GAIN AND COHERENT RADIATION FROM A SMITH-PURCELL FREE-ELECTRON LASER*

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Abstract

It has been shown recently that the electron beam in a Smith-Purcell free-electron laser interacts with an evanescent wave for which the phase velocity is synchronous with the electron velocity. However, the group velocity of this wave may be either positive or negative, and when the group velocity vanishes, the gain diverges. In addition, the bunching of the electrons by the interaction with the evanescent wave enhances the ordinary Smith-Purcell radiation due to coherent effects. In particular, the spacing of the bunches causes the Smith-Purcell radiation to peak at the harmonics (and corresponding Smith-Purcell angles) of the evanescent wave frequency. The high gain achievable near zero group velocity makes it possible to design high-power cw Smith-Purcell free-electron lasers. An experiment is being assembled to observe these effects.

INTRODUCTION

Smith-Purcell (SP) radiation is emitted when an electron passes close to the surface of a grating [1]. The virtual photons of the electron field are scattered by the grating, and the wavelength λ_{SP} of the radiation observed at the angle θ from the direction of the electron beam is

$$\frac{\lambda_{SP}}{L} = \frac{1}{\beta} - \cos\theta \qquad (1)$$

where L is the grating period, βc the electron velocity, and c the speed of light. The angular and spectral intensity of SP radiation is described by the theory of van den Berg and Tan [2,3,4]. When the current in the electron beam is sufficiently high, the interaction between the electrons and the fields above the grating becomes nonlinear. This causes bunching of the electrons in the beam, which enhances the SP radiation. Recently, a tabletop SP-FEL has been demonstrated at Dartmouth [5]. This device operated near threshold in the spectral region from 300-900 μm. To improve on this performance, it is necessary to understand how these devices operate.

GAIN

A theoretical description of the small-signal gain in a Smith-Purcell free-electron laser (SP-FEL) is discussed in detail elsewhere [6]. The electron beam is represented by a uniform plasma traveling in the positive x direction with velocity βc that fills the region $y > 0$ above the grating. The electrons in the beam interact with an evanescent wave that travels along the surface of the grating in synchronism with the electron beam. To describe the evanescent wave, Floquet's theorem is used and the periodic function is expanded in a Fourier series. In the grooves of the grating the fields are expanded in a Fourier series, but numerical computations show that it is sufficient to keep just the zeroth term in this series. Across the interface between the grating and the electron beam, the tangential component of the electric field and the tangential component of the magnetic field must be continuous. When this expansion is substituted into the Maxwell equations, we get the dispersion relation

$$R_{00} - 1 + \chi_0 S_{00} = 0 \qquad (2)$$

where

$$R_{00} = 2\tanh(\kappa_0 H) \sum_{p=-\infty}^{\infty} \frac{\kappa_0 A}{\alpha_p L} \frac{\cos\left[(k+pK)A\right]-1}{(k+pK)^2 A^2} \qquad (3)$$

and

$$S_{00} = 2\tanh(\kappa_0 H) \frac{\kappa_0 A}{\alpha_0 L} \frac{\cos[kA]-1}{k^2 A^2} \qquad (4)$$

in which A is the width of the grooves and H the depth, $K = 2\pi / L$ the grating wave number, k the x-component of the wave vector of the evanescent wave, and

$$\alpha_p^2 = (k+pK)^2 - \frac{\omega^2}{c^2} + \frac{\omega'^2_p}{c^2} \qquad (5)$$

$$\kappa_0^2 = -\omega^2 / c^2 \qquad (6)$$

where ω is the frequency of the evanescent wave and ω'_p is the plasma frequency of the electron beam in the rest frame of the beam.

Table I. Parameters for the planned experiments.

