WIGGLER EFFECTS ON THE GROWTH RATE OF A RAMAN FREE-ELECTRON LASER WITH AXIAL MAGNETIC FIELD OR ION-CHANNEL GUIDING

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Abstract

A relativistic theory for Raman backscatteing in the beam frame of electrons is used to find the growth rate of a free-electron laser (FEL), in the Raman regime. First, a one dimensional helical wiggler and an axial magnetic field are considered. The wiggler effects on the linear dispersion relations of the space-charge wave and radiation are included in the analysis. A numerical computation is conducted to study the the growth rate of the excited waves. It was found that the wiggler effects on the growth rate decreases the growth rate on both group I and group II orbits. Next, the growth rate under an ion-channel guiding, instead of an axial magnetic field, under similar condition is calculated and is studied numerically.

INTRODUCTION

Lab-frame analysis of a FEL with a one dimensional helical wiggler and an axial magnetic field was first presented by Kwan and Dawson [1] using fluid model. In linearizing the relativistic factor, they included the energy exchange between the beam and the space-charge wave but neglected the exchange of energy between the beam and the radiation. The fully relativistic treatment of this problem, in the lab-frame, was first presented by Bernstein and Friedland [2] but they did not analyze the wiggler effect on the dispersion relations of the growing waves. Mehdian *et al*. [3] derived a nonlinear dispersion relation, with the wiggler effects on the dispersion relation of the excited waves included, but they did not find the growth rate. Ion-channel guiding of the electron beam in a FEL has been proposed as an alternative to guiding by a solenoid (or quadrupole) magnetic field [4]. This type of guiding involves the formation of a positive ion core by expulsion of electrons from a preionized plasma channel into which the electron beam is injected. Jha and Kumar [5] have calculated the growth rate with the higher order relativistic terms neglected.

BEAM-FRAME ANALYSIS WITH AXIAL MAGNETIC FIELD

A relativistic and cold electron beam is passed through a uniform static axial guide magnetic field, and a static helical (wiggler) magnetic field, which is periodic along the guide axis. In the beam frame of reference, the wiggler field comprises a propagating electromagnetic (pump) wave, which undergoes stimulated Raman backscattering. This process is characterized by the parametric decay of the pump wave (ω_1, k_1) into a forward-scattered space-charge wave (ω_2, k_2) and a backscattered electromagnetic wave (ω_3, k_3). The transverse and longitudinal components of the velocity are treated as relativistic. The unperturbed state in the beam frame is characterized by $n^{(0)} = n_0/\gamma_{||}$, and

$$\mathbf{E}^{(0)} = \frac{-\gamma_{||} v_{||} B_w}{2c}(\widehat{\mathbf{x}} - i\widehat{\mathbf{y}})exp[i(k_1 z + \omega_1 t)] + c.c., \tag{1}$$

$$\mathbf{B}^{(0)} = \Big(\frac{-i\gamma_{||} B_w}{2}(\widehat{\mathbf{x}} - i\widehat{\mathbf{y}})exp[i(k_1 z + \omega_1 t)] + c.c.\Big) + \widehat{z} B_0, \tag{2}$$

$$\mathbf{V}^{(0)} = \frac{i\gamma_{||} v_w}{2}(\widehat{\mathbf{x}} - i\widehat{\mathbf{y}})exp[i(k_1 z + \omega_1 t)] + c.c., \tag{3}$$

$$v_w = \frac{\Omega_{wL} v_{||}}{(\Omega_{0L} - \gamma_0 k_w v_{||})}.$$

Perturbation composed of a longitudinal plasma wave (ω_2, k_2) and a transverse backscattered electromagnetic wave (ω_3, k_3) are considered. The frequencies and wave numbers satisfy the phase matching conditions, $\omega_3 = \omega_2 - \omega_1$ and $k_3 = k_2 - k_1$. These waves are assumed to vary as $exp[i(k_2 z + \omega_2 t)]$, and $exp[i(k_3 z + \omega_3 t)]$. The amplitudes of the longitudinal space-charge wave are E_2, V_2, and n_2 and the x and y components of the radiation are E_3, B_3, and V_3.

