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Beyond the limits of 1D coherent synchrotron radiation

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The Cockcroft Institute
of Accelerator Science and Technology

bERLin 2019

Background, Motivation and Outline

- Part of ongoing study of CSR and microbunching aimed at benchmarking codes, understanding their limitations and directly comparing to experimental data for electron beam to drive a future industrial EUV-FEL

- Collaboration between STFC, FERMI, Pulsar Physics and ASML Netherlands BV

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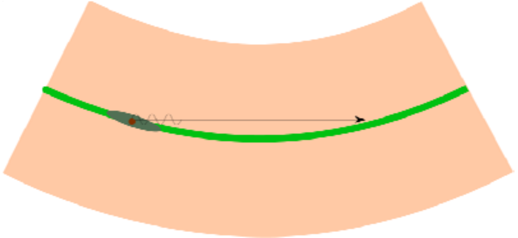
Presentation outline:

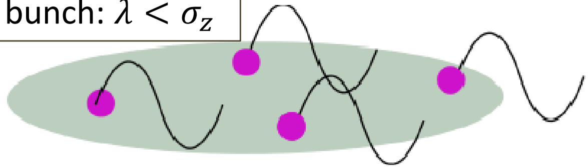
1. Theory of CSR and inclusion of transients
2. Simulation developments
3. Measurements at FERMI and comparison to simulations
4. Recent development of Stupakov 1D model affecting our conclusions
5. Teaser 1: Analysis of natural microbunching instability
6. Teaser 2: Analysis of laser heater beating induced microbunching

Coherent Synchrotron Radiation (CSR)

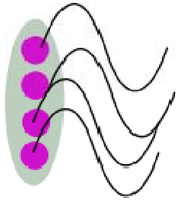


- Particles on a curved trajectory emit synchrotron radiation.
- Radiation due to a bunch of electrons in a dipole causes a wakefield which can interact with particles at the head of the bunch, causing a loss in energy and emittance growth.
- Overtaking length: $L \sim (24\sigma_z R^2)^{1/3}$ - comes from simple geometry comparing curved electron path to straight radiation path
- Power radiated causes a total energy loss in the bunch: $P \propto N^2$ - CSR is a problem for short bunches!

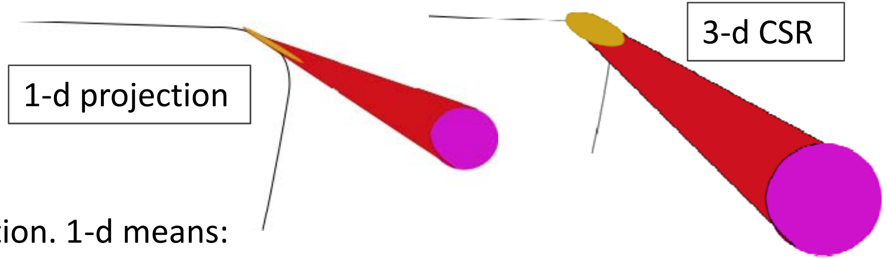
Long bunch: $\lambda < \sigma_z$



Short bunch: $\lambda \gg \sigma_z$



1D Calculations of CSR

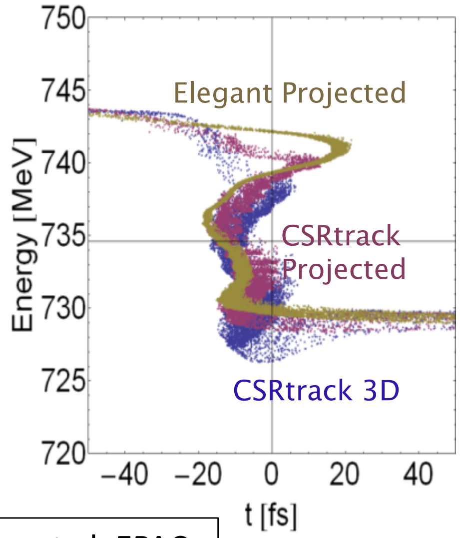


- Many codes simulate CSR energy loss in a “1-d” approximation. 1-d means:
  - The bunch distribution is projected onto the central trajectory
  - Any change in longitudinal distribution due to CSR is neglected
  - Ultrarelativistic approximation made
  - Assumes infinitely long drifts before and after the dipole = no shine from previous elements

- A bunch whose transverse size is large wrt its emitted radiation cone -> possible for CSR to “miss” due to the offset -> overestimate of deleterious effects

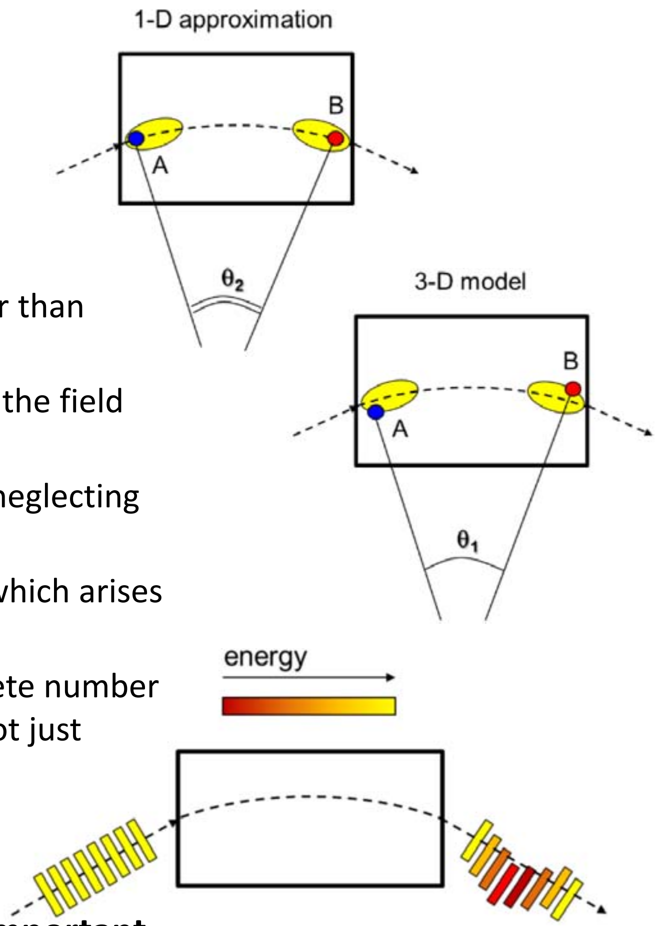
- Need to validate codes and test their region of applicability (particularly 1D), and compare with experiment.

