PROPOSAL OF LASER-DRIVEN ACCELERATION WITH BESSEL BEAM

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Abstract

A novel approach of laser-driven acceleration with Bessel beam is presented in this paper. Bessel beam, in contrast to the Gaussian beam, demonstrates "diffraction-free" characteristics in its propagation, which implies potential in laser-driven acceleration. A configuration of Bessel beam truncated by a set of annular slits makes several special regions in its travelling path, where the laser field becomes very weak and the accelerated particles are possible to receive slight deceleration as they undergo decelerating phase. Thus, multistage acceleration is realizable. With the help of numerical computation, we have shown the potential of multistage acceleration based on a three-stage model.

INTRODUCTION

The intense laser field provides an ultrastrong field gradient implying an attractive potential in particles acceleration. Laser-driven accelerator, an active current research area, has received considerable attention in recent years [1-7]. Many proposals, such as inverse free electron laser [8], plasma-wave accelerator [9], ponderomotive schemes [10], inverse Cherenkov acceleration [11], and vacuum acceleration directly by the longitudinal component of electric field [12,13] were presented and extensively studied.

In general, laser beam from a laser cavity travels in the form of Gaussian mode, exhibiting manifest transverse spreading when it is focused down to small spot size. A strongly focused Gaussian beam will show apparent divergence after it passes a short certain distance, known as Rayleigh length. Thus, even there exist schemes to keep synchronization between accelerated particles and the wave, the effective acceleration can only occur within a relative short range. In order to extend the interaction, "diffraction-free" Bessel beams are introduced to the laser driven accelerator [12,13].

Since Durnin experimentally demonstrated the nondiffracting property of Bessel beam, it has been widely studied in various applications [14-16]. Hafizi et.al analysed the vacuum beat wave accelerator with using laser Bessel beam [17], and other Bessel beam driven acceleration schemes were also discussed recently.

This paper is aimed at an innovative approach in vacuum laser-driven acceleration with using Bessel beam. We truncate the Bessel beam by a set of annular slits (hereafter, we call it slits-truncated Bessel beam), so that several special regions are formed in its propagation, where the laser field becomes rather weak, and consequently, the accelerated particles, which slide behind the wave, are possible to receive slight deceleration as they undergo the decelerating phase of the wave field by just travelling in these regions. Because of the "diffraction-free" characteristic of Bessel beam, multistage acceleration comes to be feasible.

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BESSEL BEAM

General Theory

A solution of the Helmholtz wave equation for an azimuthally symmetric wave of frequency ω that propagates in the positive z direction gives the Bessel beam expression

$$\psi(\vec{r},t) = J_0(k\sin(\alpha)\rho)e^{i(k\cos(\alpha)z-\omega t)} \tag{1}$$

An ideal Bessel beam maintains the same radial profile over arbitrary propagation distance and has maximum amplitude on-axis and hence is called "diffraction-free". The following integral representation of Bessel function is helpful to comprehend the angle α,

$$J_0(k\sin(\alpha)\rho)e^{i(k\cos(\alpha)z-\omega t)} = \frac{1}{2\pi}\int_0^{2\pi} d\phi e^{i(k\sin\alpha x\cos\phi+k\sin\alpha y\sin\phi+k\cos\alpha z-\omega t)} \tag{2}$$

which exhibits that Bessel beam is superposition of an infinite set of ordinary plane waves making angle α with respect to the z axis [18]. And this is also the principle to generate Bessel beam, i.e., superpose plane waves with equal amplitude and a common phase that make angle α to the z axis.

A simple geometrical relation tells that, if the Bessel beam is formed at z=0 plane and to propagate a distance z, extent of $R=z\tan\alpha$ at z=0. That means an ideal Bessel beam needs arbitrarily large initial radial extent and arbitrarily large energy, to retain its "diffraction-free" character over an arbitrarily large distance, which is impossible to realize in practice. The physically meaning beam is an aperture-truncated one, which possesses nearly diffraction-free properties within some axial distance. The generation of aperture-truncated Bessel beam was widely studied elsewhere [14,15].

The phase velocity of a Bessel beam relies on the angle α,i.e., $v_p= c/\cos(\alpha)$, which is larger than the light speed c. If such a beam is used for direct laser-driven acceleration in vacuum, the particle will slide behind the wave and cannot gain net energy during a wave period. Naturally, we conceive a scheme that after the particle is accelerated in the first half circle of the wave period, the laser field suddenly disappears to allow the particle avoid the following deceleration; and when the particle slides into the next accelerating phase, the laser field takes on again. In other words, we have to make "laser-on" and "laser-off" regions alternatively appearing during laser propagation and well arrange the intervals among them to realize required phase matching.

