ISBN: 978-3-95450-216-5 doi:10.18429/JACoW-eeFACT2018-TUPBB04

RESONANT DEPOLARIZATION AT Z AND W AT FCC-EE

I. A. Koop†
Budker Institute of Nuclear Physics,
also at Novosibirsk State University,
Novosibirsk State Technical University, [630090] Novosibirsk, Russia,

Abstract

Both future 100 km in circumference electron-positron colliders CEPC and FCC-ee need know beams energies with the extreme precision of 1–2 ppm. This can be done only with the help of Resonant Depolarization (RD) technique. Still, some beam parameters of these machines, like energy spread and damping decrements, are so high near 80 GeV per beam, that it is required special consideration and tricks to overcome the difficulties.

The author has written simple spin tracking code, which simulates main features of the RD process in presence of continuous energy diffusion due to synchrotron radiation fluctuations.

It was shown by this study, that the applicability of the RD method is limited by the effect of widening of the central and side band peaks of the spin precession spectrum when the synchrotron tune Qs is chosen too low, say below Q_s=0.05. In this case spin precession lost its resonant nature due to overlap of the wide central spectrum peak with the nearby synchrotron side bands.

Dependencies of the spectrum peaks width from various beam parameters and a new RD-procedure recipe are presented.

INTRODUCTION

Beam emittances in FCCee are so small, that all spin resonances with the betatron motion frequencies are suppressed and their influence on the spin motion is negligible. Truly, at 80 GeV beam energy (spin tune $\nu_0 = \gamma a = 181.5$) the horizontal and the vertical beam emittances are expected to be $\varepsilon_x = 0.84$ nm and $\varepsilon_y = 1.7$ pm, respectively [1]. At Z ($\nu_0 = 103.5$) these emittances are of the same order, but optics for Z is slightly different [2].

The code ASPIRRIN [3, 4] provides reliable estimation of the strengths of the so-called “intrinsic” resonances $\nu_0 = \nu_{\pm k} = Q_y \pm k$. In Fig.1 are presented the modules of few resonance harmonics in the vicinity of Z-peak, while same plot for the W energy range is shown in Fig.2.

One can see that the maximal spin perturbation value does not exceed $w_k = 8 \cdot 10^{-5}$. This is much below of the critical level $(w_k)_{crit} = 0.01$ - a value which may shift the fractional part of the spin tune by $\delta\nu \approx (w_k)_{crit}{}^2 = 0.0001$ —the wanted accuracy of the fractional part of the tune measurement ($\Delta\nu = \nu_0 \cdot \Delta E / E = (100 \div 180) \cdot 10^{-6} \approx 10^{-4}$).

Taking all this into account, we concentrate now our attention on a study of integer resonances and their synchrotron side band satellites. These resonances are driven by the closed orbit distortions or by the longitudinal magnetic field. The last is just the case in the detector region.

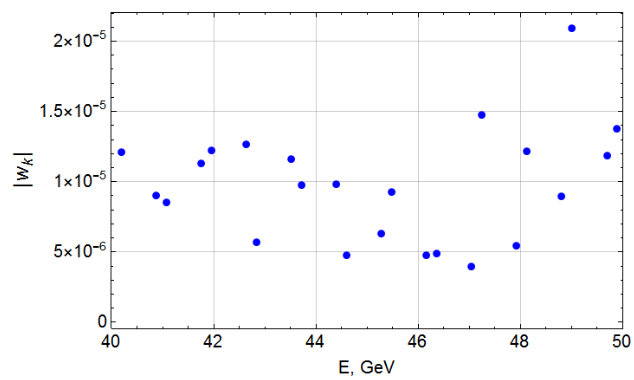

Figure 1: The strengths of few intrinsic resonances in vicinity of Z-peak beam energy.

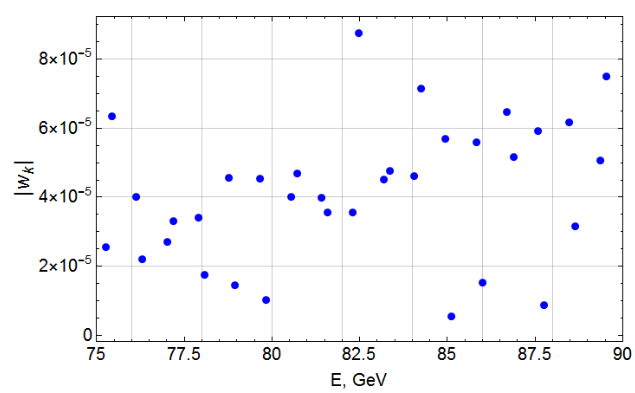

Figure 2: The strengths of few intrinsic resonances near the W-threshold beam energy.