Grating period	200 μm
Groove width	100 μm
Groove depth	175 μm
Grating length	12.7 mm
Electron energy	45-65 keV
Electron-beam current	1 mA
Electron-beam diameter	50 μm

In the absence of the electron beam, the last term in (2) vanishes, and we are left with the dispersion relation for the empty grating. Some simple computations carried out

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using MathCad are shown in Figure 1 for the grating parameters summarized in Table I. In Figure 1, the operating point of the laser is the point where the beam line, βk intersects the dispersion curve. As shown there, for 56-keV electrons the intersection occurs at a point $k/K \approx 0.5$, where $d\omega/dk \approx 0$. Below this point the group velocity is positive, and above this point the group velocity is negative, in the manner of a BWO.

When the effect of the electron beam is nonvanishing but small, the solution to (2) can be expanded about the no-beam case. Since the susceptibility diverges near the synchronous point, the gain is largest there and the amplitude growth rate is found to be

$$\mu = \frac{\sqrt{3}}{2}\left|\frac{\omega_p^2}{\gamma^3\beta^2c^2}\frac{S_{00}(\omega_0,k_0)}{R'_{00}(\omega_0,k_0)}\right|^{1/3} \tag{7}$$

where $\gamma = 1/\sqrt{1-\beta^2}$. The growth rate for the power is twice this. The total power gain per pass is then

$$g = e^{2\mu Z} \tag{8}$$

where Z is the overall length of the grating.

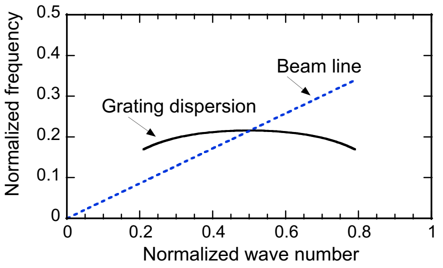

Figure 1. Dispersion diagram for a SP-FEL; beam line drawn for a 56-kV beam.

As shown in Figure 2, the gain has a strong peak near 56 kV, and actually diverges where the group velocity vanishes. In a fundamental sense, the net gain for the evanescent wave is a balance between the energy absorbed from the electron beam and that lost by energy flow along the grating. But the energy in the evanescent wave travels at the group velocity, $d\omega/dk$, and this depends on the wave number of the wave, as indicated in Figure 1.

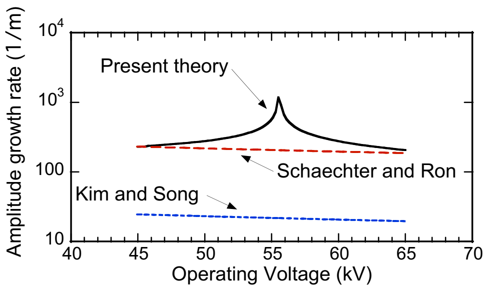

Figure 2. The gain of a SP-FEL.

Other theories have been proposed to describe the operation of a SP-FEL [7,8,9]. Schaechter and Ron analyze the interaction of an electron beam with a wave traveling along the grating, and include waves that are emitted by the beam and reflected off the grating [8]. Kim and Song consider an electron beam that interacts with a Floquet wave traveling along the surface of the grating, somewhat like the present theory [9]. However, they assume that at least one of the Fourier components of the Floquet wave is radiative, rather than evanescent. That is, at least one component of the wave is not exponentially decreasing away from the grating surface. The results of computations using these alternative theories are shown for comparison in Figure 2. In the present theory, the gain peaks near 56 kV due to the dispersion of the grating, which is not explicitly included in the other theories.

COHERENT RADIATION

The frequency of the synchronous evanescent wave is found from the dispersion relation (2). Some computations for the parameters of Table I are shown in Figure 3. As shown there, the free-space wavelength of the evanescent wave is always longer than that of the SP radiation. Since it does not radiate, the evanescent wave is coupled out from the SP FEL only at the ends of the grating. When the group velocity vanishes, this radiation disappears. The evanescent wave was not observed in the Dartmouth experiments.

