FEL SIMULATION CODE FOR UNDULATOR PERFORMANCE ESTIMATION

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Abstract

An FEL simulation code called SIMPLEX is introduced, which has been developed to investigate the effects of the undulator field error on the FEL gain. It can perform FEL simulations with the magnetic field distribution actually measured along the undulator axis so as to check the performance of the undulator as an FEL driver. Basic equations are derived that enable the numerical implementation of the FEL equations with the error fields taken into account. Practical examples for investigation of the undulator field error are also presented.

INTRODUCTION

The undulator is one of the most important components of an FEL. In particular, a quite large number of periods is required to achieve saturation in SASE-based x-ray FELs. Because the permanent magnets in the undulator are not perfect, the undulator field has necessarily error components. Needless to say, the tolerance on the undulator field error is more severe for larger number of periods. It is thus important to check the performances after construction of an undulator by measuring the magnetic field and calculating optical properties with it.

As a spontaneous-emission synchrotron radiation (SR) source, the performances of an undulator are in general specified by a quantity called the phase error. It is also possible to compute the intensity of SR by Fourier transforming the electric field generated by an electron moving in the undulator field. On the other hand, it is not easy to check the performances of the undulator as an FEL driver. The most promising way is to perform simulations of the FEL processes driven by the undulator with error fields. We have recently developed an FEL simulation code called SIMPLEX that can perform FEL simulations with the magnetic field distribution actually measured along the undulator axis. The details of which are described in the following sections.

FEL EQUATIONS

The amplification process in the FEL is represented by three equations, i.e., the wave, energy and phase equations.

The wave equation of the radiation field under the paraxial approximation is written as

$$\mathrm{e}^{i\omega(z/c-t)}\left[\nabla^2+2i\frac{\omega}{c}\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\right]\tilde{E}+\text{c.c.}=\mu_0\frac{\partial j_x}{\partial t},$$

where $\tilde{E}$ is the complex amplitude of the radiation field, j_x is the horizontal component of the beam current density, μ_0 the permeability of vacuum. Averaging over the period of radiation, $T=2\pi/\omega$, and integrating by part, we have

$$\left[\nabla^2+2i\frac{\omega}{c}\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\right]\tilde{E} = \frac{i\mu_0\omega e}{T}\sum_{j=1}^{N}\beta_{jx}\mathrm{e}^{ik_u z}\mathrm{e}^{-i\psi_j}\delta(\boldsymbol{r}-\boldsymbol{r}_j), \qquad (1)$$

with

$$\psi_j(z)=k_u z+\frac{\omega}{c}\left(z-\int^z\frac{dz'}{\beta_{\perp j}(z')}\right),$$

where $\lambda_u=2\pi/k_u$ is the periodic length of the undulator, N is the number of electrons existing in the averaging area, and $\boldsymbol{\beta}_j$, $\boldsymbol{r}_j$, and ψ_j are the velocity, position and phase of the j-th electron, respectively. The phase equation can be modified as

$$\frac{d\psi_j}{dz}=k_u\left(\frac{2\Delta\gamma_j}{\gamma_0}-\frac{\Delta\omega}{\omega_0}-\frac{\gamma_j^2\beta_{\perp j}^2-K^2/2}{1+K^2/2}\right), \qquad (2)$$

with

$$\Delta\gamma_j=\gamma_j-\gamma_0,\quad \Delta\omega=\omega-\omega_0,$$

where γ_j is the energy of the j-th electron, γ_0 is the average energy of the electron beam at the undulator entrance, ω_0 is the fundamental energy of the undulator radiation, and K is the deflection parameter of the undulator.

The energy of each electron changes by interaction with the radiation field, which is described by the equation

$$\frac{d\gamma_j}{dz}=\frac{e}{mc^2}(\beta_{jx}\mathrm{e}^{-ik_u z}\mathrm{e}^{i\psi_j}+\text{c.c.}). \qquad (3)$$

The differential equations (1) $\sim$ (3) can be solved numerically with a certain longitudinal step Δz. In most FEL simulations, a multiple length of the undulator period λ_u is chosen for Δz. This is because the electric field $\tilde{E}$ and electron energy γ_j do not change drastically over such a length, and the equations can be averaged over Δz with the periodic condition of the magnetic field for ideal undulators. For real undulators, however, the periodic condition cannot be applied to modify the FEL equations because of the field error intrinsic to the real undulators.

NUMERICAL IMPLEMENTATION

The most straightforward way to solve precisely the FEL equations with the field-error effects taken into account is to adopt a fine integration step, i.e., $\Delta z \ll \lambda_u$. Needless to say, it takes a lot of computation time. Because $\tilde{E}$ and γ_j are slowly varying functions compared to λ_u even in the case with the real undulator, averaging is again a promising way for practical simulation. It should be noted, however, that the averaging should be performed at every integration step, i.e., $z = \Delta z, 2\Delta z, \ldots, n\Delta z, \ldots$. In the following sections, the FEL equations are modified to the forms that are convenient for averaging.

