HARMONIC AMPLIFIER FREE ELECTRON LASER

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Abstract

The harmonic optical klystron (HOK) in which the second undulator is resonant on the higher harmonic of the first undulator is analysed as a harmonic amplifier. The optical field evolution equation of the HOK is derived analytically for both CHG mode (Coherent Harmonic Generation, the quadratic gain regime) and HGHG mode (High Gain Harmonic Generation, the exponential gain regime), the effects of energy spread, energy modulation and dispersion in the whole process are considered.

INTRODUCTION

One way of free electron laser (FEL) developing toward short wavelength is using harmonic. In the coherent harmonic generation (CHG) [1,2] scheme an optical klystron(OK) has been used, an external laser pulse is focused into the first undulator, the wavelength of the laser is resonated with the fundamental radiation of the optical klystron, with optimized system parameters the harmonic radiation in the second undulator is coherently enhanced. Analyze shown that if make second undulator of OK resonant on the higher harmonic, namely the wavelength of fundamental radiation of the second undulator matches with *n*th harmonic optical field in the first undulator, it will be more beneficial to the harmonic generation [3]. To distinguish it from the normal optical klystron, we temporarily call such optical klystron the "harmonic optical klystron"(HOK). A similar configuration was proposed and used for high gain harmonic generation (HGHG) [4].the scheme evolved from many earlier ideals (e.g. ref. [5]). In the HGHG mode the optical power grows exponentially while in the CHG mode the optical power grows quadratically, both modes are harmonic amplifier. Cascaded optical klystron [6,7] and cascaded harmonic optical klystron [8-9] for X-ray FEL are also proposed and discussed.

So far the theory of optical klystron amplifier has all approximated it as the scheme with separate functions: the energy modulation only be considered in the first undulator, the dispersive effect only occur in dispersive section, and the gain section generate radiation. In Ref [4] the HGHG problem is solved for the small energy spread limit, in the second undulator the electron beam is assumed be mono-energetic and dispersive effect is ignored. The amplifying process of optical klystron (and harmonic optical klystron) have been analysed mostly by calculating the bunching factor at the entrance of the second undulator (the techniques developed for microwave klystron). But such approximation treatments are not always appropriate. In this paper I derive the optical field evolution equation complete analytically for HOK, the energy spread effect and the dispersive effect in the whole process will be considered in the derivation.

OPTICAL FIELD EVOLUTION EQUATIONS

We use the one-dimensional FEL theory and start from the paraxial optical field equation and the electron phase equation:

$$\frac{d\tilde{a}_s}{dz} = \lambda_s r_e a_u \delta_p n_e \left\langle \frac{e^{-i\phi}}{\gamma} \right\rangle \tag{1}$$

$$\frac{d^2\phi}{dz^2} = -\frac{2k_u k_s a_u \delta_p}{\gamma^2} \mathrm{Re}(\tilde{a}_s e^{i\phi}) \quad , \tag{2}$$

where $\tilde{a}_s = a_s e^{i\varphi_s}$, $a_s=eE_s/(mc^2k_s)$ and $a_u=eB_u/(mc^2k_u)$ are dimensionless vector potential of the *rms* radiation field E_s and undulator field B_u, respectively; $k_s=2\pi/\lambda_s$, $k_u=2\pi/\lambda_u$ are the corresponding wave number; φ_s is the phase of radiation field; $\phi=(k_s+k_u)z-\omega_s t$ is the pondermotive phase of electron, r_e is the classical electron radius; n_e and γ is the density and energy of electrons; the angular bracket represents the average over the electron's initial phases and initial phase velocities. δ_p is the polarization modify factor: for circularly polarized helical undulator $\delta_p=1$; for linearly polarized planar undulator with even *n*th harmonic radiation $\delta_p=0$, and with odd *n*th harmonic $\delta_p=[J,J]_n$,

$$[J,J]_n = (-1)^{\frac{n-1}{2}} [J_{\frac{n-1}{2}}(\frac{na_u^2}{2(1+a_u^2)}) - J_{\frac{n+1}{2}}(\frac{na_u^2}{2(1+a_u^2)})]$$

J is integer order Bessel function.

