STABILITY OF A SHORT RAYLEIGH RANGE LASER RESONATOR WITH MISALIGNED OR DISTORTED MIRRORS

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Abstract

Motivated by the prospect of constructing an FEL with short Rayleigh length in a high-vibration environment, we have studied the effect of mirror vibration and distortion on the behavior of the fundamental optical mode of a cold-cavity resonator. A tilt or transverse shift of a mirror causes the optical mode to rock sinusoidally about the original resonator axis. A longitudinal mirror shift or a change in the mirror's radius of curvature causes the beam diameter at a mirror to dilate and contract with successive impacts. Results from both ray-tracing techniques and wavefront propagation simulations are in excellent agreement.

INTRODUCTION

Some designs for a high-power free electron laser (FEL) call for a short Rayleigh length optical resonator in order to reduce the system size while minimizing heat damage to the mirrors [1, 2]. An additional advantage of this design is improved optical beam quality, due to the small interaction region in the center of the resonator [3]. However, this design raises concerns about mode stability, in particular the sensitivity to motions of the mirrors. This paper presents a study of the effect on beam behavior of mirror motion and mirror radius change, particularly as they affect short Rayleigh length resonators.

We study the results of several cavity distortions: mirror tilt, transverse and longitudinal shifts in mirror position, and changes in mirror focal length. In order to isolate resonator effects, our results are for a resonator alone with no gain. Since mirror motions are relatively slow (∼ms) compared with the optical round trip time (∼ns), the motions are assumed to be fixed over many passes of the beam through the resonator.

In general, the optical beam in a laser resonator retraces itself — it is an eigenmode of the resonator. If a mirror is misaligned or distorted, however, the resonator eigenmode will be redefined and the existing optical beam will tend to walk around the mirrors [4]. For sufficiently large misalignment, the beam radius may increase indefinitely — i.e., the resonator may become unstable. These effects are most pronounced for short Rayleigh length resonators, which are already near the stability limit. In practical terms, the mirror misalignment and distortion will cause the beam displacement to exceed the size of the mirrors, thereby creating beam loss and lowering the resonator Q.

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SIMULATION TECHNIQUES

We start by assuming a resonator with two identical mirrors of radius of curvature R (focal length $f = R/2$) separated by distance S and enclosing a Gaussian beam which is an eigenmode of the resonator with Rayleigh length z_0 (Fig. 1). If we normalize all longitudinal distances by S, all transverse distances by $(\lambda S/\pi)^{1/2}$ and all angles by $(\lambda/\pi S)^{1/2}$, then $f = z_0^2 + 1/4$, and the $1/e$ radius of the beam at any z is $w(z) = (z_0 + z^2/z_0)^{1/2}$. In particular, the waist radius is $w_0 = z_0^{1/2}$ [5]. For a 10 m long resonator with $\lambda = 1\mu$m, the transverse scaling length is 1.8 mm and the scaling angle is 0.18 mrad.

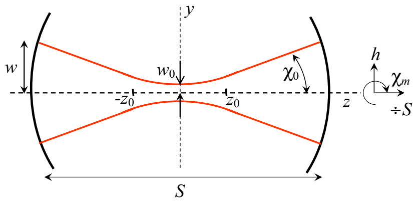

Figure 1: Resonator with Gaussian mode characterized by Rayleigh length z_0. Distortions of the right-hand mirror include tilt θ_m, transverse shift h, longitudinal shift ΔS, and focal length change Δf (not shown).

The beam is simulated using two techniques. In the *ray tracing* technique, a Gaussian beam is simulated by a random collection of rays, Gaussian distributed in both transverse position y and angle θ and set up at the beam waist [6]. For a beam whose amplitude in the y-plane is $A\exp(-y^2/w_0^2)$, the joint probability density is given by $f(y, \theta)$. Setting the distribution widths to $\delta y = w_0 = z_0^{1/2}$ and $\delta\theta = \theta_0 = z_0^{-1/2}$,

$$f(y,\theta) = \frac{1}{\pi}\, e^{-(y^2+z_0^2\theta^2)/z_0}. \qquad (1)$$

Here θ_0 is the angular spread of the beam at $z \gg z_0$ as shown in Fig. 1. Each ray is then propagated numerically with the usual ABCD ray matrices and the evolving ray density and direction is found to closely emulate the actual behavior of a Gaussian beam .

In the *wave propagation* technique [1], the spatial part of the Gaussian beam $a(x, y, z)$is set up at the beam waist and then propagated numerically by the paraxial wave equation $\partial_z a = (-i/4)\nabla_\perp^2 a$.

Both simulation methods can accommodate tilted,

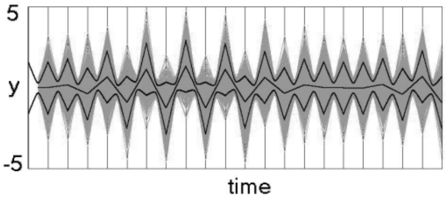

Figure 2: Evolution of an optical beam in a resonator with $z_0 = 0.1$ and $\theta_m = 0.05$. Each vertical line corresponds to a mirror, with the successive reflections unfolded to see the overall behavior. The shaded area shows the trajectories of 1000 random rays; the center line is the center of the optical beam; and the top and bottom lines, calculated from beam theory, correspond to the radius $w(z)$ for the Gaussian mode.

shifted, and distorted mirrors, while the latter method can also incorporate laser gain.