The relativistic momentum equation in the beam frame

$$m\gamma d\mathbf{V}/dt + m\mathbf{V}d\gamma/dt = -e\mathbf{E} - ec^{-1}\mathbf{V}\times\mathbf{B}, \tag{4}$$

with $d\gamma/dt = -e/(mc^2)\mathbf{V}.\mathbf{E}$ and with the use of the linearized relativistic factor, can be linearized as follows

$$m\gamma_0\gamma_{||}^{-1}\big[\frac{\partial \mathbf{V}^{(1)}}{\partial t} + \big(\mathbf{V}^{(0)}.\nabla\mathbf{V}^{(1)} + \mathbf{V}^{(1)}.\nabla\mathbf{V}^{(0)}\big)\big]$$
$$+mc^{-2}\gamma_0^3\gamma_{||}^{-3}\big(\frac{\partial \mathbf{V}^{(0)}}{\partial t}\mathbf{V}^{(0)}.\mathbf{V}^{(1)}$$

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$$+\mathbf{V}^{(0)}.\mathbf{V}^{(1)}\mathbf{V}^{(0)}.\nabla\mathbf{V}^{(0)}\big) - ec^{-2}\big(\mathbf{V}^{(0)}\mathbf{V}^{(0)}.\mathbf{E}^{(1)}$$
$$+\mathbf{V}^{(0)}\mathbf{E}^{(0)}.\mathbf{V}^{(1)} + \mathbf{V}^{(0)}.\mathbf{E}^{(0)}\mathbf{V}^{(1)}\big) = -e\big(\mathbf{E}^{(1)}$$
$$+c^{-1}\mathbf{V}^{(0)} \times \mathbf{B}^{(1)} + c^{-1}\mathbf{V}^{(1)} \times \mathbf{B}^{(0)}\big). \quad (5)$$

After substituting the first order quantities in Eq. (5), and making use of the phase matching-conditions in the coupling terms, the terms corresponding to (ω_2, k_2) and (ω_3, k_3) phases will give equations for the longitudinal space-charge wave and the transverse wave (radiation), respectively. The fluid-Maxwell equations will be

$$m\gamma_0\gamma_{||}^{-1}\omega_2 V_2 = ieE_2 - ec^{-1}\gamma_{||}V_w B_3 + ec^{-1}\gamma_{||}B_w V_3 \quad (6)$$

$$m\gamma_0\gamma_{||}^{-1}\big(2\omega_3 V_3 - k_1\gamma_{||}V_w V_2\big) - mc^{-2}\omega_1\gamma_0^3\gamma_{||}^{-1}$$
$$\times V_w^2 V_3 - ec^{-2}\big(\gamma_{||}^2 V_w^2 E_3 - c^{-1}\gamma_{||}^2 V_{||} V_w B_w V_3\big)$$
$$= -2eE_3 - 2ec^{-1}B_0 V_3 + e\gamma_{||}B_w V_2, \quad (7)$$

$$\omega_2 n_2 + k_2 n_0 V_2 = 0, \quad (8)$$

$$4\pi e n_0 V_2 = i\omega_2 E_2, \quad (9)$$

$$k_3 c E_3 = -\omega_3 B_3, \quad (10)$$

$$-k_3 c B_3 = 4\pi e n_0 V_3 - 2\pi e\gamma_{||}V_w n_2 + \omega_3 E_3. \quad (11)$$

Equations (6)-(11) form a system of linear homogenious algebraic equations. The necessary and sufficient condition for a nontrivial solution is the dispersion relation

$$\Big(\omega_2^2 - \omega_p^2\Phi\Big)\Big(k_3^2c^2 - \omega_3^2 + \frac{\omega_p^2\omega_3}{\omega_c+\omega_3} + \Psi\Big)$$
$$= -\frac{\omega_w^2 k_2\omega_1\omega_p^2}{2k_w^2\Big(\omega_1-\omega_c\Big)}\Big[\frac{k_3\omega_1}{\omega_1-\omega_c} + \frac{k_1\omega_3(1-a_1)}{\omega_c+\omega_3-a_1a_2}\Big]$$
$$\times\Big[1 - \frac{k_1\omega_2\omega_c}{k_2\omega_1\big(\omega_c+\omega_3\big)} - \frac{a_1a_2}{\omega_c+\omega_3}\Big]. \quad (12)$$