CLOSED ORBIT DISTORTIONS

Sensitivity to misalignment of quads is very high in future electron-positron colliders. This can be expressed via the so-called spin-orbit response function $F_3(\theta)$, which defines the strength of the integer spin resonance $\nu_0 = k$ via the convolution of $F_3(\theta)$ with the dimensionless vertical orbit curvature $\Delta K_x(\theta)$ [3, 4]:

$$w_k = \frac{1}{2\pi}\int_0^{2\pi} F_3(\theta) \cdot \Delta K_x(\theta) d\theta.$$

In Fig.3 as example is plotted the module of $F_3(\theta)$ function for FCC-ee at 45.6 GeV calculated by the code ASPIRRIN. Its average value in arcs is about 250 and

† koop@inp.nsk.su

ISBN: 978-3-95450-216-5 doi:10.18429/JACoW-eeFACT2018-TUPBB04

increases to about 1500 in the final focus and few other quads. With such high sensitivity the average errors of the arc's quads alignment should be much smaller than the expected level $\sigma_y = 0.05 \div 0.10$ mm to have all integer harmonics below the critical value $(w_k)_{crit} = 0.01$. But we, in fact, shall suppress only two most dangerous harmonics: say with k=103 and k=104, if $\nu_0 \simeq 103.5$. This can be done with the well-known technique of harmonic spin matching [5, 6].

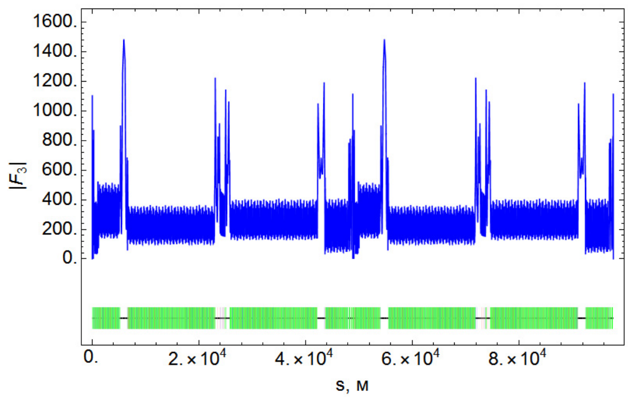

Figure 3: Spin response function $|F_3|$ for the vertical orbit distortions in FCC-ee at 45.6 GeV.

SPIN TRACKING CODE ALGORITHM

Author has written a simple code which performs a Monte-Carlo simulation of the spin motion of ensemble of particles, which are subjected to synchrotron oscillations with their amplitude variation due to radiation damping and quantum fluctuations of SR. I do not consider the betatron oscillations due to reasons discussed above. This greatly simplifies the task. Now we are interesting only by the synchrotron motion parameters and not by any details about the ring lattice.

I do in fact the turn by turn matrix transformation of the synchrotron and spin variables. The synchrotron map is described as follows. We start from the equation:

$$w_k = \frac{1}{2\pi}\int_0^{2\pi} F_3(\theta)\cdot \Delta K_x(\theta) d\theta.$$

It describes the evolution of the relative energy deviation $\delta = \Delta E / E$ with the constant damping decrement λ and the synchrotron frequency $Q_s = \sqrt{Q_{0s}^2 - \lambda^2}$. The solution at the azimuth θ is:

$$\begin{pmatrix}\delta(\theta)\\ q(\theta)\end{pmatrix} = e^{-\lambda\theta}\begin{pmatrix}\cos(Q_s\theta) & \sin(Q_s\theta)\\ -\sin(Q_s\theta) & \cos(Q_s\theta)\end{pmatrix}\begin{pmatrix}\delta(0)\\ q(0)\end{pmatrix}.$$

Here $q(\theta) = (\delta'(\theta) + \lambda\delta(\theta))/Q_s$ is the conjugate variable for the energy deviation $\delta(\theta)$. The synchrotron tune Q_s is small. Therefore we typically choose θ=2π as a period for spin and energy mapping.

We place the random Gaussian law energy jumps at the beginning of each turn. Their sigma value is adjusted so, as to receive the required equilibrium energy distribution.

