NUMERICAL MODELING OF THE NOVOSIBIRSK TERAHERTZ FEL AND COMPARISON WITH EXPERIMENTAL RESULTS

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Abstract

Recently a new high-power terahertz FEL has been put in operation at the Siberian Center for Photochemical Research in Novosibirsk. The first lasing at the wavelength near 140 micrometer was achieved in April 2003. Since then some experimental data were obtained which required theoretical explanation. In this paper we use a simple 1-D model for numerical simulations of the FEL operation. The model is based on the excitation of multiple longitudinal radiation modes of the optical resonator by the charged discs. We restrict our consideration to the transverse fundamental mode only. This approximation is valid in the case of long-wave FELs. We compare the results of numerical simulations with some analytical estimates and experimental data.

INTRODUCTION

The Novosibirsk high power FEL started operation recently [1]. It operates in CW mode at terahertz band using simple two-mirror optical resonator. The computer code was created to provide simulations for all measured data. As the beam transverse sizes are significantly less than the radiation eigenmode size, it is enough to use a one-dimensional code.

MODEL DESCRIPTION AND BASIC EQUATIONS

Electron beam in our model is presented as a set of thin rigid disks with Gaussian transverse distribution of charge, which is supposed to be the same for each disk. The total charge of a disk is equal to the total charge of the bunch, divided by the number of disks and the charge-to-mass ratio is the same as for a single electron. We neglect the influence of the radiation field on the transverse motion of a disk. It means that transverse trajectory is definitely determined by the undulator field and the disk has only the longitudinal degree of freedom. We also neglect betatron oscillations and include their contribution to the longitudinal velocity as an additional energy spread. It is convenient to use the longitudinal spatial coordinate z as an independent variable. In this case the system of equations for the longitudinal motion can be written as

$$\begin{aligned} \frac{d\tau_n}{dz} &= \frac{1}{V_z(\Delta_n)} - \frac{1}{V_z(0)} \\ \frac{d\Delta_n}{dz} &= \frac{e}{\gamma_0 m_e c^2}\langle E_x \rangle x'(z) \end{aligned} \tag{1}$$

where n is the disk number, $V_z(\Delta)$ is the longitudinal velocity, τ_n is the time delay with respect to the reference particle with energy γ_0, Δ_n is the relative energy deviation, $\langle E_x \rangle$ is radiation electrical field, averaged over disk charge distribution. $x'(z)$ is the transverse trajectory angle in x-z plane, in our approximation it depends only on z coordinate, and in the case of planar undulator $x'(z) = -\frac{K}{\gamma_0}\sin(k_w z)$, where K is the undulator deflection parameter, k_w is the undulator wave number.

The radiation field inside the optical resonator may be represented as the linear combination of the resonator eigenmodes. Taking into account small transverse size of electron beam and relatively high damping rate for high-order transverse modes, one can restrict the consideration to the fundamental transverse mode only. Then the on-axis radiation electric field may be represented as

$$E_x(0,0,z,t) = 2\,\mathrm{Re}\left[\sum_k a_k(z) e^{ik(z-ct)}\right] \tag{2}$$

For the numeric calculations it is convenient to consider the discrete spectrum wave packet with the carrier wave number k_0: $k_m = \frac{2\pi m}{cT_0} + k_0$, m is integer, and T_0 is the envelope period. T_0 have to be chosen much more, than the packet duration (typically, of the order of the electron bunch length). Using this field expansion, one can derive the FEL equations for a single pass of the wave packet and particles (charged disks) through the undulator

$$\begin{aligned} \frac{db_k}{d\theta} &= \sum_n D e^{i[\theta\delta_k + \varphi_n(1+\delta_k)]} \\ \frac{d\Delta_n}{d\theta} &= -\mathrm{Re}\sum_k b_k e^{-i[\theta\delta_k + \varphi_n(1+\delta_k)]} \\ \frac{d\varphi_n}{d\theta} &= -2\Delta_n \end{aligned} \tag{3}$$

Here we introduced the following set of dimensionless variables and constants: $\varphi_n = k_0 c \tau_n$, $\theta = k_w z$, $\delta_k = \frac{k-k_0}{k_0}$, $b_k = -\frac{eK}{m_e c^2}\frac{(JJ)_0}{\gamma_0^2}\frac{i}{k_w}a_k$,

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$D = \pi \frac{Q}{T_0 S_0 I_0 \gamma_0^3}\left[\frac{K(JJ)_0}{k_w}\right]^2$, where $(JJ)_0$ is the standard combination of Bessel functions, $I_0 = \frac{m_e c^3}{e}$ - the Alfven current, S_0 is an effective area of radiation (the power over the on-axis intensity), and $k_0 = 2\gamma_0^2 k_w/\left(1+K^2/2\right)$ is chosen.