Annular Slits-truncated Bessel Beam

A Bessel beam truncated by a set of annular slits can do the work, as schematically shown in Fig. 1, where laser

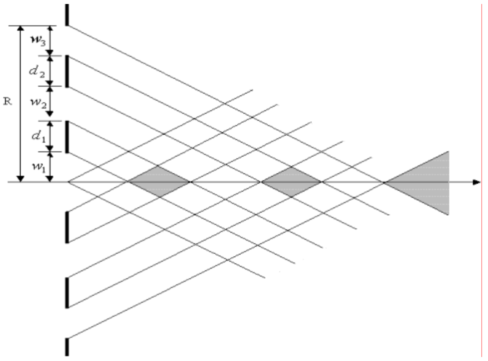

Fig. 1 schematic configuration of annular slits-truncated Bessel beam

field disappears at those shadow regions. The lengths of "laser-on" and "laser-off" regions are related to the slits' parameters, width w_i (i=1,2,3) and the interval between two slits d_i (i=1,2). Using the geometrical method we can easily work out those lengths. Actually, the "laser-off" areas are not entirely free of laser field because of the diffraction effect induced by annular slits. To precisely analysis Bessel beam diffraction, we employ scalar diffraction theory.

In the Fresnel approximation the amplitude $A(r, z)$ at a distance z can be obtained from diffraction integral. For a circularly symmetric incident amplitude A(ρ , 0), we have [19]

$$A(r,z) = \exp\left(ikz + \frac{ikr^2}{2z}\right)\left(\frac{k}{iz}\right) \times \int_0^a \rho A(\rho,0) J_0\left(\frac{k_\rho r}{z}\right) \exp\left(\frac{ik\rho^2}{2z}\right) d\rho \tag{3}$$

where a is the radius of a circular aperture. For the case of Bessel beam, we have $A(\rho,0)=J_0(k\sin(\alpha)\rho)$ as the initial amplitude distribution. If the beam is truncated by a set of annular slits, e.g., for the case as shown in Fig. 1, the incident amplitude should be modified as

$$A(\rho,0) = \zeta(\rho) J_0(k \sin \alpha\rho) \tag{4}$$

and $\zeta(\rho)$ is given by

$$\zeta(\rho) = \begin{cases} 1 & (0 \le \rho < R_1) \\ 0 & (R_1 \le \rho < R_2) \\ 1 & (R_2 \le \rho < R_3) \\ 0 & (R_3 \le \rho < R_4) \\ 1 & (R_4 \le \rho < R_5) \\ 0 & (R_5 \le \rho < \infty) \end{cases}$$

where $R_1=w_1$, $R_2=R_1+d_1$, $R_3=R_2+w_2$, $R_4=R_3+d_2$, $R_5=R_4+w_3$.

Together with the initial conditions, Eq. 3 can be numerically calculated to understand the propagation properties. As an example, here we consider the on-axis case (r=0) and employ the following parameters: laser wavelength λ =1 μ m, α =1°, and w_1 =4 mm, w_2 =4

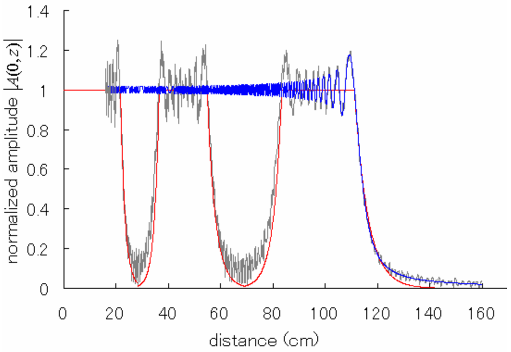

Fig. 2 Propagation of aperture-truncated (blue curve) and slits-truncated (grey curve) Bessel beam shown from z=16 cm. Synthesized function $\chi(z)$ (red curve) gives out the approximation to the grey curve

mm, w_3 =6 mm, d_1 =2 mm, d_2 =4 mm. The computation results for slits-truncated Bessel beam are illustrated in Fig. 2, and for the sake of comparison, aperture-truncated Bessel beam (with circular aperture radius of a=20 mm, summation of w_i and d_i) is also presented. The blue and grey curves represent aperture- and slits-truncated beams, respectively. It is seen that, their amplitudes characterize oscillation and decay at a finite distance ~a/tan(α). Contrast to the aperture-truncated beam, the slits-truncated one exhibits two special zones where the amplitude falls down by two orders in magnitude. With properly setting the parameters of annular slits, the injected electrons are possible to spend the decelerating phase in those regions and receive slight deceleration. The red curve is synthesized to approximate the grey curve with showing an average effect, in ignorance of the oscillation. It takes the below form, with the use of exponential functions to fit the falling and rising edges,