The electron phase in the second undulator is

$$\phi_2 = \phi_{20} + \phi_{20}' z_2 + \Delta\phi_2 \quad . \tag{3}$$

The first term of the right hand side of eq.(3) is the electron phase at the entrance of the second undulator

$$\phi_{20} = n\phi_1(z_{20}) + (k_{u2} - nk_{u1})z_{20} \quad . \tag{4}$$

The second part in the right hand side of eq.(4) is a constant for all electrons. $\phi_1(z_{20})$ is the electron phase referenced to the first undulator valued at the entrance of the second undulator and given by

$$\phi_1(z_{20}) = \phi_{10} + \phi_{10}' l_1 + \Delta\phi_1 + \Delta\phi_d \; , \tag{5}$$

where ϕ_{10} and ϕ_{10}' is the initial phase and phase velocity (detuning parameter), $\Delta\phi_1$ is the phase change due to interaction with optical field of the seed laser in the first undulator and given from eq.(2)

$$\Delta\phi_1 = -2k_{s1}k_{u1}a_{u1}\delta_{p1}\,\mathrm{Re}\int_0^{l_1}(l_1 - z_1)\frac{\tilde{a}_{s1}e^{i\phi_1}}{\gamma^2}dz_1 . \quad (6)$$

k_{s1} and a_{s1} are the wave number and the dimensionless vector potential of the seed laser field (*rms*) , respectively. $\Delta\phi_d$ is phase change in the dispersive section

$$\Delta\phi_d = \int_0^d k_s(1-\frac{1}{\beta_{II}})dz = -\frac{k_s}{2}\int_0^d (\frac{1}{\gamma^2}+\beta_\perp^2)dz = -\frac{k_s}{2\gamma^2}L_d \quad (7)$$

$$L_d = d + (\frac{e}{mc^2})^2\int_0^d (\int_0^z B_d dz')^2 dz$$

is equivalent drift length, d and B_d are the lengths and magnetic field of the dispersive section, respectively. Using resonant relation of FEL and the phase velocity expression $\phi' \cong k_u(1-\gamma_r^2/\gamma^2)$, eq.(7) can be written as

$$\Delta\phi_d = -\frac{k_u L_d \gamma_r^2}{(1+a_u^2)\gamma^2} = N_d\lambda_u\phi_1'(l_1) - 2\pi N_d \quad (8)$$

$$N_d = \frac{1}{\lambda_u(1+a_u^2)}[d + (\frac{e}{mc^2})^2\int_0^d (\int_0^z B_d dz')^2 dz], \quad (9)$$

where γ_r is the resonant energy, N_d is the dispersive section parameter, it is the scale parameter of optical klystron itself and independent on the electron beam. The physical meaning of N_d is that it is number of wavelengths of the light passing over the resonant electron in the dispersive section. For electrons with different energy N_d also can be expressed as

$$N_d = \frac{\gamma}{4\pi}\frac{\delta\Delta\phi_d}{\delta\gamma} , \quad (10)$$

obviously N_d describe the dispersive strength

The second term of eq.(3) ϕ_{20}' is electron phase velocity at the entrance of the second undulator

$$\phi_{20}' = k_{u2}(1-\frac{\gamma_r^2}{\gamma_{20}^2}) = \frac{k_{u2}}{k_{u1}}\phi_1'(l_1) \quad (11)$$

$$\phi_1'(l_1) = \phi_{10}' + \Delta\phi_1' ,$$

$$\Delta\phi_1' = -2k_{s1}k_{u2}a_{u1}\delta_{p1}\,\mathrm{Re}\int_0^{l_1}\frac{\tilde{a}_{s1}e^{i\phi_1}}{\gamma^2}dz_1 . \quad (12)$$

The third term of eq.(3) is phase variation due to the interaction with the radiation field in the second undulator

$$\Delta\phi_2 = -2k_{s2}k_{u2}a_{u2}\delta_{p2}\,\mathrm{Re}\int_0^{z_2}(z_2 - z_2')\frac{\tilde{a}_{s2}e^{i\phi_2}}{\gamma^2}dz_2' \quad (13)$$