- 1D codes: Elegant, Impact-Z, ……..
- 2D/3D codes: CSRTrack, GPT v3, …



[1] Williams et al, EPAC ‘08, MOPC034 (2008)

“3D” Calculations of CSR

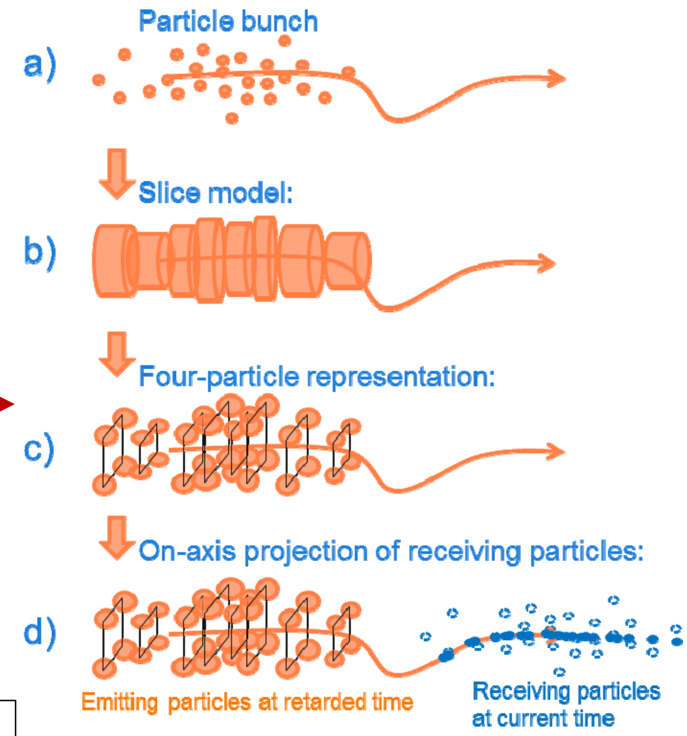


- A **full** 3D simulation of CSR needs to:
  - Account for **both** longitudinal and transverse forces
  - Take the **transverse extent** of the bunch into account **during emission**, rather than assuming that all electrons emit on-axis
  - Take the **transverse extent** of the bunch into account **during interaction**, i.e. the field experienced is a function of x **and** y
  - Self-consistently solve for the **actual trajectory** during emission rather than neglecting deviations from the nominal trajectory = **betatron motion** exists!
  - Take the **full Liénard-Wiechert field** into account rather than only the term which arises during acceleration
  - Include stochastic effects due to the **long-range interaction** between a discrete number of radiation cones = previous elements matter (especially in complex arcs, not just simple chicanes)
  - Allow the **charge density** to evolve along the nominal trajectory.
  - Include **shielding** effects

THIS IS A LOT TO ASK!!! Realistically, the 3-D codes choose which are important

Simulations of CSR

- Elegant: based on 1-d model of Saldin et al [2]. Good agreement with experimental results [8].
- CSRTrack: 2-d and 3-d models based on sub-bunches. Neglects vertical forces. CSR field calculated directly from retarded potentials. Good agreement with experiment [9].
- **GPT v3**: Developed as part of this study. Bunch is sliced longitudinally and coherent force simulated from four transverse points in each slice. Needs benchmarking! Because GPT gives us access to the fields we choose to look at these and compare with analytic formulae as one check in addition to experimental data
- Other codes exist: (e.g. IMPACT, BMAD, CSR3D) – Not used for this study

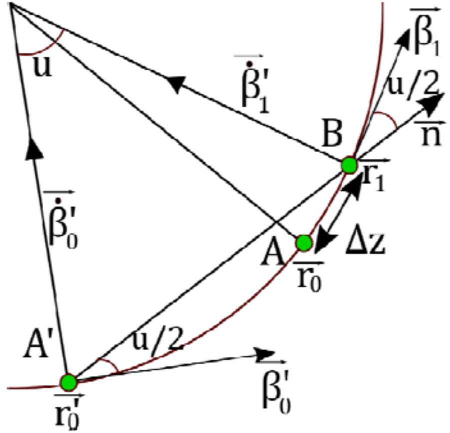


[2] Saldin et al, NIM A 398.373
[8] Bane et al, PRSTAB 12.030704
[9] Bettoni et al, PRSTAB 19.034402

Radiation in a Dipole – 3 Regimes

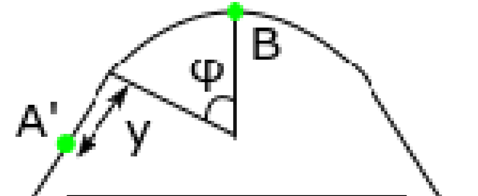
- Begin with the Liénard-Wiechert field between two particles:

$$\underline{E}\left(\underline{r},t\right)=\frac{e}{4\pi\epsilon_0}\left[\underbrace{\frac{\underline{n}-\underline{\beta}'}{\gamma^2\left(1-\underline{n}\cdot\underline{\beta}'\right)\rho^2}}_{\text{‘Coulomb’ field}}+\underbrace{\frac{\underline{n}\times\left(\left(\underline{n}-\underline{\beta}'\right)\times\underline{\dot{\beta}}'\right)}{c\left(1-\underline{n}\cdot\underline{\beta}'\right)^3\rho}}_{\text{‘Acceleration’ field}}\right]$$

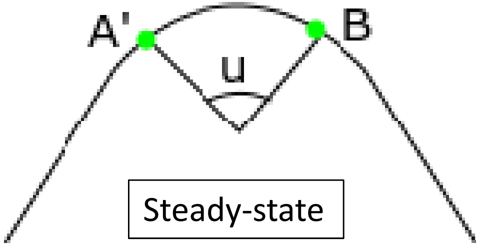


- The electric field experienced by a particle in front of an emitter is then dependent on the geometry of the lattice and the particle trajectory.

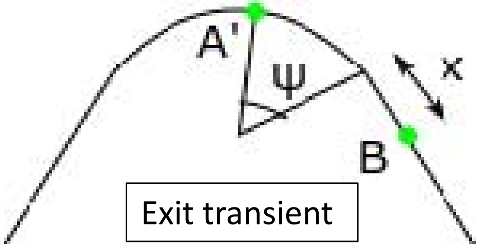
[2] Saldin et al (NIM A 398.373) (1997) distinguished 3 regimes: steady-state, entrance and exit transient



