser (HGHG-FEL) if the radiator is long enough for amplification until saturation is achieved [4].

In CHG-FEL experiment, we can obtain, on axis, the coherent (with seed laser) and incoherent (without seed laser) radiation at the wavelength λ_s/n_0n_r (n_r=1, 3, 5…) from radiator and the incoherent radiation at the wavelength λ_s/n_m (n_m=1, 3, 5…) from modulator. The aim of CHG-FEL is to gain the coherent radiation at the wavelength λ_s/n_0n_r. For example, in the experiment of CHG-FEL on the ACO storage ring at Orsay of France, they have obtained the 3rd and 5th harmonic coherent radiation of the seed laser wavelength 1064nm using a symmetrical OK (n_0=1, n_r=3, 5) [3]. And for our experiment, the OK is unsymmetrical, which benefits the HGHG study. It has two working modes: first, to obtain the 2nd harmonic coherent radiation of the seed laser wavelength 532nm (n_0=2, n_r=1); second, to obtain the 3rd harmonic coherent radiation of the seed laser wavelength 1064nm (n_0=1, n_r=3) [5].

The performances of different CHG-FELs can be effectively evaluated by the experimental spectral ratio R'_{spe} which can be approximately calculated by experimental integral ratio R'_{int} and the ratio of theoretical spectral ratio R_{spe} and theoretical integral ratio R_{int}:

$$R'_{spe} = R'_{int} \times R_{spe} / R_{int} \ . \qquad (1)$$

R'_{int} can be directly measured in experiment, and R_{spe}/R_{int} can be calculated according to the bandwidth and solid angle aperture of the measurement system and the parameters of CHG-FEL.

We give a simple method to calculate the R_{spe}/R_{int} in this paper. In the following discussion, the variables with subscript u, m, d and r represent parameters of any undulator, modulator, dispersive section and radiator respectively.

UNDULATOR RADIATION

When an electron beam traverses the periodic magnetic field provided by an undulator, it emits radiation at the well-known resonant wavelength λ and its nth odd harmonics λ/n. λ is defined as

$$\lambda = \frac{\lambda_u}{2n\gamma^2}\left(1+K_u^2/2\right). \qquad (2)$$

Here, λ_u is the undulator period, γmc^2 is the electron beam energy, m is the electron mass, c is the light speed, $K_u=eB_0\lambda_u/(2\pi mc)=93.37B_0\lambda_u$ is the undulator parameter, e is the charge on the electron, and B_0 is the maximum on-axis magnetic field strength of the undulator.

The radiation energy by a single electron per unit solid angle (dΩ) and per uint wavelength (dλ) is given by [6]:

$$\left.\frac{\mathrm{d}^2 I}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{1e^{-1}} = \frac{e^2}{4\pi\varepsilon_0 c\lambda^2}\left|\int_{-\infty}^{+\infty} \boldsymbol{n}\times\left(\boldsymbol{n}\times\boldsymbol{v}(\tau)\right)\mathrm{e}^{\mathrm{i}\frac{2\pi}{\lambda}\left(c\tau-\boldsymbol{n}\cdot\boldsymbol{x}(\tau)\right)}\mathrm{d}\tau\right|^2, \qquad (3)$$

where ε_0 is permittivity of vacuum, $\boldsymbol{n}$=(sinθcosφ, sinθsinφ, cosθ) is the unit vector from the point of emission to the observer, $\boldsymbol{v}=(v_x, v_y, v_z)$ is the electron velocity, and $\boldsymbol{x}=(x, y, z)$ is the electron trajectory. The magnetic field in an ideal planar undulator isgiven by $B_y(z)=B_0\cos(2\pi z/\lambda_u)$, for $0\le z\le N_u\lambda_u$, with $B_x(z)=B_z(z)=0$, N_u is the period number. The electron velocity and trajectory can be calculated according to B(z), and then with equation (3), the

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#chnian@mail.ustc.edu.cn

radiation energy can be calculated at any solid angle and wavelength. Here, we only give the single-electron radiation intensity on axis at the resonant wavelength:

$$\left.\frac{\mathrm{d}^2 I}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{1\mathrm{e}^{-1}}^{\theta=0,\ \lambda=\lambda} = \frac{e^2 n^2 N_\mathrm{u}^2 \gamma^2 K_\mathrm{u}^2}{4\pi\varepsilon_0 c\left(1+K_\mathrm{u}^2/2\right)^2}\mathrm{F}_n^2\left(K_\mathrm{u}\right), \quad (4)$$

with

$$\mathrm{F}_n\left(K_\mathrm{u}\right) = \mathrm{J}_{\frac{k-1}{2}}\left(\frac{kK_\mathrm{u}^2}{4\left(1+K_\mathrm{u}^2/2\right)}\right) - \mathrm{J}_{\frac{k+1}{2}}\left(\frac{kK_\mathrm{u}^2}{4\left(1+K_\mathrm{u}^2/2\right)}\right), \quad (5)$$

here $\mathrm{J}_n(x)$ is the Bessel function of nth order.