Therefore the electron phase in the second undulator (we drop the constant term) is

$$\phi_2 = n\phi_{10} + \phi_{10}'[n(l_1 + N_d\lambda_{u1}) + z_2\lambda_{u1}/\lambda_{u2}] -$$
$$-2k_{u1}k_{s1}a_{u1}\delta_{p1}\,\mathrm{Re}\int_0^{l_1}[n(l_1 - z_1 + N_d\lambda_{u1}) + z_2\frac{\lambda_{u1}}{\lambda_{u2}}]\frac{\tilde{a}_{s1}e^{i\phi_1}}{\gamma^2}dz_1$$
$$+\Delta\phi_2 \quad (14)$$

The harmonic generation problem of HOK including the electron beam quality effects and dispersive effects for whole process from beginning to saturation can be numerically solved by substituting above expression into eq.(1).

SUPERRADIANCE REGIME(CHG MODE)

Owing to the short length of first section undulator (modulator) the optical field in the modulator is approximately constant. The third term of the right hand side of eq.(14) is phase variation due to the interaction with the seeding optical field. The integral function in the term varied approximately linearly with z_1, so taking its median in the integral is a reasonable approximation

$$-\frac{2k_{u1}k_{s1}a_{u1}\delta_{p1}a_{s1}}{\gamma^2}\int_0^{l_1}[l_1 - z_1 + N_d\lambda_{u1} + \frac{\lambda_{u1}z_2}{n\lambda_{u2}}]\cos(\phi_{10} + \phi_{10}'z_1)dz_1$$
$$\approx n\Delta\xi\cos(\phi_{10} + \phi_{10}'\frac{l_1}{2}) \quad (15)$$

where

$$\Delta\xi = 4\pi(\frac{N_1}{2} + N_d + \frac{z_2}{n\lambda_{u2}})\frac{\Delta\gamma_m}{\gamma} \quad (16)$$

$$\frac{\Delta\gamma_m}{\gamma} = 4\pi N_1\frac{a_{u1}\delta_{p1}a_{s1}}{(1+a_{u1}^2)} . \quad (17)$$

$\Delta\gamma_m/\gamma$ is the maximum energy modulation induced in the first section undulator.

In coherent harmonic generation (CHG) mode, the electron beam current is low and the length of the gain section of HOK (or OK) is short, while the N_d may be very large. It have $L_2 < 3L_g$ (L_g: the power gain length) and $N_d + N_1/2 >> N_2/n \geq z_2/n\lambda_{u2}$. Therefore the z_2 in $\Delta\xi$ can be approximated with its median:

$$\Delta\xi = 4\pi(\frac{N_1}{2} + N_d + \frac{N_2}{2n})\frac{\Delta\gamma_m}{\gamma} ,$$

and the phase variation due to interaction with the radiation field (the last term of the right hand side of eq.(14)) can be neglected. Thus the optical field in CHG mode for HOK is

$$\tilde{a}_{s2} = -\frac{r_e\lambda_{s2}a_{u2}\delta_{p2}n_e}{\gamma}J_n(n\Delta\xi)$$
$$\times\left\langle i^n e^{-i\phi_{10}'\lambda_{u1}[n(\frac{N_1}{2}+N_d)+\frac{N_2}{2})]} l_2\,\mathrm{sin}\,c\frac{\phi_{02}'l_2}{2}\right\rangle \quad (18)$$

For a Gaussian initial energy distribution of the electron beam the corresponding radiation intensity is

$$\tilde{a}_{s2}^2 = (\frac{r_e\lambda_{s2}a_{u2}\delta_{p2}n_e l_2}{\gamma})^2 J_n^2(n\Delta\xi)f_\gamma^2 \quad (19)$$

$$f_\gamma = \exp\{-\frac{1}{2}[4\pi n(N_d + \frac{N_1 + N_2/n}{2})\frac{\sigma_\gamma}{\gamma}]^2\}$$

where $\delta_{p2}=[J,J]_1$. For given energy spread and energy modulation (i.e. given seed laser) we can give the optimal dispersive parameter

$$N_d^{opt.} = \frac{1}{4\pi n}\min[\frac{\sqrt{n}}{\sigma_\gamma/\gamma}, \frac{n+1}{\Delta\gamma_m/\gamma}] - \frac{N_1+N_2/n}{2} \quad (20)$$