Entrance transient

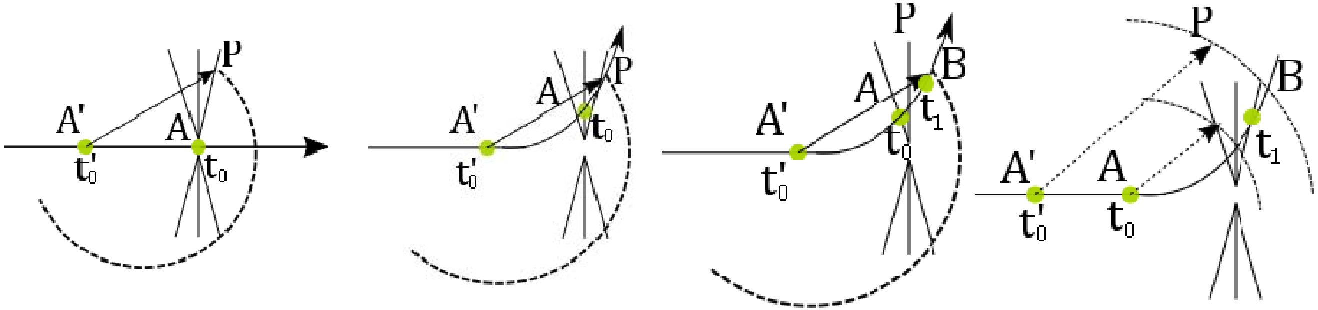


Steady-state



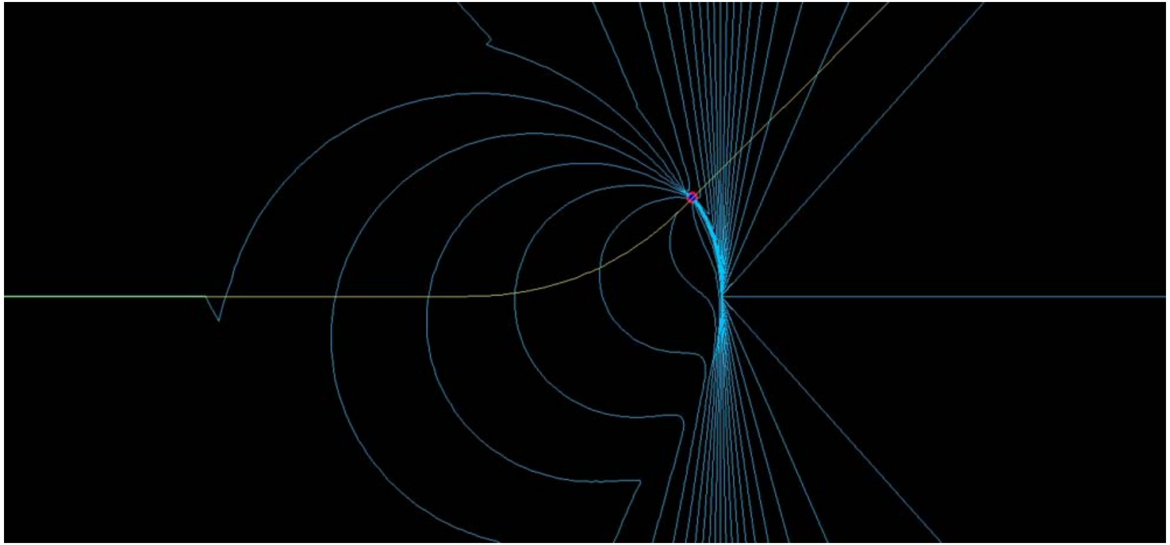
Exit transient

The Entrance Transient



- On entrance, a portion of the electrons have not yet entered the dipole, so their contribution comes from the Coulomb term only

- Since the Coulomb field continues to travel along the straight trajectory, it moves in front of the emitting particle as it enters the bend, producing a spike in the field observed at the head of the bunch



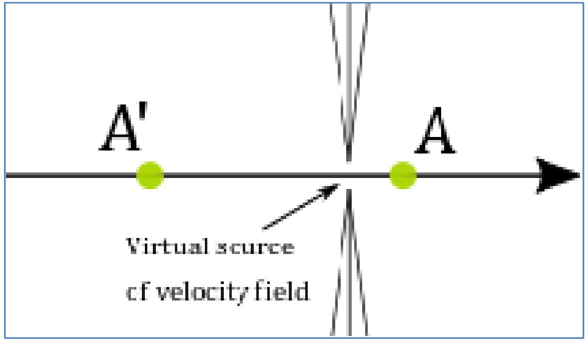
[5] Shintake, Radiation2D code (2003)

The Exit Transient

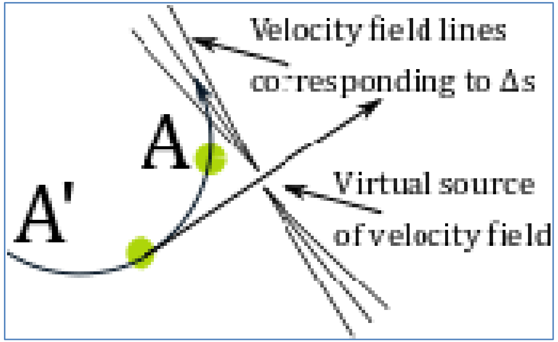
- The field lines corresponding to the velocity field of a relativistic particle are confined in a very flat pancake perpendicular to the direction of motion.

- At the end of the arc, the geometry must pass from a situation with field lines in front of the observer to a situation with field lines behind the observer, giving a spike of CSR force.