One can easily check that the system Eq. (3) conserves the total energy

$$\frac{d}{dz}\left(\sum_k \frac{|b_k|^2}{2D} + \sum_n \varDelta_n\right) = 0\,. \qquad (4)$$

It also worth noting that this system can be derived from the Hamiltonian

$$H = -\sum_n \Delta_n^2 + \sum_n\sum_k \sqrt{\frac{2DI_k}{1+\delta_k}}\sin[\theta\delta_k + \varphi_n(1+\delta_k) - \Psi_k], \qquad (5)$$

where $I_k = \frac{|b_k|^2}{2D(1+\delta_k)}$, $\varPsi_k = \arg(b_k)$; $\varPsi_k, \varphi_n$ and $I_k, \varDelta_n$ are canonical coordinates and momenta respectively. Then the existence of the integral Eq. (4) follows from the invariance of Eq. (5) with respect to the transformation

$$\varphi_n \to \varphi_n + \delta s\,,\ \varPsi_k \to \varPsi_k + (1+\delta_k)\delta s\,.$$

NUMERIC APPROXIMATION AND COMPUTER CODE DESCRIPTION

To obtain the numeric solution of Eq.(3) we used the following difference scheme

$$b_k^{j+1} = b_k^j + hD\sum_n e^{i\left[\theta\delta_k + \varphi_n^j(1+\delta_k)\right]}$$

$$\Delta_n^{j+1} = \Delta_n^j - h\,\mathrm{Re}\sum_k \frac{b_k^{j+1}+b_k^j}{2} e^{-i\left[\theta\delta_k + \varphi_n^j(1+\delta_k)\right]}, \qquad (6)$$

$$\varphi_n^{j+1} = \varphi_n^j - 2h\Delta_n^{j+1}$$

where j is the step number, h is the step size. This scheme implements a canonical transformation of the phase space. It also conserves the following value

$$\sum_k \frac{|b_k^j|^2}{2D} + \sum_n \Delta_n^j = const\,, \qquad (7)$$

which corresponds to the energy integral (4).

The computational algorithm, which directly followed from Eq.(6), was realized in a simple computer code. The code simulates reiterated passes of the radiation wave packet and the electron beam through the undulator. At the beginning of each pass a new particle initial distribution is formed, and all amplitudes of the radiation field modes b_k are reduced with accordance to the losses of the optical resonator. The center of a new bunch is delayed by the optical resonator length L detuning $\delta T = 1/f_0 - 2L/c$ (f_0 is the bunch repetition rate). We used Gaussian distribution for energy $\varDelta_n$ and uniform, parabolic or Gaussian distributions for coordinate φ_n. The typical number of radiation modes is 800 and the number of particles is 1000.

Accuracy of the code was checked for the small-signal low-gain calculations. The obtained dependences for the gain showed a very good agreement with the theoretical expressions. The results of simulation with the real FEL parameters are presented below.

SIMULATION RESULTS

Simulations were carried out with the Novosibirsk FEL parameters [1]. The dependence of the average radiation power inside the optical resonator on the pass number for different detunings is presented at Fig. 1. One can see that the radiation power reaches saturation after few hundreds of electron bunch passes through the undulator. For some detunings there was no constant saturation level and the power was non-stationary.

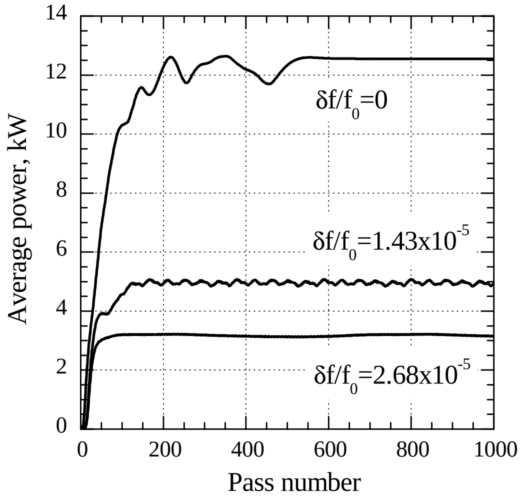

Figure 1: Dependence of average radiation power inside the optical resonator on pass number for different detunings $\delta f/f_0 = -c\delta T/(2L)$.

The maximum level of average power inside the optical resonator at zero detuning is about 12 kW, which is in a good agreement with the value obtained in the experiment.

Spectral density of radiation in the case of zero detuning is shown in Fig. 2. The presence of several bands in spectrum apparently indicates the development of side-band instability.

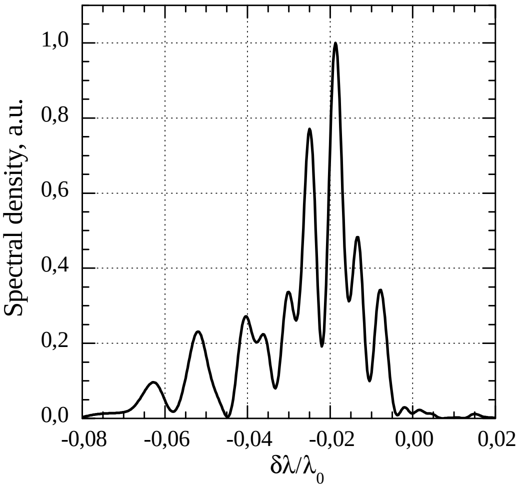

Figure 2: Spectral density distribution for zero detuning.