If we do following substitution in eq.(19)

$a_{u2}\to a_u$, $\delta_{p2}\to\delta_p=[J,J]_n$, $N_2/n\to N_2$,

then we have the nth harmonic radiation intensity for OK configuration. The advantage of HOK over OK for CHG is obvious: the energy spread effect is reduced, the radiation is also enhanced by proper selecting undulator parameters to make $(a_{u2}[J,J]_1)^2 >> (a_u[J,J]_n)^2$ [6]. Moreover besides the odd harmonic the HOK also can operated at the even harmonic of the seed laser.

EXPONENTIAL GAIN REGIME (HGHG MODE)

For the HGHG mode, the electron beam current is relatively high, the length of the second undulator must be sufficiently long to reach the exponential gain regime: $L_2>3L_g$. Therefore the condition $N_d+N_1/2>>N_2/n$ may not be satisfied. Substituting eq.(14) into eq.(1) and linearizing it, after averaging over a uniform initial phase distribution of electrons, the optical field evolution equation in linear region for HOK is given by

$$\begin{aligned}\frac{d\tilde{a}_{s2}}{dz_2} &= \frac{8k_{u2}^2\gamma^2\rho_2^3}{k_{s2}a_{u2}\delta_{p2}}\left\langle e^{-i[n\phi_{10}'(\frac{l_1}{2}+N_d\lambda_{u1})+\phi_{02}'z_2]} i^n J_n(n\Delta\xi)\right\rangle \\ &-(2k_{u2}\rho_2)^3\left\langle \frac{\partial}{\partial\phi_{02}'}\int_0^{z_2} J_0(2k_{u2}(z_2-z_2')\frac{\Delta\gamma_m}{\gamma}) \times a_{s2}e^{-i\phi_{02}'(z_2-z_2')}dz_2'\right\rangle \\ &+(2k_{u2}\rho_2)^3\left\langle ie^{-i2n\phi_{10}'(\frac{l_1}{2}+N_d\lambda_{u1})}\int_0^{z_2}(z_2-z_2') \times J_{2n}(n(\Delta\xi+\Delta\xi'))\tilde{a}_{s2}e^{-i\phi_{02}'(z_2+z_2')}dz_2'\right\rangle\end{aligned} \quad (21)$$

where ρ is Pierce parameter, ϕ_{02}' is the electron phase velocities (referenced to the second undulator) at the entrance of the first undulator (note it is different with ϕ_{20}'):

$$\phi_{02}' = \frac{k_{u2}}{k_{u1}}\phi_{10}' = k_{u2}(1-\frac{\gamma_r^2}{\gamma_0^2}) \quad (22)$$

In the right hand side of equation (21), the first term correspond to the coherent enhancement process, we can see that the dispersion effect ($\Delta\xi$) and energy-spread effect (the exponential factor) include the contribution not only from dispersive section, the modulation section, but also from gain section. The formula (18) can be obtained from this term. The second term corresponds to the usual gain process, it gives usual gain results when the seed laser is off, the Bessel function in it indicates the effect of the additional energy spread due to energy modulation. The third term contributes small and can be neglected for many cases.