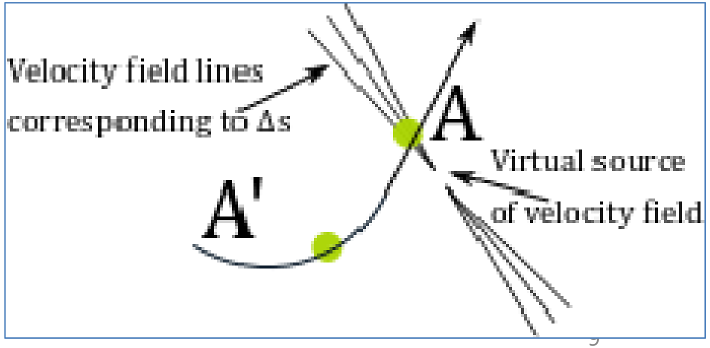
Rectilinear motion



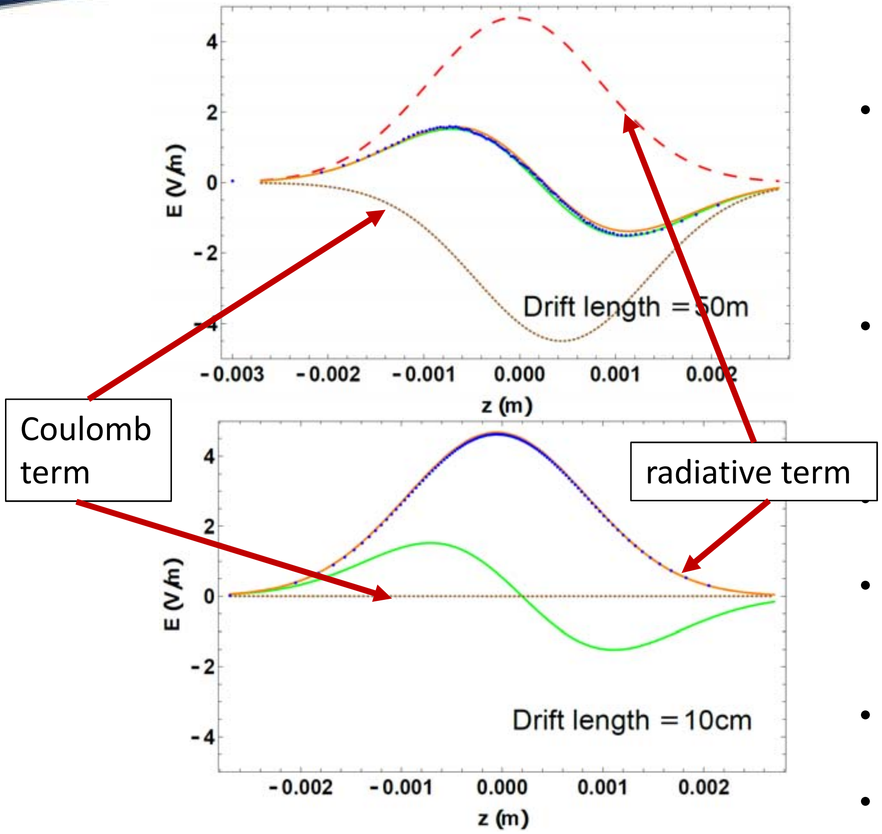
Both particles on arc



Exit transient



Validation of GPT-CSR model – Entrance Transient



- Firstly we used a test case for 1 dipole to compare with the new expressions for transients – in **green** we show the 1d entrance transient under the usual assumption that the drift beforehand is infinite
- This transient field results from a **combination** of the **radiative** term from particles in the bend and the **Coulomb** term from particles on the straight trajectory before
- These partially cancel out if the drift length is large (top plot).
- But this is not the case if the drift is small – the Coulomb term does not contribute to the overall CSR field.
- This is important when considering multi-bend systems!
- We see GPT (blue dots) reproduces this effect nicely

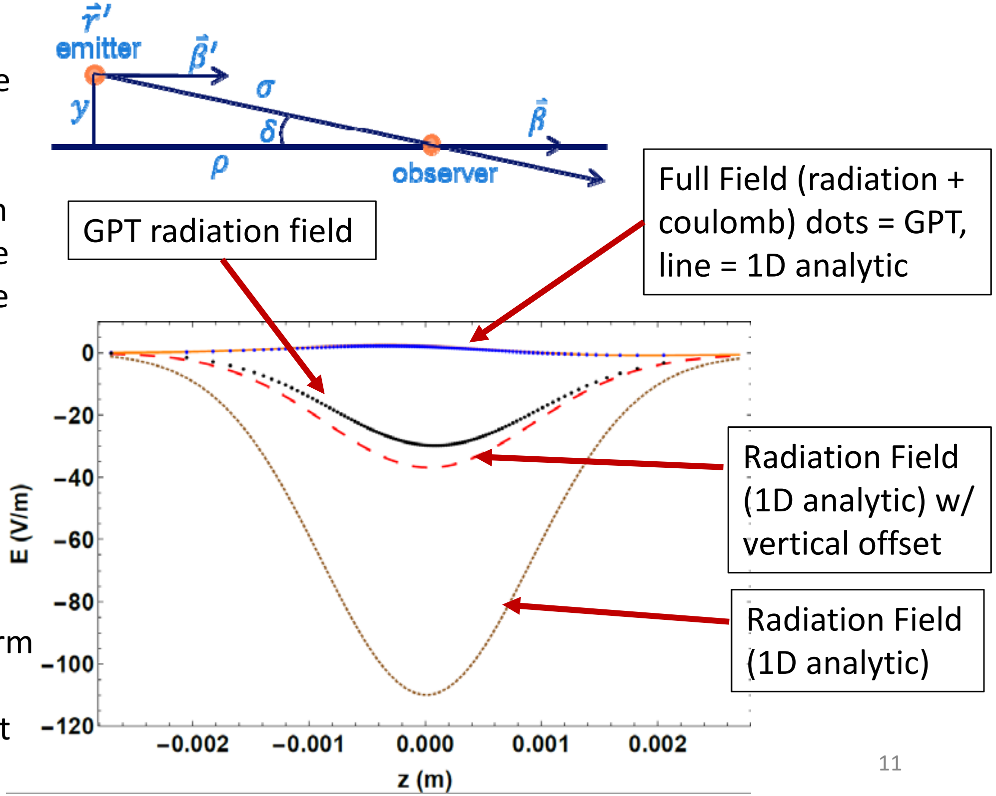
Validation of GPT-CSR model – Exit Transient – BEWARE – subtlety required

- The Coulomb and radiation terms can also be studied separately analytically, but GPT only allows access to the radiation term

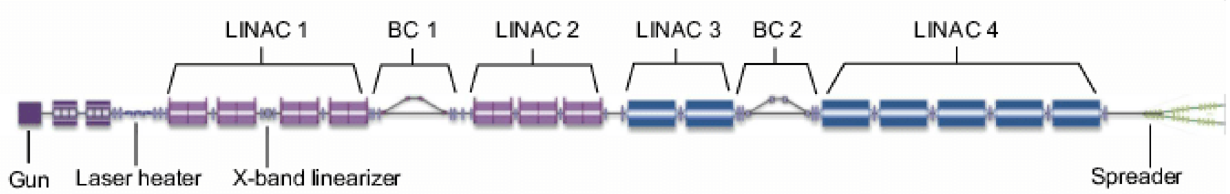
- The magnitude of the radiation term is overestimated in the 1D approximation, due to an underestimation of the retarded distance between emitter and observer i.e. the vertical dimension exists!!!

- If we offset the emitter transversely, this effect is corrected.

