STUDY OF COHERENT HEAD-TAIL INSTABILITY DUE TO BEAM-BEAM INTERACTION IN CIRCULAR COLLIDERS BASED ON CRAB WAIST SCHEME

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Abstract

Coherent motion of colliding beam-beam system has been studied mainly for transverse modes. π and σ modes are We dicuss coherent head-tail instability for beam-beam collision with a large Piwinski angle. The instability seems serious for colliders based on the crab waist scheme.

INTRODUCTION

Recent and future e^+e^- colliers adopt collision with large crossing angle $\sigma_z\theta_c \gg \sigma_x$, where θ_c is half crossing angle. The vertical beta function is squeezed smaller than the bunch length, $\beta_y^* < \sigma_z$, while the crossing angle (θ_c: half angle) is chosen $\sigma_x/\theta_c \le \beta_y$ to avoid the hourglass effect. Crab waist using based on a transformation $H = xp_y^2/2\theta_c$ at collision point suppress hourglass effect for particles with a large horizontal amplitude.

The beam-beam effects in the crab waist collision has been studied using the weak-strong simulation. Strong-strong simulation recenlty showed a strong coherent head-tail instability in the crab waist collision [**?**]. The instability is studied in detail in this paper.

STRONG-STRONG SIMULATION

A strong-strong bam-beam simulation code "BBSS" is used to study coherent effects in the crab waist collision. Two colliding bunches are represented by many macro-particles, $\approx 1,000,000$. Each bunch is sliced into several or many pieces, depending on Piwinski angle. Figure 1 shows schematic view of collision simulation for a large Piwinski angle. Typically the number of slices is chosen $n_{sl} \approx 10\sigma_z\theta_c/\sigma_z$. Collision order is given by sorting $z_{+,i} + z_{-,j}$, where z is the arrival time advance of $i(j)$-th slice of $e^{+(-)}$ beam multiplied by the light speed c. The collision point of a slice pair is given by $s_\pm = \pm(z_{+,i} - z_{-,j})/2$ for $e^\pm$ beam. A slice pair with $z_{+,i} \approx z_{-,j}$ collides at similar horizontal position, $x_{+,i} \approx x_{-,j} \approx z_{\pm,ij}\theta_c$ at $s \approx s^*$. While a pair with a large difference in z collides at $s = (z_{+,i} - z_{-,j})/2$ deviating from s^* with a large horizontal offset $(s - s^*)\theta_c$.

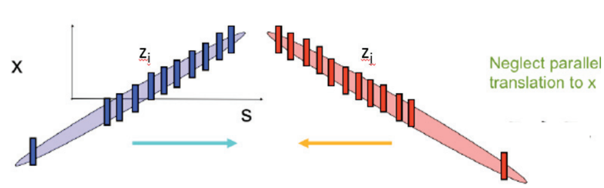

Figure 1: Schematic view of collision simulation for a large Piwinski angle.

In the strong-strong simulation, particles in a beam move with experience of electro-magnetic field induced by another beam. The motion of two beams (slice pair) are solved self-consistently. Strong-strong simulations are performed based on Particle In Cell method usually. The particle distribution is mapped on a transverse grid space (cell). Electric potential due to the particle distribution is calculated by solving Poisson equation in the two dimensional grid space. The potential calculation is simplified by assuming Gaussian distribution in transverse.

The code "BBSS" [3,4] eqipps several options to calculate electro-magnetic force between slice pair.

1. Gaussian approximation using rms value. Transverse Rms sizes of slice pair are calculated at $s_\pm = \pm(z_{+,i} - z_{-,j})/2$. Beam-beam force is evaluated by Bassetti-Erskine formula.

2. Gaussian approximation using fitting value. Transverse sizes of slice pair are calculated by Gaussian fitting at $s_\pm = \pm(z_{+,i} - z_{-,j})/2$. Beam-beam force is evaluated by Bassetti-Erskine formula.

3. Combined of PIC and Gaussian approximation. PIC is used for collision with small offset, namely $z_{+,i} \approx z_{-,j}$, while Gaussian approximation is used for collision with a large offset.

4. Full PIC using shifted Green function. Every collisions of slice pairs are evaluated by PIC method. Shifted Green function makes possible to evaluate potential for collision with a large horizontal offset.

Computing is harder for later options.

The strong-strong beam-beam simulation has been performed for FCC-ee. We discuss for Z and H, which parameters are listed in Table 1. Coherent instability has been seen in the simulation. Study of the coherent beam-beam instability is main thema of this paper.