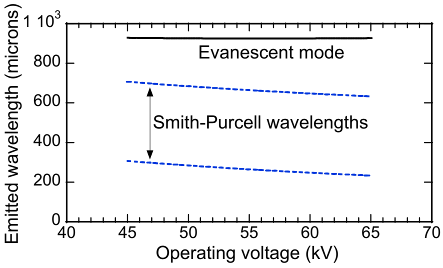

Figure 3. Wavelengths emitted from a SP-FEL.

For the grating parameters in Table I, the conventional SP radiation is emitted over a range of wavelengths from $\lambda/L = (1-\beta)/\beta$ to $\lambda/L = (1+\beta)/\beta$, as indicated by (1) and shown in Figure 3. When the electron beam interacts with the evanescent wave, it is bunched at the frequency of the evanescent wave. This bunching enhances the SP radiation at the harmonics of the evanescent wave. This can be seen in the following way. When a single electron passes over the grating, the angular spectral intensity S_{SP} of the SP radiation is proportional to q^2, where q is the electronic charge, and the total radiation from n_e electrons is proportional to $n_e q^2$. When the n_e electrons are formed into a bunch of charge $n_e q$ whose dimensions are small compared to a wavelength, the SP radiation is proportional to $(n_e q)^2$, which represents a coherent enhancement of the radiation by the factor n_e over the entire band of SP radiation. However, when the electrons appear in periodically spaced bunches, there is yet another effect. As shown in Figure 4, the bunches interfere constructively at wavelengths that satisfy the relation

$$\lambda_b + \beta n_h L \cos\theta = n_h L \quad (9)$$

where $n_h > 0$ is an integer, $\lambda_b = \beta\lambda_\infty$ is the spacing between the bunches, and λ_∞ is the free-space wavelength of the evanescent wave. But if we use the SP relation (1) we find that

$$\lambda_\infty = n_h L\left(\frac{1}{\beta} - \cos\theta\right) = n_h \lambda_{SP} \quad (10)$$

so the coherent SP radiation is enhanced at the harmonics of the evanescent wave. Relative to the intensity from the same number of electrons in randomly spaced bunches, the angular intensity of the radiation at the harmonic n_h is enhanced by the number N_g / n_h of bunches that radiate in phase with the wave over the length of the grating, where N_g is the total number of grooves in the grating. However, the angular width of the harmonic is on the order of $1/N_g$, the angular resolution of a grating with N_g grooves, so the coherent radiation is compressed into a narrow angular width but the total power is conserved. These effects are indicated schematically in Figure 5.

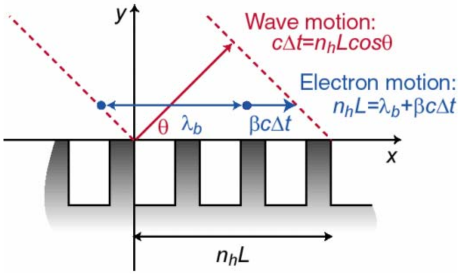

Figure 4. Coherent SP radiation is formed when successive bunches radiate in phase, where Δt is the time for a wave to catch up to the previous bunch.

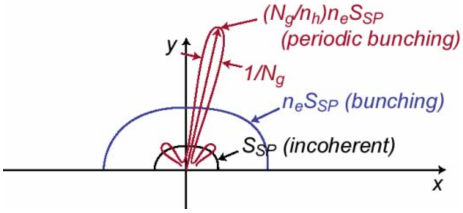

Figure 5. Incoherent and coherent SP radiation from a bunched electron beam.

For a numerical estimate of these effects we can use the parameters of Table I. The number of electrons in a bunch is on the order of

$$n_e = I_e \lambda_\infty / qc \quad (11)$$

where I_e is the total current in the electron-beam. For $\lambda_\infty = 920\ \mu$m this gives $n_e \approx 2\times10^4$, so if the electrons are perfectly bunched, the SP radiation is enhanced by this factor. Of course, even at saturation the bunching is less than perfect and the actual enhancement will be less than this. The number of periods in the grating is $N_g = 64$, so the angular spectral intensity at the peak of the second harmonic ($\lambda_{SP} \approx 460\ \mu$m) will be enhanced by the additional factor $N_g / n_h \approx 32$. But from the theory of van den Berg, the total power of the incoherent SP radiation emitted by this beam is on the order of a hundred nanowatts, so the coherent radiation should be on the order of a milliwatt at saturation. However, even with coherent enhancement, the SP radiation is still small compared with the power in the evanescent wave itself.