Particles spins are rotated two times per turn. First time at the beginning of each turn θ_j by the spin perturbation frequency w_j:

$$w_j = w_0 + w_{RF}\cdot\cos\left(\nu_{RF}\theta_j\right).$$

Here w_0 states for the static rotation around the longitudinal or radial axis, while w_{RF} is the spin rotation amplitude generated by the RF flipper/depolarizer operated at frequency tune ν_{RF}. The second rotation represents the spin transformation done in the arc. In this stage each particle is rotated individually, according to its energy, around the vertical axis.

DEPOLARIZATION RATES EVALUATIONS

Running the spin tracking code with different spin tunes, I got the depolarization rate dependence from this tune, see Fig.4. The results in this plot are presented in the form of asymptotic polarization degree attainable in presence of the depolarization effects due to radiative diffusion caused by the quantum fluctuations of SR.

The equilibrium is established between two opposite processes: the Sokolov-Ternov - polarizing one and the concurrent - depolarizing due to chromatic nature of spin perturbation, which was chosen be w=0.001. The polarization degree was calculated as the ratio:

$$P = 92.6\%\left(1 + \tau_{ST}/\tau_{Dep}\right)$$

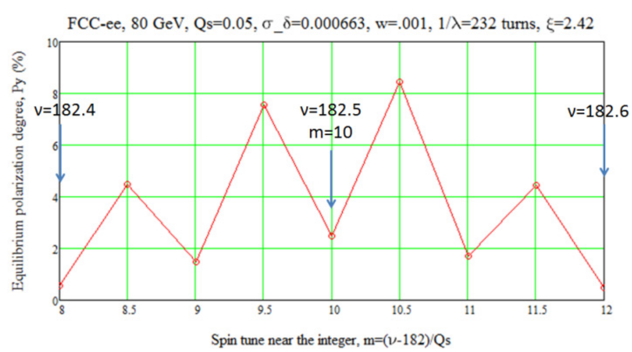

Figure 4: Equilibrium Sokolov-Ternov polarization degree in FCC-ee at 80 GeV with the spin perturbation harmonic w=0.001 and the synchrotron tune Q_s=0.05.

We see that polarization drops down at all synchrotron side band resonances $\nu_0 = k \pm mQ_s$, where the integer m is the order of a satellite. Also it is interesting that region near the half-integer spin tunes looks most favourable to get the highest polarization degree.

SINGLE PARTICLE PRECESSION SPECTRUM

One can ask the question: how looks the precession spectrum for a single particle? To answer this question I run my code for 40000 turns and got the plot presented in the Fig.5. The average particle's energy was set to 80.41 GeV, which corresponds to the spin tune $\nu_0 = 182.481$.

ISBN: 978-3-95450-216-5 doi:10.18429/JACoW-eeFACT2018-TUPBB04

This peak is the highest at the plot. But just nearby one can see the mirror symmetric side band satellite $\nu=\{\nu_0\}+Q_s=0.531\rightarrow 0.469$. This complicates a task of finding of the central peak in measurements with RD.

Other beam parameters in this run are: energy spread $\sigma_\delta=0.000663$, synchrotron tune $Q_s=0.05$, the decrement of synchrotron oscillations $\lambda=1/232$.

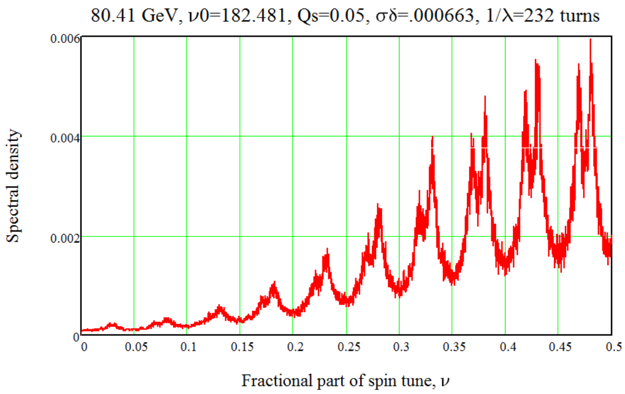

Figure 5: Spectrum of spin precession of a single particle which makes 40000 revolutions around the ring.

Besides of difficulties with presence of many peaks in the spectrum, there appears another problem for RD – each peak became very wide and beam became depolarized before RF- flipper's frequency crossed the centre of the spectrum central peak.

To overcome this difficulty, I propose to change slightly the standard RD procedure. Instead of continues scan by the RF-flipper's frequency, we can switch it on for short time periods to depolarize a beam partially. Then one shifts the flipper's frequency by a large step and turns it on again, and so on and forth. The result of the simulation of this depolarization procedure is shown in Fig.6.