Coherent beam-beam mode has been studied for a long time. Typical mode is π and σ modes, in which two beams collide with corrective frequency of transverse betatron frequency shifted by the coherent beam-beam tune shift. Here we discuss head-tail mode induced by beam-beam interaction with a large Piwinski angle. Two beams oscillate coherently with a head-tail mode.

The simulation calculate luminosity and beam distribution turn-by-turn Beam-beam parameter, which is normalized luminosity, is used for index of the beam-beam limit. The

ISBN 978-3-95450-187-8

beam-beam parameter is saturated at a certain value at the limit,

$$\xi_L = \frac{2r_e\beta_y}{N_e\gamma f_{rep}}L. \quad (1)$$

COHERENT HEAD-TAIL INSTABILITY IN THE SIMULATION

FCC-ee/H

FCC-ee-H targets beam-beam parameter $\xi_y \sim \xi_L = 0.14$ under radiation damping time 150 turns. FCC-ee has 2IP. In the simulation, we consider a half ring model with the circumference 50,000 km. The effective damping time is 300 turns. Perfect superperiodicity is assumed in this model. Luminosity is converted to 2-IP with 100,000 km circumference.

Figure 2 shows evolutions of the beam-beam parameter and $\langle xz\rangle$ given by full PIC based strong-strong simulation at tune operating point (0.513,0.57). The beam-beam parameter is saturated at around $\xi_L \approx 0.15$. For higher nominal tune shift $\xi_0 \geq .239$, luminosity oscillates turn-by-turn. $\langle xz\rangle$, which also oscillates, seems the source of the beam-beam limit. $\langle xz\rangle$ oscillates in phase for two beams. The beam-beam parameters $\xi_L = 0.11$ and 0.125 are achieved at noimnal value $\xi_0 = 0.12$ (design) and 0,179 (1.5x design) without coherent instability. Therefore FCC-ee-H is feasible but somewhat critical for the instability in this operating point (0.513,0.55).

Figure 3 shows evolution of the beam-beam parameter for different conditions. Left plot depicts at operating point (0.54,0.61), and right plot depicts for two limes longer damping time ($\tau_x/T_0 = 300$ turns) at tune operating point (0.51,0.55). The coherent oscillation is seen in every x_0 at the operating point (0.54,0.61). The design operating point is (0.54, 0.59-0.61). Therefore operating point should be reconsidered in FCC-ee-H. For slower damping time, coherent oscillation is seen at $\xi_0 = 0.12$. The beam-beam parameter is saturated at $\xi_L \approx 0.12$.

Figure 5 shows the beam-beam parameter for Gaussian strong-strong at tune (0.513,0.57). The beam-beam parameter oscillates turn-by-turn, though not seen in the left plot. (the luminosity is calculated every 10 turns.) $\langle xz\rangle$ oscillation is depicted in the right plot. FCC-ee-H is critical for the instability.

Figure 4 summarizes the beam-beam parameter limitation. Three kind of points is depict for operating points (0.51,0.55), (0.54,0.57) and (0.54, 0.61). The error bars correspond to amplitude of coherent fluctuation of luminosity.

Figure 5 shows evolution of $\langle xz\rangle$ and turn-by-turn change of x-z distribution for simulation using the Gaussian option. Luminosity fluctuation is also seen in the Gaussian option. Gaussian approximation is more robust for choice of grid. point and treatment of particles outside of grid space.

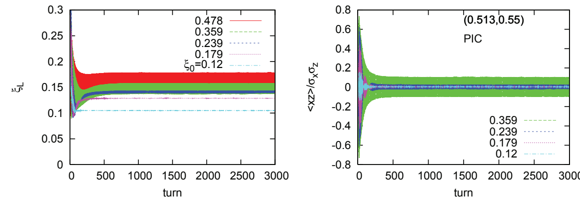

Figure 2: Evolution of the beam-beam parameter (left) and $\langle xz\rangle$ (right) given by strong-strong simulation (full PIC).

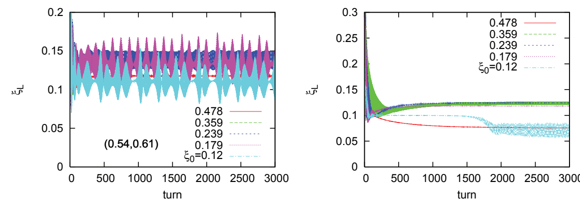

Figure 3: Evolution of the beam-beam parameter for tune (0.54,0.61) in left and twice (slower) damping time 300 turns in right. Both are given by full PIC.