We can estimate the power in the evanescent wave at saturation by the following argument. The electrons continue to lose energy to the evanescent wave until they lose enough energy to drop out of synchronism. For small changes in energy, the change in the velocity of an electron is

$$\Delta\beta = \Delta\gamma / \beta\gamma^3 \quad (12)$$

and the phase shift in one gain length is

$$\Delta\phi = \frac{2\pi\Delta\beta}{\mu\beta\lambda_b} = \frac{2\pi\Delta\beta}{\mu\beta^2\lambda_\infty} \quad (13)$$

The total power lost by the beam is then

$$P = \frac{mc^2}{2\pi q} I_e \beta^3\gamma^3 \mu\lambda_\infty \Delta\phi \quad (14)$$

Saturation corresponds to $\Delta\phi = O(1)$. For the parameters in Table 1, the saturated power at 56 kV is on the order of a watt, which is much larger than the coherent SP radiation on the second harmonic of the evanescent wave.

When the group velocity is large, this power comes out from the evanescent wave at the ends of the grating. However, when the group velocity is sufficiently small, the power put into the evanescent wave by the electrons is lost by scattering from imperfections in the grating and by dissipation in the grating surface.

EXPERIMENT

At the present time, an experiment is being assembled to test these predictions. The parameters of the experiment are summarized in Table I, and a schematic diagram of the experiment is shown in Figure 6. The electron beam is emitted from the tip of a blunt needle illuminated by 3-ns pulses from the fifth harmonic of a Nd:YAG laser pulsed at 20 Hz [10,11]. The electron beam is focused by a solenoidal lens and aligned above the surface of the grating by two pairs of steering coils.

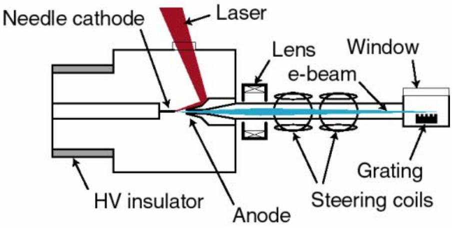

Figure 6. Schematic diagram of the experiment

The wavelength range of greatest interest is 400-1000 μm, as indicated in Figure 3, so a quartz window close to the surface of the grating can be used to provide a large collection solid angle. The radiation is detected using a liquid-He-cooled InSb bolometer, which is sensitive (above the noise level) to signals as small as nanowatts. Therefore, it should be possible to observe both the incoherent and coherent SP radiation. Measurements will be performed at voltages ranging from 45 to 65 kV, which spans the region of zero group velocity. It should be possible to observe the peak in the output power at the zero-group-velocity point, as well as the wavelength of the evanescent wave (about 930 μm) and the enhanced coherent emission at the second harmonic (around 420 μm).

CONCLUSIONS

The detailed theory of gain in a SP-FEL shows the importance of taking into account the dispersion relation for the evanescent wave. It has long been recognized that the electron velocity must be synchronous with the phase velocity in slow-wave devices, but in a SP-FEL the gain has a maximum near the condition where the group velocity vanishes. In a BWO, the output vanishes for this condition since the energy flows at the group velocity. However, in a SP-FEL an alternative source of output is provided by the coherent SP radiation at the harmonics of the evanescent frequency. Although this is radiated at lower power, the high gain makes it possible to envision low-current devices operating in cw mode with significant average power. In addition, if the group velocity is not too small, power from the evanescent wave can be directly coupled out from the ends of the grating. This would produce watts of average power from the device.

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