- However, because of the cancellation between Coulomb and radiation terms, this 3D effect is masked. We must assume that the Coulomb term also overestimated! Are there any cases where this cancellation is spoiled? If so this could also be important



CSR Degradation of Projected Emittance Measurements: Procedure



- Measurements taken at the exit of BC1 in the FERMI FEL
- Maintain "nominal" parameters:
  - BC1 @ 105mrad.
  - Linac 1 on crest.
  - Beam matched @ exit of BC1.
  - Peak current up to 1.5kA.
  - Compression factor 8 – 65.
  - LH at full power (21uJ)
- Scan through each of BC1 angle and L1 phase
- Measure projected emittance on OTR

Electron beam parameters	Value
Energy (MeV)	300
Charge (pC)	100
Peak current (A)	560
Initial bunch length (ps FW)	1.8
Slice emittance (um-rad, norm)	<1.0
Proj. emittance (um-rad, norm)	<2.0
Uncor. e-spread (keV, rms)	150
Total e-spread (rms)	0.1%
Pulse-to-pulse energy stability	0.1%
Timing jitter (fs, rms)	<150

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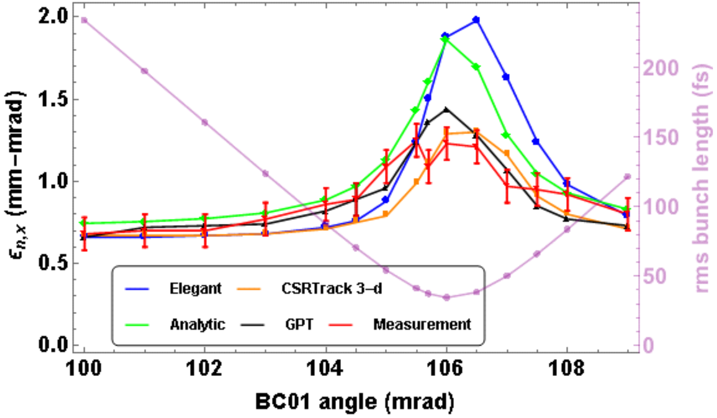
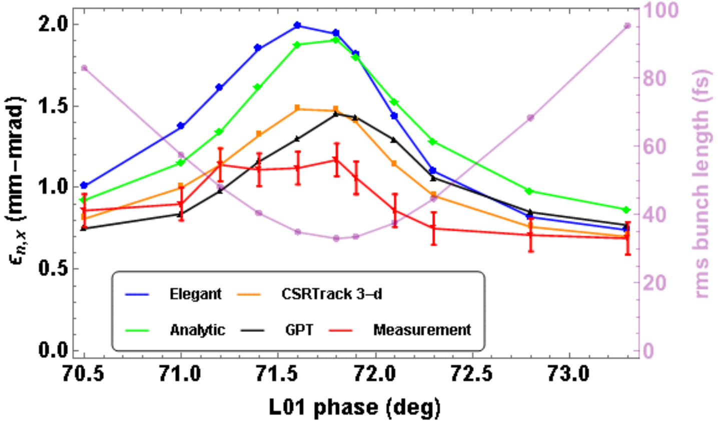
CSR Degradation of Projected Emittance Measurements: Results





- Projected emittance measured, and compared with **1-D** analytic approximation, **1-D** Elegant, **3-D** GPT and **3-D** CSRTrack

- Analytic: projected emittance growth from the longitudinal and transverse CSR force in each dipole to be [10,11]:

$$\Delta\epsilon_n \approx 7.5\times 10^{-3}\frac{\beta}{\gamma}\left(\frac{N r_e L_b^2}{R^{5/3}\sigma_z^{4/3}}\right)+\frac{-3+2\sqrt{3}}{24\pi}\frac{\beta}{\gamma}\left(\frac{\Lambda N r_e L_b}{R\sigma_z}\right)^2$$

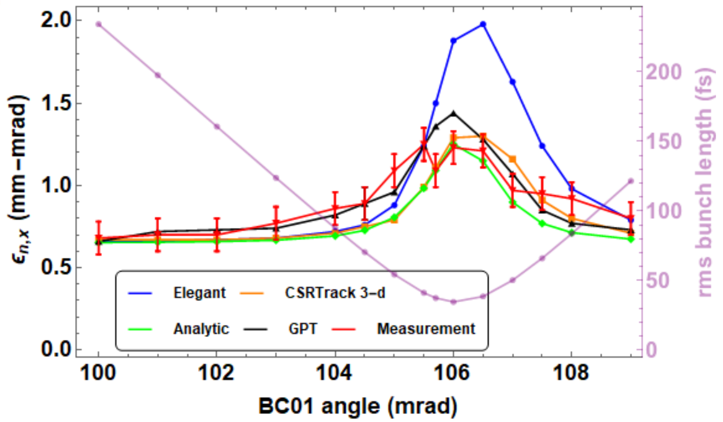
$$\Lambda = ln\left(\frac{(R\,\sigma_z^2)^{2/3}}{\sigma_x^2}\left(1+\frac{\sigma_x}{\sigma_z}\right)\right)$$

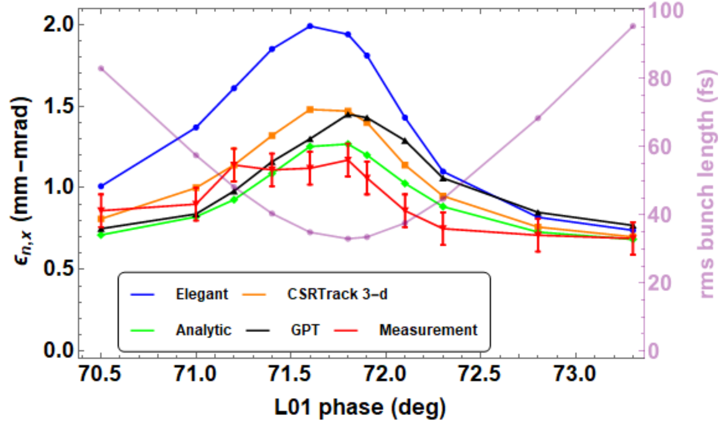
- All agree for compression factor (CF) < 40, but the **1-D results diverge as we approach maximal compression**

- **3-D** CSRTrack and GPT manage to capture the trend and the values agree quite well for both BC angle and linacs phase scans

[10] Stupakov, SLAC-PUB-8028 (1999)
[11] Cai, PRAB 20.064402 (2017)

CSR Degradation of Projected Emittance Measurements: STOP PRESS!!! UPDATE!





