PARAMETER ANALYSIS FOR A HIGH-GAIN HARMONIC GENERATION FEL BY NUMERICAL CALCULATION BASED ON 1D THEORY

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Abstract

High-gain harmonic generation (HGHG) free-electron laser (FEL) is an important candidate for fourth-generation light source. Lots of research works about it have been done all along. Recently a further 1D theory about HGHG FEL has been developed. It considers the effects of different parameters for the whole process. An initial program based on this theory has been made. In this paper, a brief compare of the results from this 1D program and from TDA (3D code) is discussed and it also analyses the parameters for Shanghai deep ultra violate free electron laser source (SDUV-FEL), including electron beam energy spread, seed laser power, strength of dispersion section etc.

INTRODUCTION

The high-gain harmonic generation (HGHG) scheme is one of leading candidates for VUV to X-ray FELs. Ordinarily, a modified TDA3D code, which takes into account the effect of dispersive section, is used to simulate the process of HGHG-FEL.

Recently, a 1D theory about HGHG-FEL has been developed [1]. Different from the previous theories, it accounts for the energy modulation and density modulation (bunching) for a whole process. In this paper, we present some analysis about the choice of seed laser power and dispersive strength based on it.

BRIEF REVIEW OF THE 1D THEORY

The 1D paraxial optical field equation and the election phase equation are:

$$\frac{d\tilde{a}_s}{dz} = \lambda_s r_e a_u \delta_p n_e \left\langle \frac{e^{-i\phi}}{\gamma} \right\rangle \tag{1}$$

$$\frac{d^2\phi}{dz^2} = -\frac{2k_u k_s a_u \delta_p}{\gamma^2} \mathrm{Re}(\tilde{a}_s e^{i\phi}) \tag{2}$$

The symbols in the equations express the common meaning. Especially, δ_p is the polarization modify factor: for circularly polarized helical undulator $\delta_p=1$; for linearly polarized planar undulator with even nth harmonic radiation $\delta_p=0$, and with odd nth harmonic $\delta_p=[J,J]_n$.

As well known, the process of HGHG-FEL is the three steps: the electron beam first pass an undulator (modulator) with a resonant seed laser for energy modulation. Next, the beam travels through a dispersive section, to form spatial bunch. Finally the beam enters a second undulator (radiator), which is tuned in resonance to the harmonic wavelength of seed laser, to achieve coherent radiation at this higher. The pondermotive phase of electron in the second undulator can be calculated by the equation below[1]:

$$\phi_2 = n\phi_{10} + \phi_{10}'[n(l_1 + N_d\lambda_{u1}) + z_2\lambda_{u1}/\lambda_{u2}]$$
$$-2k_{u1}k_{s1}a_{u1}\delta_{p1}\,\mathrm{Re}\int_0^{l_1}[n(l_1 - z_1 + N_d\lambda_{u1}) + z_2\lambda_{u1}/\lambda_{u2}]\frac{\tilde{a}_{s1}e^{i\phi_1}}{\gamma^2}dz_1 + \Delta\phi_2 \quad \text{---------------- (3)}$$

where ϕ_{10} and ϕ_{10}' is the initial phase and phase velocity; k_{s1} and a_{s1} are the wave number and the dimensionless vector potential of the seed laser field (*rms*) , respectively; z_1 , z_2 are the coordinate in the two section undulator; $\Delta\Phi_2$ is phase variation due to the interaction with the radiation field (a_{s2}) in the radiator:

$$\Delta\phi_2 = -2k_{s2}k_{u2}a_{u2}\delta_{p2}\,\mathrm{Re}\int_0^{z_2}(z_2 - z_2')\frac{\tilde{a}_{s2}e^{i\phi_2}}{\gamma^2}dz_2' \tag{4}$$

From equations (1)-(4), the optical field evolution equation for linear region can be given by:

$$\begin{aligned}\frac{d\tilde{a}_{s2}}{dz_2} = \frac{8k_{u2}^2\gamma^2\rho_2^3}{k_{s2}a_{u2}\delta_{p2}} &\left\langle e^{-i[n\phi_{10}'(\frac{l_1}{2}+N_d\lambda_{u1})+\phi_{02}'z_2]} i^n J_n(n\Delta\xi)\right\rangle \\ &-(2k_{u2}\rho_2)^3\left\langle \frac{\partial}{\partial\phi_{02}'}\int_0^{z_2}J_0(2k_{u2}(z_2-z_2')\frac{\Delta\gamma_m}{\gamma}) \right. \\ &\left. \times a_{s2}e^{-i\phi_{02}'(z_2-z_2')}dz_2'\right\rangle \\ &+(2k_{u2}\rho_2)^3\left\langle ie^{-i2n\phi_{10}'(\frac{l_1}{2}+N_d\lambda_{u1})}\int_0^{z_2}(z_2-z_2') \right. \\ &\left. \times J_{2n}(n(\Delta\xi+\Delta\xi'))\tilde{a}_{s2}e^{-i\phi_{02}'(z_2+z_2')}dz_2'\right\rangle\end{aligned} \tag{5}$$