The wiggler effects on the linear dispersion relations of the space-charge wave and radiation are contained in Φ and Ψ, respectively. For zero wiggler, they reduce to an ordinary longitudinal plasma oscillations and a transverse electromagnetic wave in a magnetized plasma. If coupling to the radiation is removed Φ becomes unity and the wiggler has no effect on the space-charge wave. On the other hand, if coupling to the space-charge wave is removed Ψ becomes nonzero, which shows the direct effect of wiggler on the radiation.

The real parts of the frequencies and wave numbers satisfy the linear dispersion relations for the space-charge wave and radiation, respectively, with all of the effects of the wiggler and guide magnetic fields included. The imaginary parts of the complex frequencies and wave numbers of the growing waves may be expressed in terms of the lab-frame growth rate Δ_L. This will give the lab-frame spatial growth rate of the space-charge wave and radiation with the lab-frame temporal growth rate taken to be zero;

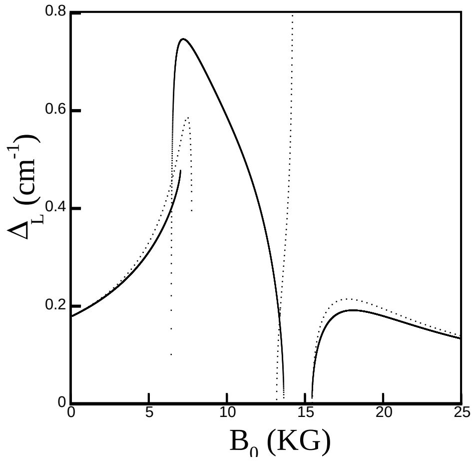

Figure 1: Lab-frame spatial growth rate Δ_L as a function of axial magnetic field B_0. With wiggler effects (solid line); without wiggler effects (dotted line).

$$\Delta_L^2 = \frac{\omega_w^2 k_2\omega_p^2}{8k\gamma_{||}\Big(\omega_1-\omega_c\Big)}\Big[\frac{k_3\omega_1}{\omega_1-\omega_c} + \frac{k_1\omega_3(1-a_1)}{\omega_c+\omega_3-a_1a_2}\Big]$$
$$\times\Big[1 - \frac{k_1\omega_2\omega_c}{k_2\omega_1\big(\omega_c+\omega_3\big)} - \frac{a_1a_2}{\omega_c+\omega_3}\Big]\Big[k_3c^2 - v_{||}\omega_3$$
$$+\frac{\omega_p^2 v_{||}\omega_c(1-a_1)}{2\big(\omega_c+\omega_3\big)^2} - \frac{a_1a_2k_3c^2}{\omega_c+\omega_3} + \frac{a_1a_2\omega_c\omega_c v_{||}}{2(\omega_c+\omega_3)^2}$$
$$+\frac{a_1a_2v_{||}(k_3^2c^2+\omega_3^2)}{2(\omega_c+\omega_3)^2}\Big]^{-1}\Big[\omega_2 - \big(k_1^2c^2a_1\omega_c(\omega_c$$
$$-\omega_1)\big)\frac{(\omega_c-\omega_1-a_1a_2)}{2\omega_1^2(\omega_3+\omega_c-a_1a_2)^2}\Big]^{-1}. \quad (13)$$

The wiggler effects, through Φ and Ψ, on Δ_L are shown in Fig. 1 (solid lines). Dotted lines show Δ_L when the wiggler effects are neglected. Lab-frame values for the unperturbed electron density, wiggler wavelength(period), and Lorentz factor were taken to be $n_0 = 10^{12}\ cm^{-3}$, $\lambda_w = 2\ cm$, and $\gamma_0 = 2.5$, respectively. The wiggler is assumed to be $B_w = 1500\ G$. It can be observed that the wiggler effects on the growth rate decreases the growth rate on both group I and group II orbits.