For a bunch with N_e electrons whose distribution is $\rho(x, y, z)$, according to equation (3), the radiation intensity of the electrons is given as follows:

$$\left.\frac{\mathrm{d}^2 I}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{N_\mathrm{e}\mathrm{e}^{-1}} = \left.\frac{\mathrm{d}^2 I}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{1\mathrm{e}^{-1}} \times \left|\iiint \rho\left(x,y,z\right)\mathrm{e}^{-\mathrm{i}\frac{2\pi}{\lambda}\left(\theta\cos\varphi\cdot x+\theta\sin\varphi\cdot y+z\right)}\mathrm{d}x\mathrm{d}y\mathrm{d}z\right|^2. \quad (6)$$

INCOHERENT RADIATION OF CHG-FEL

The incoherent radiation at measured wavelength $\lambda_\mathrm{s}/n_0 n_\mathrm{r}$, not only comes from radiator but also from modulator when n_0 is an odd number. From the principle of CHG-FEL and resonant relation (1), we obtain $\lambda_\mathrm{r}(1+K_\mathrm{r}^2/2)=\lambda_\mathrm{m}(1+K_\mathrm{m}^2/2)/n_0$. And combining this equation with equation (4), the relation between the single-electron radiation intensity on axis at the wavelength $\lambda_\mathrm{s}/n_0 n_\mathrm{r}$ from modulator $\mathrm{d}^2 I_\mathrm{m}/\mathrm{d}\lambda\mathrm{d}\Omega$ and that from radiator $\mathrm{d}^2 I_\mathrm{r}/\mathrm{d}\lambda\mathrm{d}\Omega$ can be calculated as follows:

$$\left.\frac{\mathrm{d}^2 I_\mathrm{m}}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{1\mathrm{e}^{-1}}^{\theta=0,\lambda=\frac{\lambda_\mathrm{s}}{n_0 n_\mathrm{r}}} = P\times\left.\frac{\mathrm{d}^2 I_\mathrm{r}}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{1\mathrm{e}^{-1}}^{\theta=0,\lambda=\frac{\lambda_\mathrm{s}}{n_0 n_\mathrm{r}}}, \quad (7)$$

with

$$P = \begin{cases} \dfrac{\lambda_\mathrm{m}^2 N_\mathrm{m}^2 K_\mathrm{m}^2 \mathrm{F}_{n_0 n_\mathrm{r}}^2\left(K_\mathrm{m}\right)}{\lambda_\mathrm{r}^2 N_\mathrm{r}^2 K_\mathrm{r}^2 \mathrm{F}_{n_\mathrm{r}}^2\left(K_\mathrm{r}\right)} & n_0 = 1,3,5,7\ldots \\ 0 & n_0 = 2,4,6,8\ldots \end{cases}. \quad (8)$$

Without seed laser, the energy and spatial density of electrons are not modulated. Thus, electrons can be regarded as uniformly distributed over the radiation wavelength and the radiation fields of two individual electrons are incoherent. The incoherent radiation intensity is proportional to the number of electrons in the bunch, we get

$$\left.\frac{\mathrm{d}^2 I_\mathrm{inc}}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{N_\mathrm{e}\mathrm{e}^{-1}}^{\theta=0,\lambda=\frac{\lambda_\mathrm{s}}{n_0 n_\mathrm{r}}} = N_e\left(1+P\right)^2\left.\frac{\mathrm{d}^2 I_\mathrm{r}}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{1\mathrm{e}^{-1}}^{\theta=0,\lambda=\frac{\lambda_\mathrm{s}}{n_0 n_\mathrm{r}}}. \quad (9)$$

The bandwidth and solid angle aperture of incoherent radiation are far larger than those of the measurement system. So, within bandwidth and solid angle aperture of the measurement system, the incoherent radiation intensity can be regarded as invariable which is equal to the value on axis at the wavelength $\lambda_\mathrm{s}/n_0 n_\mathrm{r}$.