Magnetic Field Model

In order to specify the electron trajectory in arbitrary magnetic fields $\boldsymbol{B}$, the equation of motion should be solved with the Lorentz force taken into account, which needs 3-dimensional field mapping. In the undulator line, however, the electron trajectory can be decomposed into two components, which considerably simplifies the problem.

One is the the sinusoidal (or more generally, quasi-periodic) orbit that cause the interaction between the electrons and radiation field. For real undulators, it also contains random walks both in the horizontal (x) and vertical (y) directions due to error fields. In general, it hardly depends on the transverse coordinate of the injected electron at the entrance of the undulator and is thus determined by the magnetic field distribution $\boldsymbol{B}_u$ measured along the undulator axis.

The other is the betatron oscillation induced by the focusing force on the electron beam generated by quadrupole magnets and (natural) focusing in the undulator. It is reasonable to assume that the focusing field $\boldsymbol{B}_f$ is uniform over the distance occupied by each magnetic device as shown in Fig. 1.

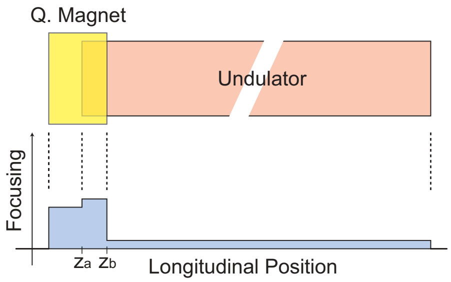

Figure 1: Example of a magnetic device configuration and focusing force distribution in the undulator line.

The above discussion arrows us to introduce an effective magnetic field $\boldsymbol{B}_{\text{eff}}$ that has no longitudinal components and the form

$$\boldsymbol{B}_{\text{eff}}(\boldsymbol{r}) = \boldsymbol{B_u}(z) + \boldsymbol{B_f}(x, y, z),$$

which can be used to calculate the electron trajectory instead of the real field $\boldsymbol{B}$. The field $\boldsymbol{B_f}(x, y, z)$ can be regarded to be uniform with respect to z within a limited range, e.g., $z_a < z < z_b$ as shown in Fig. 1.

Electron Motion

Now let us consider an electron travelling from z_0 to z with an initial condition $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ and $\boldsymbol{r} = (x_0, y_0, z_0)$ at the longitudinal position of z_0. Assuming that the travelling distance $\Delta z = z - z_0$ is much shorter than the period of the betatron oscillation, the transverse position of the electron, x and y, does not change significantly. Thus, the focusing field $\boldsymbol{B_f}$ can be regarded to be constant. Then, the velocity and position of the electron are obtained just by integrating the field distribution along z axis

$$\begin{aligned} \beta_x(z) &= \beta_{0x} + \gamma^{-1}[I_{uy}(z_0, z) + I_{fy}(z_0, z)], \\ x(z) &= \beta_{0x}\Delta z + \gamma^{-1}[J_{uy}(z_0, z) + I_{fy}(z_0, z)\Delta z/2], \end{aligned} \tag{4}$$

with

$$\begin{aligned} I_{uy}(z_0, z) &= \frac{e}{mc}\int_{z_0}^{z} B_{uy}(z')dz', \\ I_{fy}(z_0, z) &= \frac{e}{mc}(z - z_0)B_{fy}, \\ J_{uy}(z_0, z) &= \int_{z_0}^{z} I_{uy}(z_0, z')dz', \end{aligned}$$

and similar expressions for the vertical components β_y and y.

Interaction Factor

In the differential equations (1) and (3), there exists an identical factor $\beta_x \mathrm{e}^{ik_u z}\mathrm{e}^{-i\psi}$ (and its complex conjugate), which is regarded to be an efficiency of interaction between the electrons and radiation field. It is easy to show that this factor is almost periodic with a period of $\lambda_u/2$. It is therefore necessary to average it over the distance of the integration step Δz, which, in most cases, longer than $\lambda_u/2$. In averaging, we can omit the trajectory components related to the betatron oscillation because the period of the betatron oscillation is much longer than λ_u, In other words, we replace as follows

$$\beta_x \to I_{uy}/\gamma, \ \ (\gamma\beta_\perp)^2 \to I_{uy}^2 + I_{ux}^2.$$