FEL WITH ION-CHANNEL GUIDING

As an alternative to guiding by an axial magnetic field, focusing of the electron beam can be accomplished by an

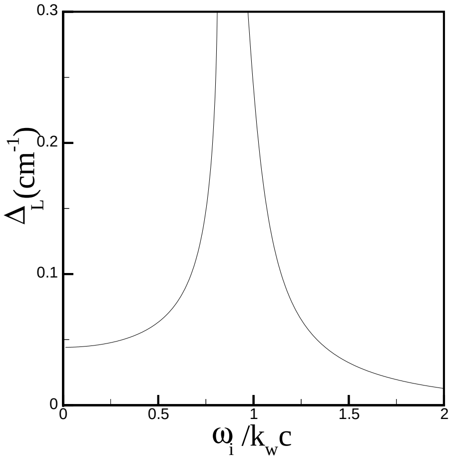

Figure 2: Lab-frame spatial growth rate Δ_L as a function of ion-channel frequency ω_i/k_wc with wiggler effects included.

ion-channel. The transverse electrostatic field generated by the ion-channel is

$$\mathbf{E}_i = 2\pi e n_i\left(\widehat{\mathbf{x}}x + \widehat{\mathbf{y}}y\right), \tag{14}$$

and the wiggler induced velocity will be given by Eq. (3) with v_w given by

$$v_w = \frac{\Omega_{wL}k_w v_{||}^2}{(\omega_i^2 - \gamma k_w^2 v_{||}^2)}, \tag{15}$$

where $\omega_i^2 = 2\pi n_i e^2/m$ is the betatron frequency squared. With similar procedure as in the axial magnetic field case, the dispersion relation can be found as

$$\begin{aligned}&\left(\omega^2 - \omega_p^2\Phi\right)\left(k_3^2c_3^2 - \omega_3^2 + \omega_p^2 + \Psi\right)\\ &\quad= \frac{-k_2k_w^2\omega_w^2\omega_p^2 v_{||}^3}{2\left(\omega_i^2 - k_w^2v_{||}^2\right)^2}\left[k_3v_{||} - \frac{\omega_3(1-a_1)\left(\omega_i^2 - k_1^2v_{||}^2\right)}{k_1v_{||}\left(\omega_3 - a_1a_2\right)}\right]\\ &\quad\times\left[1 - \frac{a_1a_2}{\omega_3} - \frac{\omega_2\omega_1^2}{k_1k_2\omega_3v_{||}^2}\right].\end{aligned} \tag{16}$$

Expressing imaginary parts of the complex frequencies and wave numbers of the growing waves in terms of the lab-frame growth rate Δ_L will give the lab-frame spatial growth rate of the space-charge wave and radiation

$$\begin{aligned}\Delta_L^2 &= \frac{k_2k_w^2\omega_w^2\omega_p^2v_{||}^3}{8\gamma_{||}^2\omega_2\left(\omega_i^2 - k_w^2v_{||}^2\right)^2}\left[1 - \frac{a_1a_2}{\omega_3} - \frac{\omega_2\omega_1^2}{k_1k_2\omega_3v_{||}^2}\right]\\ &\times\left[k_3v_{||} - \frac{\omega_3(1-a_1)\left(\omega_i^2 - k_1^2v_{||}^2\right)}{k_1v_{||}\left(\omega_3 - a_1a_2\right)}\right]\left[\omega_2 + a_1\omega_i^2c^2\right.\\ &\left.\times\frac{\left(\omega_i^2 - k_1^2v_{||}^2\right)\left(\omega_2 + a_1a_2\right)}{2k_1^2v_{||}^4\left(\omega_3^2 - a_1a_2\right)^2}\right]^{-1}\left[k_3c^2 - \omega_3v_{||}\right.\\ &\left.+\frac{a_1a_2v_{||}}{2\omega_3^2}\left(\omega_2 + a_1a_2\right)\right]^{-1}.\end{aligned} \tag{17}$$

Variation of the growth rate Δ_L with the ion-channel guiding frequency ω_i/k_wc is shown in Fig. 2 with wiggler effects on the excited waves included. Growth rate in both group I and group II orbits increase sharply as the resonance is approached.

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