- Recently, Stupakov corrected an error in the 1-D analytic approximation [12]

- He demonstrated that the **transverse** CSR kick was **much smaller** than previously thought, due to a cancellation of the CSR field with the transverse field of the bunch itself, resulting in the following formula for emittance growth:

$$\Delta\epsilon_n \approx 7.5\times10^{-3}\,\frac{\beta}{\gamma}\left(\frac{N r_e L_b^2}{R^{5/3}\sigma_z^{4/3}}\right)+2.5\times10^{-2}\,\frac{\beta}{\gamma}\left(\frac{N r_e L_b}{R\sigma_z}\right)^2$$

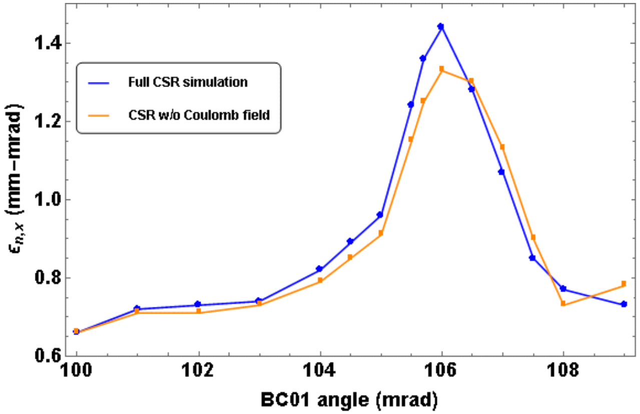
- This new analytic estimate (which is valid only in the steady-state regime) produces good agreement with the 3D codes and with the experimental results across all compression factors

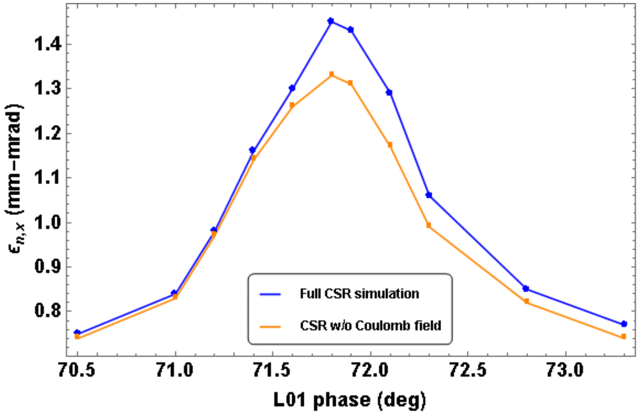
- This leaves only Elegant as the outlier – we are as yet not sure why because Elegant neglects the transverse kick anyway … we have some ideas, but suggestions welcome!

[12] Stupakov, arXiv.1901.10745 (2019)

Impact of the Coulomb Field

- GPT-CSR has the option to include/exclude the Coulomb field in CSR calculations.
- FERMI parameter scans were simulated with both the full CSR field and with only the radiation field.
- We see an increase of ~10% at full compression when the Coulomb field is included.
- For systems with multiple bends (i.e. ERL arcs), this cannot be neglected even for ultrarelativistic systems!





Analytic Model of a Microbunched Beam

- My view is that there is a confused understanding of how to treat the microbunching instability in the literature: uBI is usually referred to as an amplification of some **density modulation** during the acceleration and compression of high brightness beams

- But the remedy is to heat up the cold beam – i.e. Landau damp through a manipulation of the **energy spread**, so to me a quantitative treatment must operate in the **2-d z-δ phase space**. This is handy as FERMI has a TDC + spectrometer section that allows imaging in this 2-d plane

- Indeed the two dominant impedances driving the gain **couple density and energy**, these being CSR and LSC, CSR can be viewed as a **shearing** in that space, and LSC a true **rotation** (as it's a plasma oscillation). So let's play with an analytic toy model of a microbunched beam

- We parametrise in terms of: Modulation frequency ω, Bulk rotation of the bunch φ, Skew of modulations wrt bulk rotation θ, Intensity of the modulations represented by the bunching factor b

- Defining the following terms for a bunch with size σ_x and σ_y :

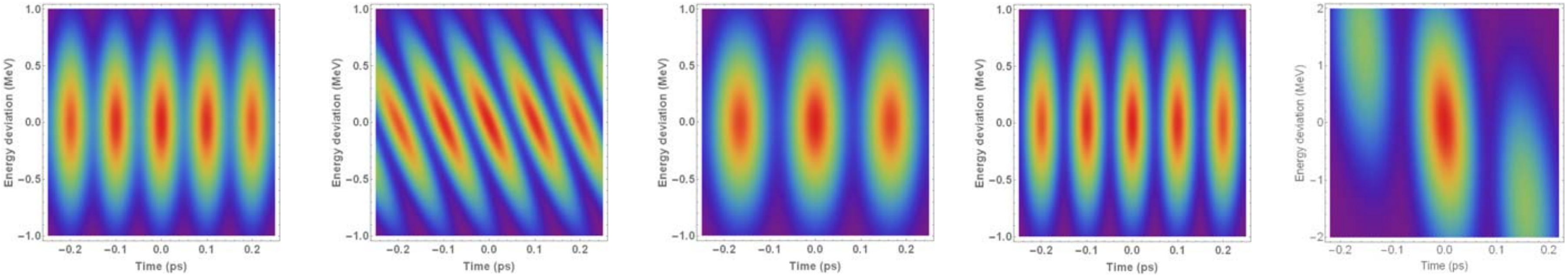
$$x_n(x, y, \theta) = x\cos(2\pi\theta) + y\sin(2\pi\theta),$$
$$y_n(x, y, \phi) = -x\sin(2\pi\phi) + y\cos(2\pi\phi),$$

we show a sinusoidally modulated two-dimensional Gaussian function:

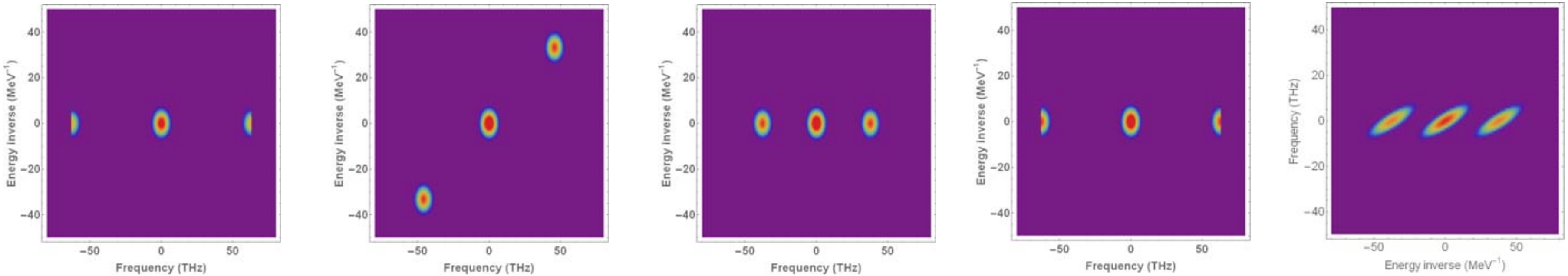
$$\mathbf{X}(x, y, \theta, \phi, \sigma_x, \sigma_y, b, \omega) = exp\left[-\left(\frac{x_n(x, y, \theta)^2}{2\sigma_x^2} + \frac{y_n(x, y, \phi)^2}{2\sigma_y^2}\right)\right]\left(1 + b\cos[2\pi\omega x_n(x, y, \theta)]^2\right).$$