$$\begin{aligned} \chi(z) = & \left[H(z-0) - H(z-21.6)\right] \\ & + \left[H(z-21.6) - H(z-28.3)\right]\exp(-0.056\times(z-21.6)) \\ & + \left[H(z-28.3) - H(z-36.5)\right]\exp(0.056\times(z-36.5)) \\ & + \left[H(z-36.5) - H(z-54.8)\right] \\ & + \left[H(z-54.8) - H(z-69.2)\right]\exp(-0.032\times(z-54.8)) \\ & + \left[H(z-69.2) - H(z-83.4)\right]\exp(0.032\times(z-83.4)) \\ & + \left[H(z-83.4) - H(z-111.2)\right] \\ & + \left[H(z-111.2)\right]\exp(-0.02\times(z-111.2)), \end{aligned} \tag{5}$$

where H is the Heaviside function.

Laser Field

All field components satisfying Maxwell's equation are derived in reference [18] for ideal Bessel beam. For the convenience, we made a summary as below,

$$E_\rho = E_0 \cot(\alpha) J_1(k_\rho \rho)\cos(\eta)$$

$$E_z = -E_0 J_0(k_\rho \rho)\sin(\eta)$$

$$B_\varphi = \frac{E_0}{c\sin(\alpha)} J_1(k_\rho \rho)\cos(\eta)$$

where E_0 is the peak amplitude, and $k_\rho = k\sin(\alpha)$ $k_z = k\cos(\alpha)$ $\eta = k_z z - \omega t$. The other components are zero. In our proposal, we directly apply the longitudinal electric field E_z to accelerating the particles.

DYNAMICS

We first consider accelerating a single electron in the slits-truncated Bessel beam field. The motion of the electron in an electromagnetic field is governed by Lorentz equation

$$d\vec{p}/dt = -e\left(\vec{E} + \vec{v}\times\vec{B}\right) \quad (6)$$

where $\vec{p} = \gamma m_0 \vec{v}$ represents the momentum, $\vec{v}$ is the electron velocity, and γ is the Lorentz factor. At first, let us see the case of on-axis acceleration. According to the above field expressions, only the longitudinal electric field exists at ρ =0, while the others are cancelled out . For a slits-truncated Bessel beam, the longitudinal field could be rewritten as

$$E_z = E_0 |A(0,z)| \sin(\eta) \quad (7)$$

where $A(0, z)$ is the combination of Eqs. (3) and (4). However, the numerical calculation of $A(0, z)$ was proven to be terrible time-consuming, leading to inconvenience in processing phase-matching optimization. In order to simplify computation, we take a function ε (z), which is in the form of χ (z), to substitute $A(0, z)$, and we therefore have

$$E_z \cong E_0 \varepsilon(z)\sin(\eta) \quad (8)$$

This approximation will not damage the essential of this acceleration approach, but only affects the details in determining slits' parameters. The concrete form of ε (z) concerning a given initial energy of an electron will be optimized and figured out as one of simulation results.

Apparently, the present approach inclines to accelerating the relativistic electrons. The static or slow electrons cannot make effective interaction with the accelerating field holding a phase velocity larger than the light speed. As an example, we assume an electron with initial energy of 150 MeV, only having axial velocity, is injected together with the laser Bessel beam by an appropriate phase at z=0. The angle α is supposed to be 0.2 degree, in order to get lower phase velocity. We choose the peak amplitude of field E_0=10^{10} *V/m*, resulting in that ~100 TW power should be contained in the central lobe of Bessel beam. After carefully optimizing the lengths of "laser-on" and "laser-off" regions, a three-stage acceleration scheme is shown in Fig. 3. From Fig. 3 (a) we know that the electron is accelerated from 150 MeV to more than 1 GeV just over ~90 cm distance. The

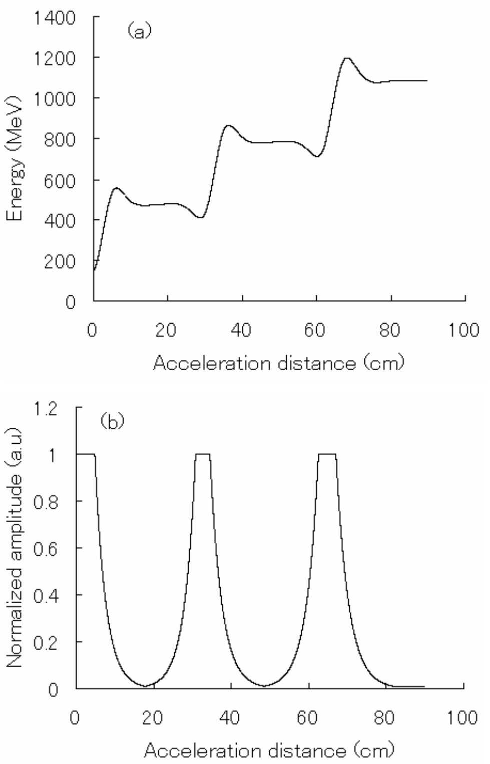

Fig. 3 Three-stage acceleration mechanism. (a) total energy of the accelerated electron shown as a function of distance, (b) corresponding ε (z) function

corresponding ε (z) function is given by Fig.3 (b), from which the slits parameters could be roughly deduced.