COHERENT RADIATION OF CHG-FEL

With seed laser, the distribution of electrons is no longer uniform after the energy and spatial density modulation. The initial distribution of electrons can be approximately expressed as Gaussian distribution:

$$\rho_0\left(x,y,z\right) = \frac{N_\mathrm{e}}{\left(2\pi\right)^{3/2}\sigma_x\sigma_y\sigma_z}\mathrm{e}^{-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}-\frac{z^2}{2\sigma_z^2}}, \quad (10)$$

where σ_x and σ_y are transverse sizes of bunch, σ_z is the longitudinal size of bunch. The distribution of electrons at the exit of the dispersive section can be expressed as

$$\rho\left(x,y,z\right) = \rho_0\left\{1+\sum_{n=1}^{\infty}2\mathrm{J}_n\left(n\eta\right)\mathrm{f}_n\cos\left(\frac{2n\pi}{\lambda_\mathrm{s}}z\right)\right\}, \quad (11)$$

with $\eta=4\pi(N_\mathrm{m}+N_\mathrm{d})\Delta\gamma_\mathrm{m}/\gamma$ and $\mathrm{f}_\mathrm{n}=\exp\{-8[n\pi(N_\mathrm{m}+N_\mathrm{d})\sigma_\gamma/\gamma]^2\}$, where N_d is the parameter of dispersive section, $2\pi N_\mathrm{d}$ represents the dephasing created by the dispersive section between an electron and the seed laser wave, $\Delta\gamma_\mathrm{m}/\gamma=eN_\mathrm{m}\lambda_\mathrm{m}K_\mathrm{m}E_s\mathrm{F}_1(K_\mathrm{m})/(2\gamma^2 mc^2)$ is the maximum relative shift of the electron induced by the seed laser, E_s is the electromagnetic wave amplitude of seed laser, σ_γ/γ is the energy spread of the electron beam [1]. As a consequence of Equations (6) and (11), we get the following expression for the coherent radiation intensity of CHG-FEL:

$$\left.\frac{\mathrm{d}^2 I_\mathrm{coh}}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{N_\mathrm{e}\mathrm{e}^{-1}} = N_\mathrm{e}^2\,\mathrm{J}_{n_0 n_\mathrm{r}}^2\left(n_0 n_\mathrm{r}\eta\right)\mathrm{f}_{\mathrm{n_0 n_r}}^2 \times \mathrm{e}^{-\frac{4\pi^2}{\lambda^2}\left(\theta^2\cos^2\varphi\sigma_x^2+\theta^2\sin^2\varphi\sigma_y^2\right)} \times \mathrm{e}^{-4\pi^2\sigma_z^2\left(\frac{1}{\lambda}-\frac{n_0 n_\mathrm{r}}{\lambda_s}\right)^2}\left.\frac{\mathrm{d}^2 I_\mathrm{r}}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{1\mathrm{e}^{-1}}. \quad (12)$$

According to the above equation, we give the bandwidth $\Delta\lambda_\mathrm{coh}$ and solid angle aperture $\Delta\Omega_\mathrm{coh}$ of coherent radiation:

$$\Delta\lambda_\mathrm{coh} = \frac{\sqrt{\ln 2}}{\pi\sigma_z}\lambda^2,\quad \Delta\Omega_\mathrm{coh} = \frac{\lambda^2}{2\pi\sigma_x\sigma_y}. \quad (13)$$

Within bandwidth and solid angle aperture of the measurement system, the coherent radiation intensity can not be regarded as invariable because $\Delta\lambda_\mathrm{coh}$ and $\Delta\Omega_\mathrm{coh}$ are very small.

EXPERIMENTAL SPECTRAL RATIO CALCULATION

According to equations (9) and (12), we can get the theoretical spectral ratio as follows:

$$R_\mathrm{spe} = \left.\frac{\mathrm{d}^2 I_\mathrm{coh}}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{N_\mathrm{e}\mathrm{e}^{-1}}^{\theta=0,\lambda=\frac{\lambda_\mathrm{s}}{n_0 n_\mathrm{r}}} : \left.\frac{\mathrm{d}^2 I_\mathrm{inc}}{\mathrm{d}\lambda \mathrm{d}\Omega}\right|_{N_\mathrm{e}\mathrm{e}^{-1}}^{\theta=0,\lambda=\frac{\lambda_\mathrm{s}}{n_0 n_\mathrm{r}}} = \frac{1}{\left(1+P\right)^2}N_\mathrm{e}\,\mathrm{J}_{n_0 n_\mathrm{r}}^2\left(n_0 n_\mathrm{r}\eta\right)\mathrm{f}_{n_0 n_\mathrm{r}}^2. \quad (14)$$