Analytic Model of a Microbunched Beam

- … which looks like Picture 1) below. Then Picture 2) is the result of varying the skew parameter; Picture 3) is the result of decreasing the frequency; Picture 4) is the result of increasing the bunching factor and; Picture 5) is the result of changing the bulk rotation



- Given that we wish to analyse real beam images that are more complex than these, we need a simple way to go from images to our 4 parameters, or sets of 4 parameters in case of multiple modulation frequencies… therefore undertake 2-d Fourier analysis



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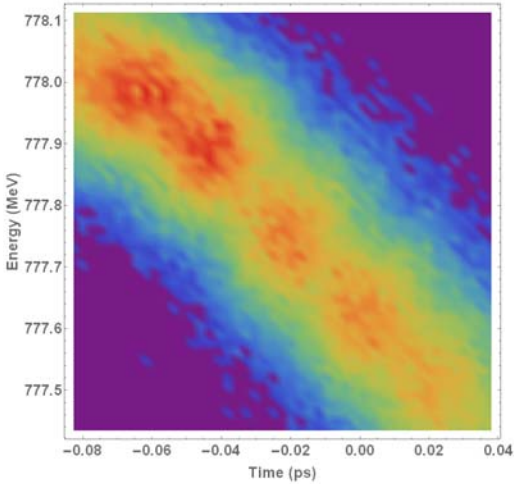
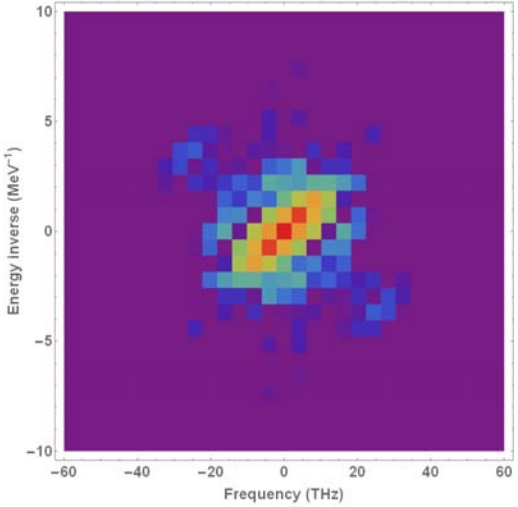
Applying Analytic Model to FERMI Beam

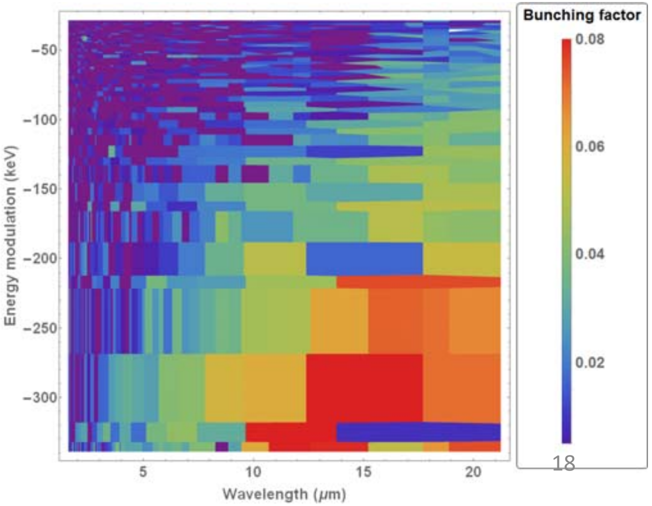
- 100 pC FERMI bunch compressed by factor 30 in BC1 only and accelerated to 787 MeV, then streaked and dispersed, imaged and converted to z-δ phase space
- Laser heater OFF = shot-noise / PI laser induced uBI



- 2-d FT of previous image in frequency – inverse energy phase space
- One pair of satellites clearly visible = a discrete modulation!
- In fact these is another pair closer in to the DC term …

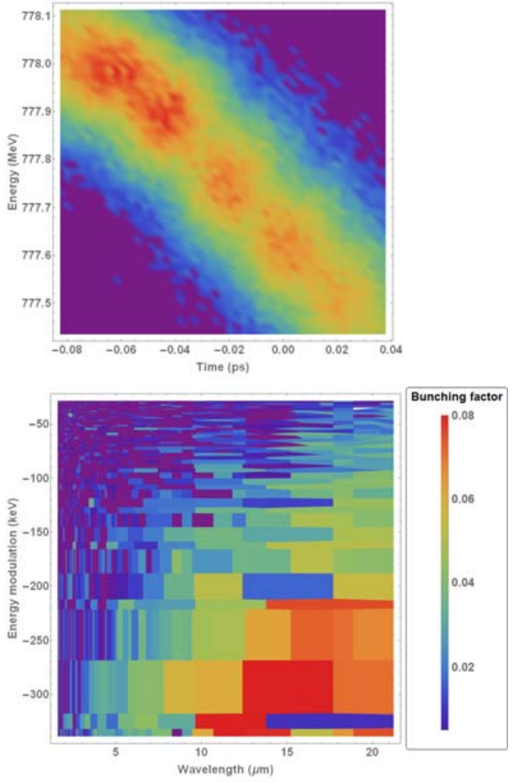


- Selecting lower right quadrant of previous image and inverting axes shows features in wavelength – energy modulation amplitude phase space
- At (12 um , 330 keV) peak of strongest modulation. A weaker modulation at (17 um, 220 keV), suggesting there are two modulation features present in this bunch