Where

$$\Delta\xi = 4\pi(\frac{N_1}{2} + N_d + \frac{z_2}{n\lambda_{u2}})\frac{\Delta\gamma_m}{\gamma} \tag{6}$$

$\Delta\gamma_m/\gamma$ is the energy modulation induced by interacting with the seed laser:

$$\frac{\Delta\gamma_m}{\gamma} = 4\pi N_1 \frac{a_{u1}\delta_{p1}a_{s1}}{(1+a_{u1}{}^2)} \tag{7}$$

The radiation power in the radiator can be described by eqs (1)-(4). But from the equations, analytical results are difficult to get without various approximations and simplifications. Therefore, a program is written to give us a numerical calculation.

The angular bracket in eq (1) represents the average over the electron's initial phase (Φ_0) and initial phase velocities (Φ_0'). The program calculates it by means of numerical integration. We assume Φ_0 has a uniform distribution and Φ_0' has a Gauss distribution (Gaussian beam). The integrate region of Φ_0' is set between $-6\sigma \sim 6\sigma$, σ is the standard deviation. The seed laser is assumed to be a Gaussian wave.

RESULT COMPARE WITH TDA3D

First, to check the program, we make a comparison between the results from the program and from the simulation code TDA3D. Three set of parameters are used in this comparison, including the Shanghai deep ultraviolet FEL source (SDUV) [2], the Accelerator Test Facility at BNL [3] and the Sincrotrone Trieste (ELETTRA) [4].

To simplify the program, we directly use a numerical integration for the process of average Φ_0 and Φ_0'. The energy spread is taken into account but the emittance is ignored to save computation time. To compare the results under same condition, we set the emittance a very small value for TDA3D.

From Figs1-3, the computation by our program gives a good agreement with the simulation by the code TDA3D.

Table 1: Parameters for SDUV

Electron beam parameters for HGHG:
γ:538.117 Peak current : 400*A*
Energy spread : 0.05%
Seed laser beam parameters:
Wavelength: 266*nm*
Input seed power: 16*MW*
Rayleigh range: 0.8*m*
Magnet parameters:
Modulative section
Length : 0.98*m*
Undulator period: 3.5*cm*
Number of periods: 28
Peak magnetic field: 0.8*T*
Dispersive section
Induced dispersion : 0.75 (dψ/dγ)
Radiative section
Undulator Period: 2.5*cm*
Peak magnetic: 0.62*T*

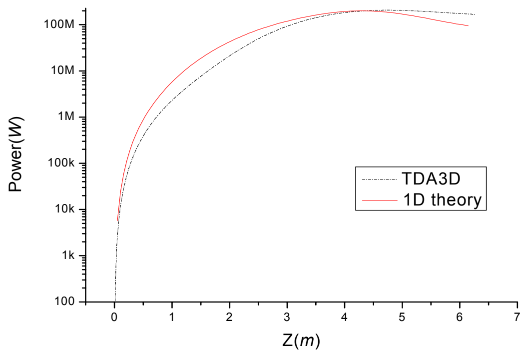

Fig1 Results of the program and TDA3D for SDUV

Table 2: Parameters for the BNL FEL

Electron beam parameters for HGHG:	
γ	82
Peak current	110*A*
Energy spread	0.043%
Seed laser beam parameters:	
Wavelength	10.6*μm*
Input seed power	0.7*MW*
Rayleigh range	0.76*m*
Magnet parameters:	
Modulative section	
Length	0.684*m*
Undulator period	7.2*cm*
Number of periods	9
Peak magnetic field	0.202*T*
Dispersive section	
Induced dispersion	1.5(dψ/dγ)
Radiative section	
Undulator Period	3.3*cm*
Peak magnetic	0.494*T*

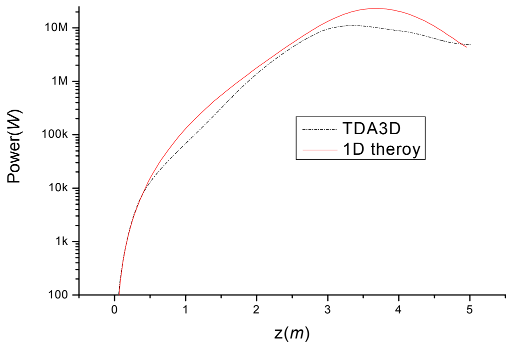

Fig2 Results of the program and TDA3D for the BNL FEL

Table 3: Parameters from ELETTRA

Electron beam parameters for HGHG:	
γ	1956.947
Peak current	600*A*
Energy spread	0.05%
Seed laser beam parameters:	
Wavelength	200*nm*
Input seed power	150*MW*
Rayleigh range	3*m*
Magnet parameters:	
Modulative section	
Length	2.964*m*
Undulator period	5.7*cm*
Number of periods	52
Peak magnetic field	1.35*T*
Dispersive section	
Induced dispersion	0.4($d\psi/d\gamma$)
Radiative section	
Undulator Period	4*cm*
Peak magnetic	0.977*T*