In a third analytical method [5], the Gaussian beam is represented by complex beam radius $q(z) = z + iz_0$. Propagation is then accomplished using the ABCD matrix elements in the form $q_2 = (Aq_1 + B)/(Cq_1 + D)$ and extracting the beam front curvature $R(z)$ and beam radius $w(z)$from $1/q = 1/R - i/w^2$. This method will accommodate longitudinal mirror shift and focal length change only. However, when coupled with ray tracing, can also describe the effects of tilt and transverse shift of the mirrors.

MIRROR TILT AND SHIFT

We now let the right-hand mirror undergo tilt θ_m and/or transverse shift h and investigate the subsequent behavior of the Gaussian beam. The immediate effect is that the reflection angle of any ray incident on the mirror will be increased by $2\theta_m + h/f$. The resonator *will* remain stable, but a new resonator axis will be defined which tilts with respect to the old axis by amount ϕ, where

$$\phi = -[(1 + 4z_0^2)\theta_m + 2h]/(8z_0^2). \tag{2}$$

The optical beam, which initially was an eigenmode of the old resonator, now becomes tilted with respect to the new axis and is no longer an eigenmode of the realigned resonator. Consequently, with each reflection, its angle with respect to the old axis will change in a rocking fashion, depending on the value of z_0.

The effect of the rocking over many passes n is to make the beam position on the mirror walk sinusoidally up and down. If y_n is the beam position on the mirror after n reflections [5],

$$y_n = C_1[1 - \cos(\alpha n)] + C_2 \sin(\alpha n) \tag{3}$$

where

$$\alpha = \cos^{-1}\left(\frac{2f^2 - 4f + 1}{2f^2}\right), \tag{4}$$

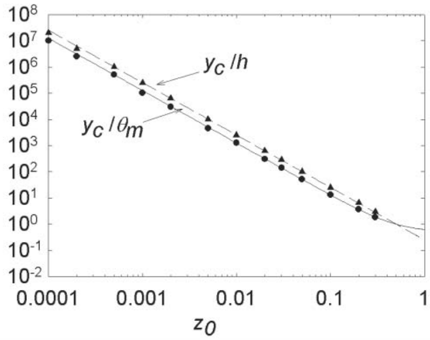

Figure 3: z_0 dependence of the maximum excursion y_c of the beam center from the original cavity axis when a mirror tilts by θ_m or undergoes transverse shift h. Tilt and shift are plotted separately. The lines are beam theory; the points are from wave simulations. For an FEL with S = 10 m and λ = 1 μm, $y_c = 10$ corresponds to 1.8 cm.

$$C_1 = \left(\frac{2f - 1}{4f - 1}\right)(h + 2f\theta_m), \tag{5}$$

$$C_2 = \frac{-(h + 2f\theta_m)}{\sqrt{4f - 1}}. \tag{6}$$

Figure 2 shows the result of mirror tilt. The beam is started on axis with the right mirror tilted. Successive reflections of the beam are unfolded, so that the horizontal axis is time. The beam angle changes continually, depending on z_0 and, for this figure, θ_m. In general, the maximum deflection of the beam center y_c is proportional to θ_m and h, as calculated from Eq.(3):

$$y_c = \frac{(4z_0^2 + 1)\theta_m + 2h}{8z_0^2}. \tag{7}$$

The rocking period n_0 can also be calculated: $n_0 = 2\pi/\alpha$.

For the small z_0 case we are concerned with here, y_c is a strong function of z_0. We show this dependence in Fig. 3 where y_c/θ_m and y_c/h are plotted separately against z_0. As z_0 becomes smaller, the transverse excursions become comparable to the mirror diameter and the beam will walk off the mirrors.

LONGITUDINAL MIRROR SHIFT

Let the resonator contain a Gaussian beam which is a resonator eigenmode with Rayleigh length z_0. Since z_0 is small, the mirror focal lengths $f = z_0^2 + 1/4$ are already only slightly larger than the resonator stability limit $f_{min} = 1/4$. Let the right-hand mirror shift by (normalized) amount ΔS in the z-direction. Successive reflections of the beam will remain on axis, but the Rayleigh lengths of the beam and the resonator eigenmode will no longer be equal. If ΔS is positive (cavity length increases), the focal lengths decrease to $f' = f/(1 + \Delta S)$, and if $f' < 1/4$,

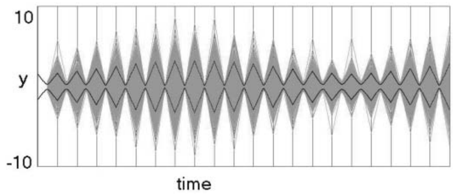

Figure 4: Evolution of an optical beam in a resonator with $z_0 = 0.1$ and right mirror shift $\Delta S/S = 0.031$. The vertical lines represent mirrors, with successive reflections unfolded to see the overall behavior. The gray areas are the trajectories of 1000 random rays; the dotted lines, calculated from beam theory, correspond to the radius $w(z)$ of the Gaussian mode. The beam remains on axis, but expands and contracts with successive reflections.

the resonator will become unstable and the beam will expand without limit. The maximum allowable value for ΔS is therefore $\Delta S_{max} = 4f - 1 = 4z_0^2$.