In addition, we can neglect the terms $\Delta\gamma$ and $\Delta\omega$ in the phase equation because they are nearly constant over Δz. Then, we have

$$\langle \beta_x \mathrm{e}^{ik_u z}\mathrm{e}^{-i\psi}\rangle = i\frac{J_a(z_0)}{\gamma}\mathrm{e}^{-i\psi_j(z_0)},$$

with

$$J_a(z_0) = \frac{1}{i\Delta z}\int_{z_0}^{z} I_{uy}(z')\exp\left[\frac{ik_u\rho(z_0, z')}{1 + K^2/2}\right], \tag{5}$$

and

$$\rho(z_0, z) = \int_{z_0}^{z}[I_{uy}^2(z_0, z') + I_{ux}^2(z_0, z') - K^2/2]dz'$$

For ideal undulators without any field errors, $I_{ux} = 0$ and $I_{uy}(z_0, z) = K \sin k_u z$, thus we have

$$J_a(z) = \frac{K}{2}\left[J_0\left(\frac{K^2/4}{1+K^2/2}\right) - J_1\left(\frac{K^2/4}{1+K^2/2}\right)\right].$$

Substituting this formula into (1) gives an ordinary wave equation to describe the amplification process in the FEL driven by an ideal undulator.

Electron Phase

Because the electron energy does not change significantly over Δz, the electron phase can be calculated by integrating equation (2) directly. Substituting (4) into (2) and integrating between z_0 and z, we have

$$\begin{aligned}\psi(z) &= \psi(z_0) + k_u \Delta z\left(\frac{2\Delta\gamma}{\gamma_0} - \frac{\Delta\omega}{\omega_0}\right) \\ &+ \frac{k_u(R_x + R_y - K^2\Delta z/2)}{1+K^2/2}, \end{aligned} \tag{6}$$

with

$$\begin{aligned}R_x = &\int_{z_0}^{z} I_{uy}^2(z')dz' - \frac{2I_{fy}(z_0,z)}{\Delta z}\int_{z_0}^{z} J_{uy}(z_0,z')dz' \\ &+\Delta z[(\gamma\beta_{0x})^2 + \gamma\beta_{0x}I_{fy}(z_0,z) + I_{fy}^2(z_0,z)/3] \\ &+2J_{uy}(z_0,z)[\gamma\beta_{0x} + I_{fy}(z_0,z)], \end{aligned} \tag{7}$$

and similar expression for R_y. Note that the subscript j has been omitted in the above equation for simplicity.

Processing of the Undulator Field Data

The above modifications on the FEL equations arrow us to process the undulator field data convenient for numerical implementation, i.e., the entire undulator line is divided into adequate integration steps and the quantities such as I_{uy}, J_{uy} and J_a are calculated in each step before starting the simulation. In most x-ray FELs, the gain length is much longer than the undulator period, which requires a great number of periods to achieve saturation. Thus, longer integration step is necessary to save the computation time. The data processing scheme described here ensures a good reproducibility of the electron motion and interaction efficiency regardless of the length of the interaction step, as far as the electron energy and radiation field does not change drastically within the step.

After the data processing, the FEL equations can be solved easily. The electron energy and phase can be calculated by substituting (5) and (7) into (2) and (3). The wave equation (1) that describes the growth of the radiation field can be modified to a more convenient form for numerical computation, by means of spatial and temporal Fourier transforms [1].

EXAMPLES

An FEL simulation code, SIMPLEX, has been developed at SPring-8 using the numerical schemes described above. Let us now show several examples of results to investigate the effects due to the undulator field error using SIMPLEX. Accelerator and undulator parameters used in the simulations are summarized in Table 1.

Electron Energy	250 MeV
Normalized Emittance	1πmm. mrad
Energy Spread	2×10^{-4}
Peak Current	2 kA
Average betatron function	7 m
Undulator Period	15 mm
Undulator Length	4.5 m
Undulator K Value	1.3
Wavelength	60 nm
Gain Length	0.16 m

Table 1: Example of a magnetic device configuration and focusing force distribution in the undulator line.

Performance of SCSS Prototype Undulator

SCSS stands for "SPring-8 Compact SASE Source" and aims at an X-ray FEL in the Angstrom wavelength region [2]. A prototype undulator for the SCSS project has been constructed in 2003, the details of which are described in [3]. Here we check the magnetic performance of the prototype. Figure 2(a) shows the phase errors as functions of the longitudinal position calculated with the magnetic field measured before and after the undulator field correction. We can see a drastic improvement between the two. In order to check the actual performance as an FEL driver, we performed FEL simulations, the results of which are shown in Fig. 2(b) as the radiation power growth along the undulator. As in the phase error, the FEL gain curve is improved significantly by the field correction, and we can expect the radiation power close to the ideal value with the constructed undulator.