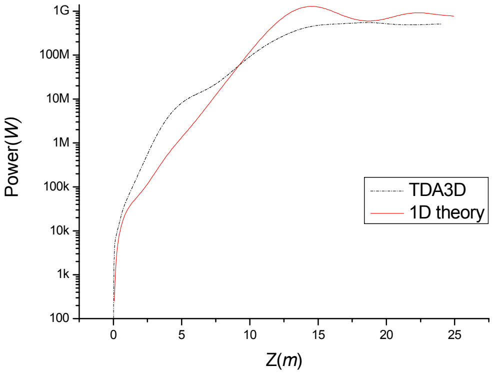

Fig3 Results of the program and TDA3D for ELETTRA

PARAMETER OPTIMIZING FOR SDUV

The HGHG-FEL experiment involves lots of parameters, which are relevant to the magnet field, the electron beam and the seed laser. Among them only few ones are tuneable during the experiment. The seed laser power and the strength of dispersive section are most important of all.

Energy modulation term $\Delta\gamma_m/\gamma$ is relevant to the seed laser power, it represents an additional energy spread and should be less than the Piece parameter. This gives an upper limit for the seed laser power. For the case considered here, it is about 50*MW*.

Using the parameters listed in Table 1, we calculate the variation of the saturation length and the saturation power with seed laser for different N_d. The results are given in Fig4 and Fig5.

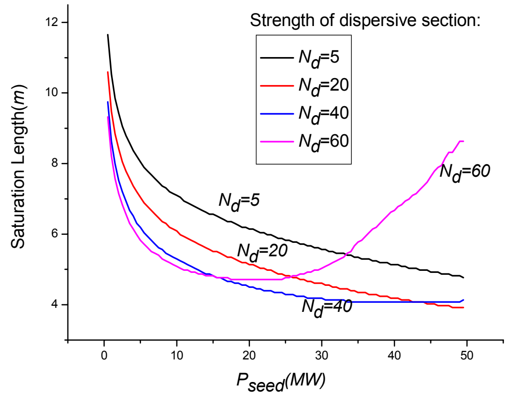

Fig4 Saturation length as function of seed laser power for different N_d

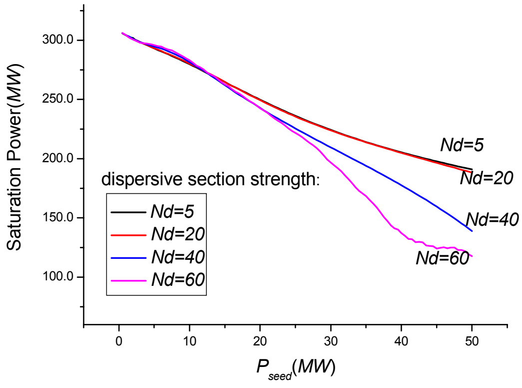

Fig5 Saturation energy as function of seed laser power for different N_d

From Fig 4, with increase of the input power, the saturation length drops dramatically initially for low input power and then the descending speed slows down. For a given input power, the saturation length also drops off with increase of N_d. But when N_d becomes sufficiently large, the saturation length ascends instead of dropping. For N_d = 60, saturation length has minimum value when the seed laser power is about 20*MW*.

Fig5 shows, as seed laser power increases, the saturation power decreases. For a given laser power less than 15*MW*, the saturation power has little difference for different N_d.

Therefore, we choose 15*MW* seed laser power and dispersive strength N_d=40 for SDUV. To illustrate this choice clearly, we give the power development along radiator with seed laser to be 15*MW* for different N_d (Fig6) and the power development with N_d=40 for different seed laser power (Fig7) respectively:

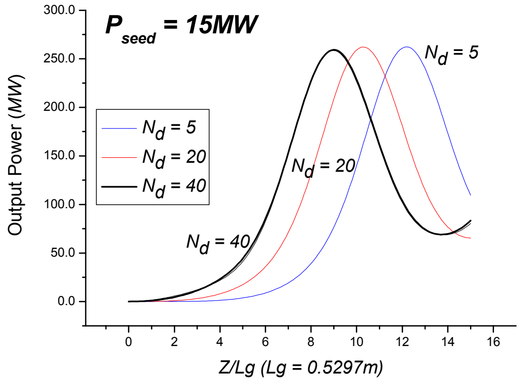

Fig6 Power development along radiator for different N_d (P_{seed} =*15MW*)