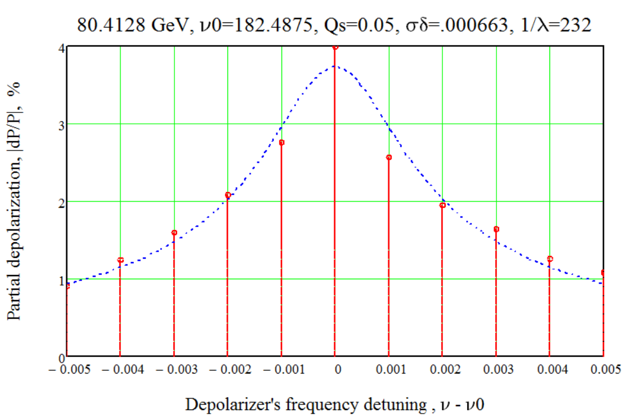

Figure 6: Tune scan by the depolarizer, which produces only the partial depolarizations at the selected frequency points.

The line shape fitting function (dotted blue) is:

$$f(\nu)=A\frac{\Delta}{\Delta^2+(\nu-\nu_0)^2}$$

It has three free parameters A,Δ,ν_0. But we are mainly interesting only by the centre of a peak position ν_0.

The overall accuracy of presented here procedure is highly determined by the sensitivity of the polarimeter and from a level of the initial polarization degree. We will not discuss it here.

Finally, I will present below the functional dependence of the spectral line width Δ from the main beam parameters. I do not pretend on very high precision of the found fitting function, but it shows more or less well - which parameters are most important to make the spectrum peaks as narrow as possible.

$$\Delta=0.0035\frac{\lambda}{0.000686}\left(\frac{\nu_0\sigma_\delta}{182.425\cdot 0.00663}\right)^{2.5}\left(\frac{0.05}{Q_s}\right)^3$$

Obviously, the peak's width is proportional to the synchrotron decrement λ, but depends much stronger from the spin tune ν_0, beam relative energy spread σ_δ and from the synchrotron tune Q_s. For given accelerator without wigglers the energy dependence is very strong:

$$\Delta\sim E^8$$

Therefore, this effect plays important role only at *W* threshold and not at *Z*!

Unfortunately, we have not a freedom to vary the machine and beam parameters as we want. They have very large influence also on the other collider's properties, like the attainable luminosity et cetera. Therefore it is very important to support any RD measurement by the full computer model of that procedure.

CONCLUSION

Spin tracking of the motion of a single particle reveals the dependence of the spectrum line width from the synchrotron tune and other beam parameters. This width becomes very large for chosen value of the synchrotron tune *Qs*=0.05 at *W* and the standard RD procedure becomes not applicable. The proposed above new RD procedure (by steps) works well even in cases when a width of the spin resonance became very large. That is just the case with Qs=0.05. Still the accuracy of a method needs to be studied further. The second order terms in orbital motion also contribute to the line width [7] and even can lead to systematic errors in beam energy determination by the RD [8].

ACKNOWLEDGEMENTS

We thank A.V. Otboev for help with the ASPIRRIN code calculations.

REFERENCES

[1] M. Boscolo, talk at FCC Weak 2018, Amsterdam, 2018.

[2] K. Oide, talk at FCC Weak 2017, Berlin, 2017. Also see: K. Oide *et al.* "Design of Beam Optics for the FCC-ee Collider Ring", DOI: 10.18429/JACoW-IPAC2016-THPOR018 Conference: C16-05-08, p. THPOR018 Proceedings.

ISBN: 978-3-95450-216-5 doi:10.18429/JACoW-eeFACT2018-TUPBB04

[3] V.I. Ptitsyn, PhD thesis (in Russian), BINP, Novosibirsk, 1997.

[4] V.I. Ptitsyn, Yu.M. Shatunov, S.R. Mane "Spin response formalism in circular accelerators", *NIM A,* 608 (2009) 225–233.

[5] R. Assmann *et al*., "Calibration of centre-of-mass energies at LEP1 for precise measurements of Z properties", *Eur. Phys. J*. C 6, 187, 223 (1999).

[6] D. P. Barber *et al*., Preprint DESY 85-044 (1985).

[7] I. Koop, Yu. Shatunov, in *proc. EPAC'88*, Rome, pp. 738-739.

[8] A. Bogomyagkov, talk at this conference.