Effects due to the Ambient Field

The ambient field means weak and uniform magnetic fields that cause a parabolic electron orbit. The main source for this is the geomagnetic field and/or an offset voltage of the Hall probe that is usually used in the undulator field measurement. In addition, the electron beam injected into the undulator with a vertical position offset with respect to the undulator axis experiences a parabolic-like orbit due to the natural focusing, which brings undesirable effects similar to those by the ambient field. The examples of the effects due to the ambient field ΔB_y in the vertical direction are shown in Fig. 3 in terms of the electron trajectory and power-growth curve. It is found that ΔB_y of 0.5 Gauss has little effects on the FEL gain, while those of 1.0 Gauss are not negligible.

Effects due to Inhomogeneous Demagnetization

It is well known that permanent magnets can be damaged by irradiation of the electron beam, which cause an

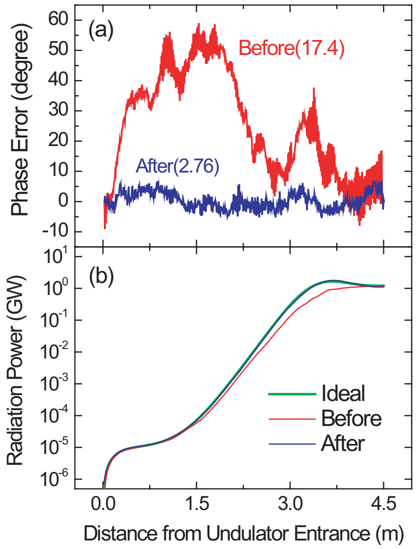

Figure 2: Magnetic performances of the prototype undulator for the SCSS in terms of the (a) phase error and (b) power-growth curve.

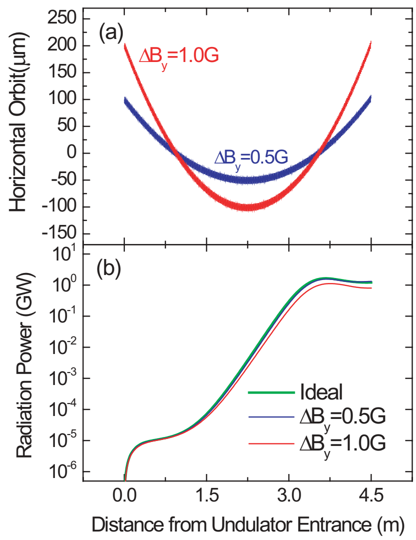

Figure 3: Effects due to the ambient fields of 0.5 and 1.0 Gauss on the (a) electron trajectory and (b) FEL gain.

inhomogeneous field variation along the undulator. As an example, let us consider the case when the magnetic field distribution is given as

$$B_y(z) = B_0 \sin(k_u z) F(z),$$

with

$$F(z) = 1 - A\mathrm{e}^{-z/D},$$

being an field envelope function to describe the radiation-induced demagnetization, where A and D denote the maximal fraction and typical depth of demagnetization, respectively. The examples are shown in Fig. 3 for two values of A. The depth D is fixed at 1.0 m in each simulation. We find a significant gain degradation even for the 0.5 % demagnetization.

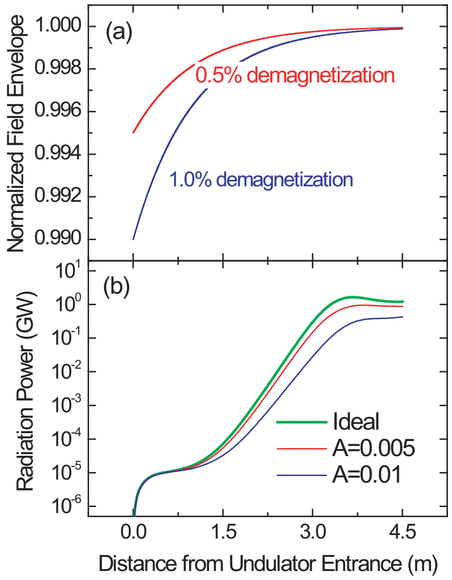

Figure 4: Effects of radiation-induced demagnetization on the (a) field envelope and (b) FEL gain for two values of the maximal demagnetization fraction A.

SUMMARY

The FEL simulation code, SIMPLEX, has been introduced. SIMPLEX has functions for FEL simulations as follows: steady-state simulation with seeding; time-dependent simulation with shot noise implementation for SASE regime; wakefield implementation; error sources related to phase slippage between undulator segments, field discrepancy, and trajectory straightness; graphical output. SIMPLEX is freely available from the web site [4]. It is equipped with a full graphical user interface for pre- and post-processing and does not need any other commercial softwares or libraries. Because of the portability of the graphical library used in SIMPLEX, it works on many platforms such as Microsoft Windows, Mac OS X, and unix-based OS such as Linux.

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