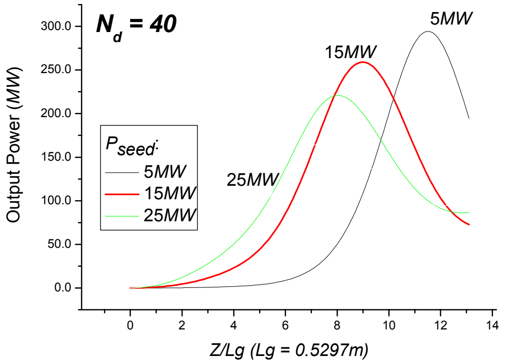

Fig7 Power development along radiator for different seed laser power (N_d=40)

As mentioned above, for N_d = 60, the saturation length reaches a minimum value when laser power is about 20*MW* (Fig4), Here we make some analysis for the phenomena.

In the right hand side of linear approximate equation (5), the first term denotes the coherent enhancement process, the Bessel function $J_n(n\Delta\xi)$ is important to it; the second term denotes the usual exponential gain process and the energy modulation $\Delta\gamma_m/\gamma$ affects it greatly; the contribution of the third term is small, can be neglect. Therefore, $J_n(n\Delta\xi)$ and $\Delta\gamma_m/\gamma$ should be emphasized to this problem.

It has been mentioned that we constrain the seed laser less than 50*MW* to guarantee the energy modulation $\Delta\gamma_m/\gamma$ less than the Piece parameter (ρ=0.217%). Therefore the additional energy spread induced by energy modulation is relatively small (at 20*MW* seed laser, $\Delta\gamma_m/\gamma$ =0.14%) that it shouldn't be the leading reason for the increase of saturation length. Thus the Bessel function $J_n(n\Delta\xi)$ should be dominant for this phenomena. We calculate the variation of Bessel function $J_n(n\Delta\xi)$ with seed laser power for N_d = 60. $\Delta\xi$ denotes the total dispersion that consists of the contribution from the modulator, the dispersive section, as well as the radiator. So it is function of radiator distance z_2. Because the coherent enhancement process is dominant in the first four gain length of the radiator [5], z_2 in $\Delta\xi$ is approximated with its median over $4L_g$ i.e.2 L_g .

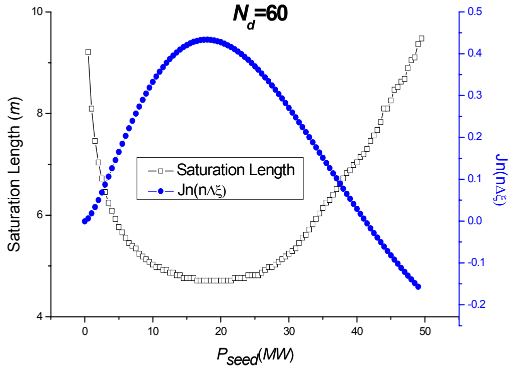

Fig8 Saturation length and $J_n(n\Delta\xi)$ as function of different seed laser power.

From Fig8, we can see that when seed laser power is about 20*MW*, where the saturation length has the minimum value, the Bessel function $J_n(n\Delta\xi)$ just reaches its maximum value. The coincidence verifies our analysis above.

SUMMARIZING

A program based on a 1D HGHG theory is introduced. First, the program is checked with modified code TDA3D. Then the seed laser power and the dispersive strength are optimized for SDUV. Finally the affect of the seed laser power and the dispersive strength to the saturation length and saturation power are analyzed and discussed. For a further work, we can add in the effect of emittance as an equivalent energy spread. And use a Monte Carlo process for the numerical integration to save the computation time.

REFERENCES

[1] Jia Qika,"One-dimensional theory of high gain harmonic generation free electron laser" Nucl.Instr. & Meth. A**519**,pp489-492,2004

[2] Z.T.Zhao *et.al,* "The Shanghai High Gain Harmonic Generation DUV Free Electron Laser" Nucl.Instr. & Meth.A528, p591-594, 2004

[3] S.G.Biedron, H.P.Freund, L.-H. Yu, "Parameter analysis for a high-gain harmonic generation FEL using a recently developed 3D polychriomatic code" Nucl.Instr.&Meth. A445. pp95-100,2000

[4] Li Hua Yu, Juhao Wu, "Simulations for the FERMI@ELETTRA proposal to generate 40- and 10-nm coherent radiation using the HGHG scheme" Nucl.Instr.&Meth. A507. pp455-458,2003

[5] Jia Qika,"harmonic amplifer free electron laser" ,this proceeding