The corresponding power distribution in time domain is presented at Fig. 3. It can be seen, that the radiation wave packet in this case consists of three short pulses. The distance between adjacent pulses is slightly shorter then total slippage at the undulator length. The mechanism of such short pulse generation was described analytically in [2].

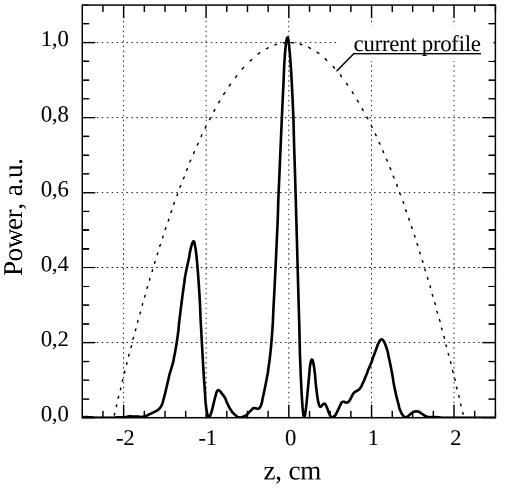

Figure 3: Power distribution over the wave packet length for zero detuning. The dot line corresponds to the beam current profile.

For large enough nonzero detuning the spectrum of radiation is narrow as it shown on Fig. 4. Similar spectrum with FWHM ~ 0.3% has also been observed in the experiment for some regimes.

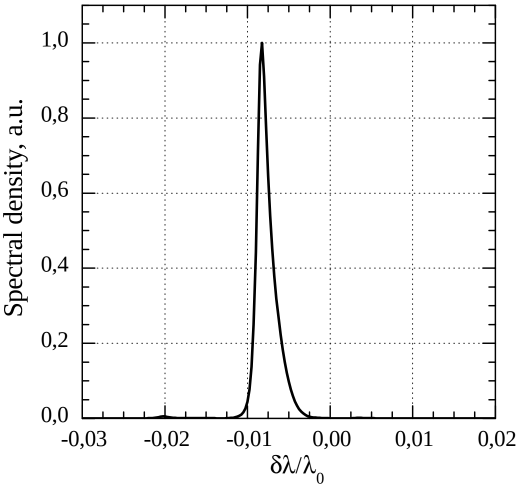

Figure 4: Spectral density distribution for the detuning $\delta f / f_0 = 2.65\times10^{-5}$.

Power distribution in time domain for nonzero detuning is presented at Fig. 5. The wave packet length in this case is much longer then in the previous one.

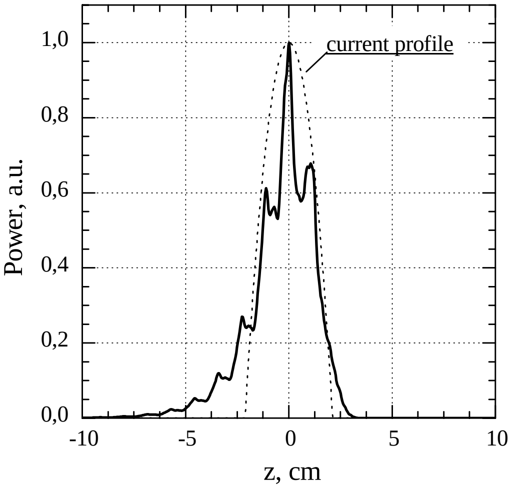

Figure 5: Power distribution for the detuning $\delta f / f_0 = 2.65\times10^{-5}$

In Fig. 6 one can see so-called detuning curve. Dependence, presented at this figure, shows, that there is a narrow peak near the zero detuning. The similar results were reported in [3].

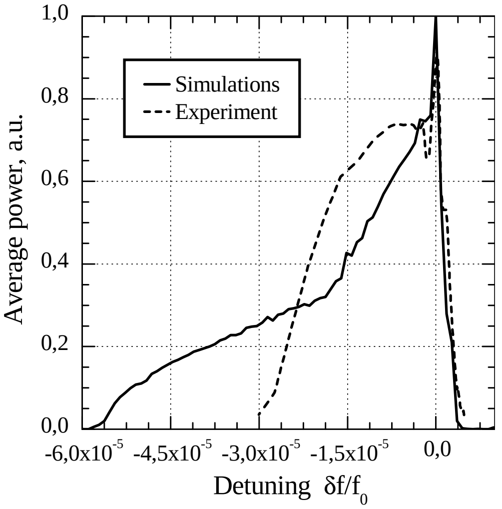

Figure 6: The average power vs. the detuning.

CONCLUSION

The results of calculations demonstrate a good agreement with the measured ones. The lack of information about the particle distribution in the real electron beam is, probably, the main limiting factor for the comparison of the experimental data and the simulation results. Calculations for more advanced FEL magnetic systems (multisection and tapered undulators, optical klystron, electron outcoupling, etc.) are planned.

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