If $\Delta S < \Delta S_{max}$, the resonator remains stable but the beam will no longer retrace itself in succeeding passes, as shown in Fig. 4. With each pass, the beam width at the mirrors will expand and contract, depending on both ΔS and z_0. Figure 5 shows the effect of varying ΔS for several z_0. For $\Delta S < \Delta S_{max}$, the effect on y_{max} is small. However, as ΔS approaches ΔS_{max}, y_{max} increases and finally diverges at ΔS_{max}.

For $\Delta S < 0$ (resonator length decreases), the resonator remains stable but the beam width again expands dramatically as the difference between the Rayleigh lengths of the beam and the resonator eigenmode becomes large.

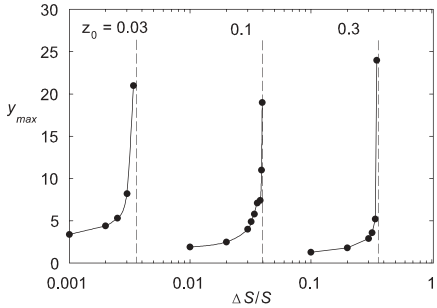

Figure 5: Maximum beam radius y_{max} for right-hand mirror shift ΔS at several values of z_0. The vertical dashed lines show the limits of resonator stability at $\Delta S_{max} = 4z_0^2$. The data points are taken from ray and beam simulations; the solid lines are guides to the eye. For an FEL with S = 10 m and λ = 1 μm, $y_{max} = 10$ corresponds to 1.8 cm.

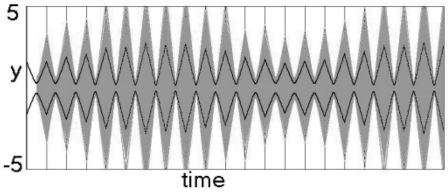

Figure 6: Evolution of an optical beam in a resonator with $z_0 = 0.1$ and right mirror shift $\Delta f/f = -0.05$.

MIRROR DISTORTION

Now let the focal length f of the right-hand mirror in the previously undistorted resonator change by amount $\Delta f/f$. Since the mirror focal lengths are unequal, the mode waist of the resonator eigenmode will move away from the resonator center. The effect is to change the resonator eigenmode so that it no longer corresponds to the original beam. Consequently the beam radius on the mirror will expand and contract with each subsequent reflection, as shown in Fig. 6. In addition, if $\Delta f/f$ is negative (a decrease in the mirror focal length) and made too large, the resonator will no longer be stable and the beam will diverge indefinitely. The stability criterion is $\Delta f > -8z_0^2/(1 + 4z_0^2)$.

Figure 7 shows the results from our simulations. The beam radius at the mirror is y_{max}, as before. As Δf is made increasingly negative, y_{max} increases slowly as the threshold for resonator instability (vertical dashed lines) is approached, and then diverges sharply at the threshold.

DISCUSSION

We have shown that for a short Rayleigh length resonator with no gain, the effects of mirror tilt, shift, and focal length

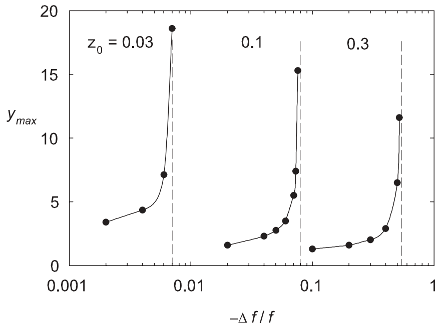

Figure 7: Maximum beam radius y_{max} for focal length change $\Delta f/f$ of the right-hand mirror at several values of z_0. The minus sign in front of Δf indicates the focal length is decreasing. The points are taken from ray simulations and beam calculations; the solid lines are guides to the eye; and the vertical dashed lines show the limits of resonator stability at $\Delta f = -8z_0^2/(1 + 4z_0^2)$.

change can produce dramatic changes in the beam direction and width. In the cases of mirror tilt and transverse shift, the effect is to cause the beam to rock up and down on the mirrors and, if the rocking amplitude is sufficiently large, to cause the beam position to exceed the mirror radius. In the cases of longitudinal mirror shift and focal length change, the beam will remain on axis but the beam radius at the mirror will expand and contract with successive reflections. If the beam radius becomes too large, portions of the beam may exceed the mirror radius. In either case beam power can be lost, or, equivalently, the cavity Q will be reduced. For comparison with actual mirrors, the y-axes in Fig. 3, 5, and 6 can be converted to real values by multiplying by the transverse scaling length $(\lambda S/\pi)^{1/2}$. For a laser with $S = 10$ m and $\lambda = 1$ μm, $y = 10$ corresponds to an actual y of 1.8 cm.

ACKNOWLEDGEMENT

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