For mono-energetic electron beam and weak modulation we can obtain [10]

$$a_{s2}(z_2)\approx \frac{4k_{u2}\gamma^2\rho_2^2}{3k_{s2}a_{u2}\delta_{p2}}J_n(n\Delta\xi^*)e^{\sqrt{3}k_{u2}\rho_2 z_2} \quad (23)$$

where $\Delta\xi^* = \Delta\xi(z_2^*)$, Z_2^*: $0<Z_2^*<l_2$

$$\int_0^{l_2} J_n(n\Delta\xi(z_2))e^{-i\phi_{02}'z_2}dz_2 = J_n(n\Delta\xi(z_2^*))\int_0^{l_2} e^{-i\phi_{02}'z_2}dz_2$$

DISCUSSION

From eq.(18) and eq.(23) the division of the CHG mode and HGHG mode for mono-energetic electron beam can be estimated: $z_2\approx 3.73L_g$, therefore the length of the second undulator for HGHG mode should be at least four time longer than the power gain length: $N_2>4N_g$. Figure 1 is a numerical result of equation (21) compared with the result given by numerically solving the equation (1) and eq.(14). It shows that the linear approximation is valid from start up to near saturation (linear region). It also shows that the quadratic gain regime (the CHG mode) is for $z_2<4L_g$.

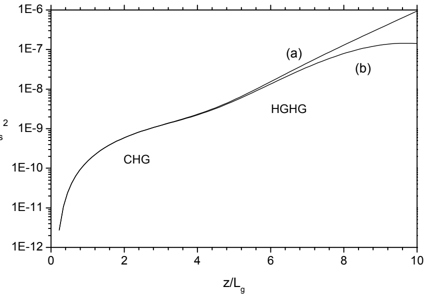

Figure 1, (a) the linear approximation (equation (21))
(b) the result given by numerically solving the equation (1) and (14)

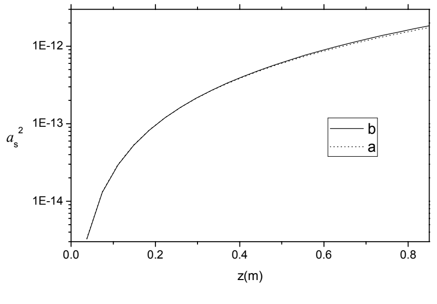

Figure 2, CHG (a) a result of analysis formula (eq. (19))
(b) the result of the linear theory (eq.((21))

Figure2 is a result of analytical formula (equation (19)) compared with the result of the linear theory (the equation (21) for the CHG. It can be seen that the agreement between them is very well.

From eq.(21) we noted that for the linear region the additional energy spread due to energy modulation only affects usual gain term but not the coherent enhancement term. Therefore, for CHG scheme, in which the coherent enhancement term is dominant, one should chose seeding laser filed a_{s1} and despersive field N_d to make $n\Delta\xi \cong n+1$ so that $J_n=J_{n\,max}$, and at the same time a large a_{s1}(strong modulation) and a small N_d (weak dispersion, to reduce the effect of energy spread) are preferred. For HGHG scheme, the additional energy spread effect due to energy modulation must be considered, this gives the up limit for seeding laser filed a_{s1}:

$$\Delta\gamma_m/\gamma < \rho, \quad a_{s1} < \frac{(2+K_1^2)}{4\pi N_1 K_1 \delta_{p1}}\rho \qquad (24)$$

For high harmonic, the optimal $\Delta\xi=(n+1)/n$ not changed much so does the energy modulation $\Delta\gamma_m/\gamma$ and the seeding optical filed (a_{s1}). But as harmonic number increase, the energy spread effect factor and the Bessel function term J_n decrease, both them make the gain degradation. The energy spread factor is more important by comparison. To reduce the energy spread effect we can reduce the dispersive field strength (N_d), but that $J_n(\Delta\xi)$ may also be decreased. The best way is reducing the energy spread itself, this can be achieved by adopting the local (slice) energy spread of electron bunch, namely adopting ultra-short pulse of seeding laser.

In summary we have derived optical field evolution equations complete analytically for harmonic optical klystron. By numerically solving the equation (1) and (14) the harmonic generation problem including the effects of energy spread, energy modulation and the dispersion in whole process can be easily described. Both CHG mode and HGHG mode are analysed as harmonic amplifier. The linear theory is given and analysed for HGHG mode. For CHG mode the analytical formula is given and the advantages of HOK over OK were demonstrated. The optimal parameters for harmonic amplifier are discussed briefly.

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