The radiation damping effect is not involved in above computation. Making use of the Larmor's formula

$$P = \left(2r_e/3m_0c\right)\left[\left(d\vec{p}/d\tau\right)^2 - \left(1/c^2\right)\left(d\varepsilon/d\tau\right)^2\right]$$

the instantaneous radiated power from an accelerated electron is calculable. Here r_e is the classical radius of the electron, $d\tau = dt/\gamma$, ε indicates the electron energy. For the case of Fig. 3, the total radiated energy is ~0.042 eV, which means the radiation influence is unimportant by the parameters treated in this paper.

Actually, by such an approach, acceleration with more stages is possible to reach. The maximum number of acceleration stages relies on the laser pulse length, the realizable Bessel beam propagation distance, and the technology on manufacturing annular slits.

OFF-AXIS INJECTION

Only a small fraction of the electrons in a bunch of transverse dimensions of a few microns will be interact with laser field with precisely zero initial transverse coordinates. The rest will enter the interaction region with transverse distance from the axis. The field components E_ρ and B_ϕ lead to radial force, which probably induces transverse spreading of electron bunch during acceleration. To understand this effect, we conducted computation for off-axis injection.

We consider injecting 11 electrons with different initial radial positions: the first one is on-axis, while the others are uniformly distributed along radial direction with an interval of 1 μm. These electrons are orderly numbered 0,1,2…10, and they have the same initial energy of 150 MeV, only with the longitudinal velocity, while the initial transverse velocity components are set to zero. The $\varepsilon(z)$ function shown in Fig. 3 (b) is adopted in this calculation, i.e., the on-axis electron is chosen for reference. The laser parameters are the same as used above.

After numerically solving the motion equation with full fields, we got the results as shown in Fig. 4. Note that the final energies of off-axis injecting electrons are lower than that of on-axis injecting one, and this therefore raises energy spread. For the case of Fig. 4a, the maximum energy deviation is ~92 MeV. Electron trajectories are depicted in Fig. 4b, which illustrates that the radius of electron bunch spreads from the original 10 μm to the final 60 μm. The outer electrons receive slighter acceleration than the inner ones, which is due to the transverse distribution of the longitudinal electric field, described by the zero-order Bessel function.

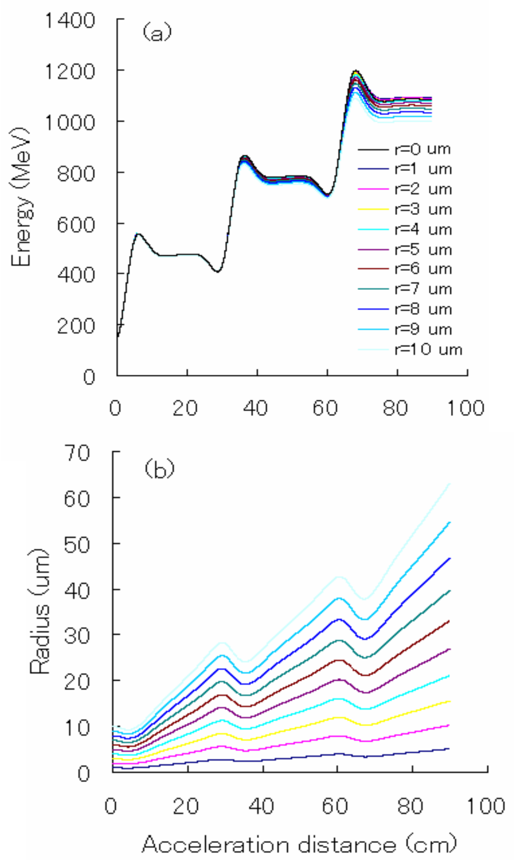

Fig. 4 Acceleration of off –axis injection electrons. (a) energy evolution, and (b) electrons' trajectories.

CONCLUSION

We proposed a novel method for laser-driven acceleration. Annular slits-truncated Bessel beam forms some special regions, where the accelerated particles are allowed to "avoid" deceleration when they slide in the decelerating phase of wave. We analysed the propagation properties of annular slits-truncated Bessel laser beam and demonstrated the acceleration mechanism by numerically solving the motion equation. The results show this method considerable potential in laser-driven acceleration.

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