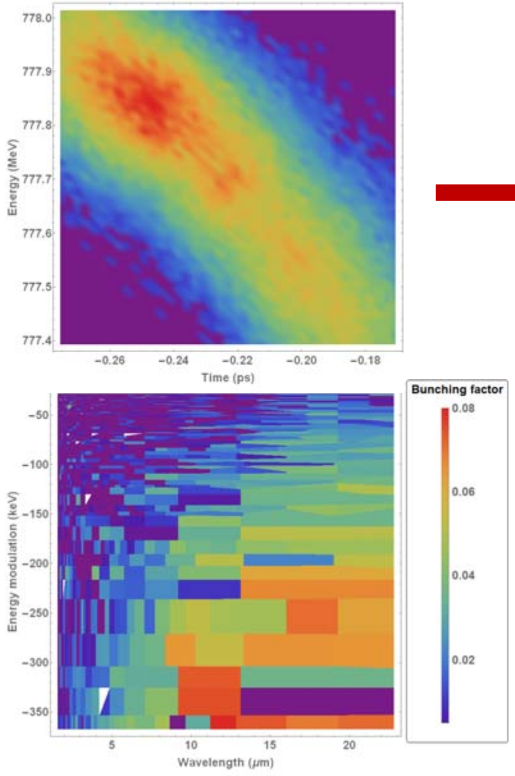


Applying Analytic Model to FERMI Beam – Turning on Laser Heater

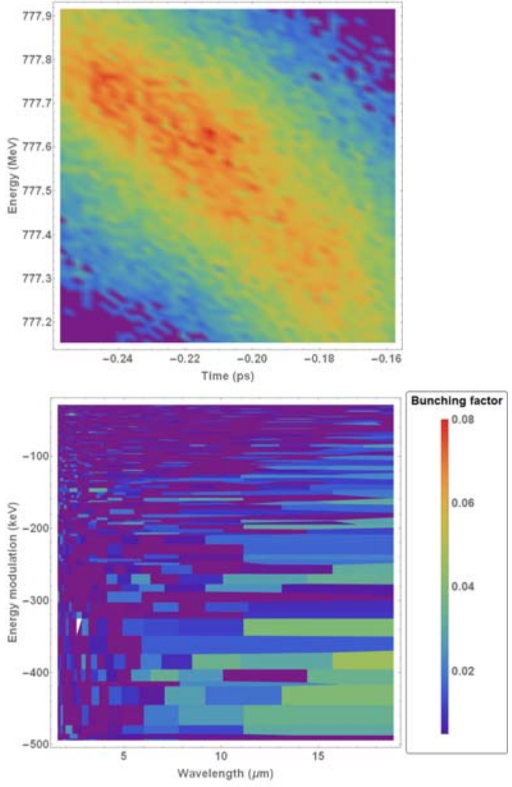
0 keV heating



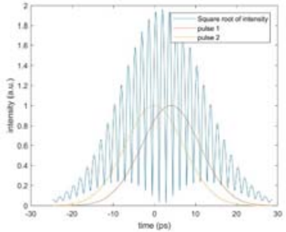
5 keV heating



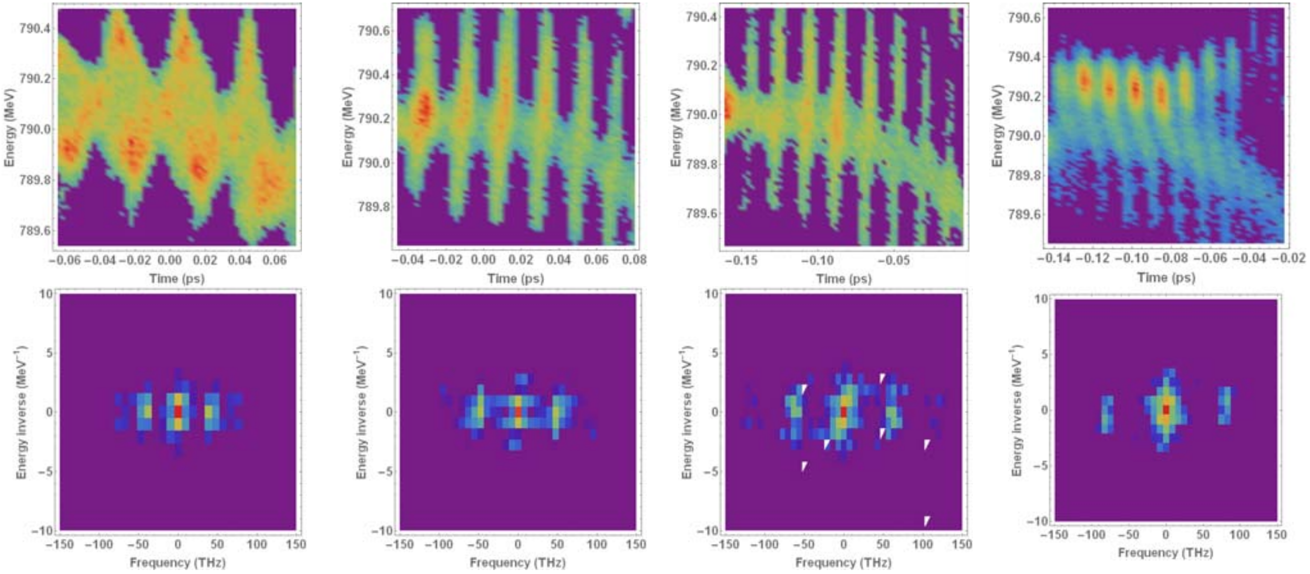
26 keV heating



Applying Analytic Model to FERMI Beam – Benchmarking Gain by Tracking known modulations imposed by Laser Heater Beating



Top: Phase space; **Bottom**: FT
Delay (ps) from L to R: 8, 12, 16, 20



- Work in progress: Aim is to produce full correspondence between measurements, simulations, semi-analytic and fully analytic uBI considerations

- Do this for three different ways of achieving same compression factor (BC1 only, BC2 only, BC1+BC2) and directly show / predict the parameters (bunching factor, frequency peak, skewness / plasma oscillation phase)

Conclusions

- **An extension to the 1D theory of CSR has been developed:**
  - Taking full account of the Lienard-Wiechert field demonstrates the interplay between Coulomb and radiation terms.
    - Neglecting this term can underestimate CSR-induced emittance growth. (Up to ~10% in our case.)
  - The exit and entrance transient effects must take account of the Coulomb term to be fully accurate. (Particularly important for systems involving compressive arcs with many dipoles close together!)
- **Measurements of the CSR-induced emittance growth have been made:**
  - Comparisons with 3D simulation codes show good agreement over the full parameter range
  - The ratio between transverse and longitudinal bunch size gives an estimate for when 3D simulations are necessary.
- **Agreement between theory, simulation and experiment is good!**

- **Work in progress:** Developed simple analytic model of microbunched beams to extract features of modulations in 2-d longitudinal phase space – apply to both experimental data and simulation, compare to analytic and semi-analytic models for natural and induced microbuncing

Thanks

- Daresbury & Cockcroft Institute: **Alex Brynes**, Andy Wolski
- Elettra Sincrotrone Trieste: Simone Di Mitri, Giuseppe Penco, Simone Spampinati and all of the FERMI team
- ASML: Irwan Setija, Peter Smorenburg, Seth Brussaard, Iwan Akkermans
- Pulsar Physics: Bas van der Geer, Marieke de Loos

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2. Saldin et al, NIM A 398.373 (1997)
3. Stupakov, SLAC-PUB-9242 (2002)
4. **<u>Brynes et al, New J Phys 20.073035 (2018)</u>**
5. Shintake, Radiation2D code (2003)
6. CSRTrack, http://www.desy.de/xfel-beam/csrtrack/
7. GPT, http://www.pulsar.nl
8. Bane et al, PRSTAB 12.030704 (2009)
9. Bettoni et al, PRSTAB 19.034402 (2016)
10. Stupakov, SLAC-PUB-8028 (1999)
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