From equations (9) and (12), we can also get the theoretical integral ratio in bandwidth $\Delta\lambda$ and the solid angle $\Delta\Omega$ of the measurement system. Here, we skipped the rather tedious details of the integral and reported the final result only as equations (15)-(17):

$$R_{\text{int}} = \frac{\int_{\Delta\lambda}\int_{\Delta\Omega}\left(\left.\frac{\mathrm{d}^2 I_{\text{coh}}}{\mathrm{d}\lambda\mathrm{d}\Omega}\right|_{N_e e^{-1}}\right)\mathrm{d}\Omega\mathrm{d}\lambda}{\int_{\Delta\lambda}\int_{\Delta\Omega}\left(\left.\frac{\mathrm{d}^2 I_{\text{inc}}}{\mathrm{d}\lambda\mathrm{d}\Omega}\right|_{N_e e^{-1}}\right)\mathrm{d}\Omega\mathrm{d}\lambda} = R_{\text{spe}} I_{\Delta\Omega} I_{\Delta\lambda}\,, \quad (15)$$

where

$$I_{\Delta\Omega} = \frac{\left(\lambda_{\text{s}}/n_0 n_{\text{r}}\right)^2}{4\pi\sigma_x\sigma_y\Delta\Omega} \times\left[1-\frac{\sigma_x\sigma_y}{2\pi}\int_0^{2\pi}\frac{e^{-\frac{4\pi\Delta\Omega}{\left(\lambda_{\text{s}}/n_0 n_{\text{r}}\right)^2}\left(\cos^2\varphi\sigma_x^2+\sin^2\varphi\sigma_y^2\right)}}{\cos^2\varphi\sigma_x^2+\sin^2\varphi\sigma_y^2}\mathrm{d}\varphi\right] \quad (16)$$

and

$$I_{\Delta\lambda} = \frac{\left(\lambda_{\text{s}}/n_0 n_{\text{r}}\right)^2}{2\sqrt{\pi}\sigma_z\Delta\lambda}\int_{-\frac{\sqrt{2}\pi\sigma_z\Delta\lambda}{\left(\lambda_{\text{s}}/n_0 n_{\text{r}}\right)^2}}^{\frac{\sqrt{2}\pi\sigma_z\Delta\lambda}{\left(\lambda_{\text{s}}/n_0 n_{\text{r}}\right)^2}}\frac{e^{-\frac{\tau^2}{2}}}{\sqrt{2\pi}}\mathrm{d}\tau\,. \quad (17)$$

Here, we list four particular cases and give their approximate expressions.

If $\Delta\lambda>\Delta\lambda_{\text{coh}}$ and $\Delta\Omega>\Delta\Omega_{\text{coh}}$, then:

$$R_{\text{int}} = R_{\text{spe}}\frac{\left(\lambda_{\text{s}}/n_0 n_{\text{r}}\right)^4}{8\pi^{3/2}\sigma_x\sigma_y\sigma_z\Delta\lambda\Delta\Omega}\,; \quad (18)$$

If $\Delta\lambda>\Delta\lambda_{\text{coh}}$ and $\Delta\Omega<<\Delta\Omega_{\text{coh}}$, then:

$$R_{\text{int}} = R_{\text{spe}}\frac{\left(\lambda_{\text{s}}/n_0 n_{\text{r}}\right)^2}{2\sqrt{\pi}\sigma_z\Delta\lambda}\,; \quad (19)$$

If $\Delta\lambda<<\Delta\lambda_{\text{coh}}$ and $\Delta\Omega>\Delta\Omega_{\text{coh}}$, then:

$$R_{\text{int}} = R_{\text{spe}}\frac{\left(\lambda_{\text{s}}/n_0 n_{\text{r}}\right)^2}{4\pi\sigma_x\sigma_y\Delta\Omega}\,; \quad (20)$$

If $\Delta\lambda<<\Delta\lambda_{\text{coh}}$ and $\Delta\Omega<<\Delta\Omega_{\text{coh}}$, then: $R_{\text{int}}=R_{\text{spe}}$.

For our experiment, the reference [7] gives a set of its parameters: σ_x=0.518mm, σ_y=0.051mm, σ_z=11.3mm, n_0=2, n_r=1, λ_s=532nm and λ=266nm. According to equation (13), $\Delta\lambda_{\text{coh}}$=0.0017nm, $\Delta\Omega_{\text{coh}}=0.43\times10^{-6}\text{rad}^2$. For the actual measurement system, $\Delta\lambda$=0.15nm, $\Delta\Omega=0.12\times10^{-6}\text{rad}^2$, we can get $R_{\text{spe}}/R_{\text{int}}$=192 using equation (15)-(17). If $\Delta\lambda$=0.15nm, $\Delta\Omega=1.2\times10^{-6}\text{rad}^2$, $R_{\text{spe}}/R_{\text{int}}$=478 using equation (18) and $R_{\text{spe}}/R_{\text{int}}$=678 using equation (15)-(17). If we have measured the experimental integral ratio R'_{int}, then, using equation (1), we can calculate the experimental spectral ratio R'_{spe}.

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