FCC-ee/Z (parameters at Apr 2016)

Z factory was designed larger Piwinski angle $\sigma_z\theta_c/\sigma_x = 2.7$, where the bunch length was $.\sigma_z = 2$ mm including beamstrahlung. The design beam-beam parameter is $(\xi_x, \xi_y) = (0.13, 0.17)$ with transverse radiation damping time 3000 turns. Though latest parameter adopts longer bunch length and larger Piwinski angle, systematic studies were performed using the old parameters. Figure 6 shows evolution of the beam-beam parameter for full PIC and fitted Gaussian model. The beam-beam parameter is summarized in Figure 7. The beam-beam parameter is saturated at $\xi_L \approx 0.05$, while the design is 0.17. Approaching half integer, beam-beam parameter increases to 0.07, but still insufficient. Figure 8 shows tune scan for the beam-beam parameter. Two type of tune scan are tried, one is changing tune of both beams, the other is changing electron tune with keeping positron tune $\nu_x = 0.54$. The coherent instability was not suppressed by separation of tune. Even if the coherent instability is weak, the beam-beam parameter is very low.

Figure 9 shows evolution of the beam-beam parameter with chromaticity $\nu_x' = \nu_y' = 5$. Chromaticity little suppresses the oscillation.

SuperKEKB

Strong-strong simulation for SuperKEKB has been done using Gaussian approximation and cobined method with PIC, because Piwinski angle is very large $\sigma_z\theta_c/\sigma_x = 20$. Figure 10 shows evolution of luminosity, horizontal beam size and $\langle xz\rangle$. Very strong coherent head-tail instability is induced by beam-beam interaction at tune (0.525,0.57), where the synchrotron tune is 0.025. The instability arises at the condition, $2(\nu_x - \nu_s)$ = integer. The instability is not seen in design operating point (0.53,0.57). The instability is considerd safe for SuperKEKB due to the narrow stop band,

ISBN 978-3-95450-187-8

Table 1: Parameters for FCC-ee (ver. 21, June 2016).

Parameter		SuperKEKB	Z	Z	W	H	tt
Energy	E (GeV)	4	45.5	45.5	80	120	175
Bunch population	$N_{\pm}(10^{10})$	9	37.1	3.3	6	8	17
Number of bunch		2500	30180	90150	5260	780	81
Emittance	$\varepsilon_{x/y}$ (nm/pm)	3.2/8.64	0.2/1	0.09/1	0.26/1	0.61/1.2	1.3/2.5
Beta at IP	$\beta^*_{x/y}$ (m/mm)	0.032/0.27	0.5/1	1/2	1/2	1/2	1/2
Bunch length	σ_z (mm)	6	6.7	3.8	3.1	2.4	2.5
Energy spread	σ_δ (%)	0..08	0.22	0.09	0.10	0.12	0.17
Synchrotron tune	ν_z	0.025	0.036	0.025	0.037	0.056	0.075
Luminosity per IP	L (10^{34} cm^{-2}s^{-1})	80	207	90	19.1	5.1	1.3
Beam-beam	$\xi_{x/y}$	0.0028/0.088	0.025/0.16	0.05/0.13	0.07/0.16	0.08/0.14	0.08/0.12
Piwinski angle	$\sigma_z\theta_c/\sigma_x$	20	10	6	2.9	1.5	1.0

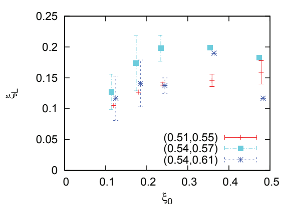

Figure 4: Achieved beam-beam parameter as function of its nominal value.

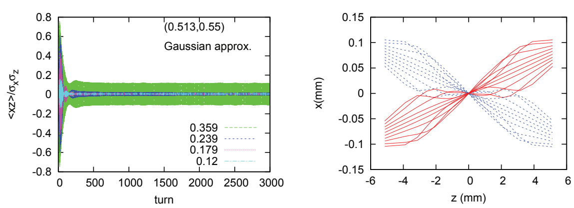

Figure 5: Evolution of $\langle xz \rangle$ given by strong-strong simulation (rms Gaussian).

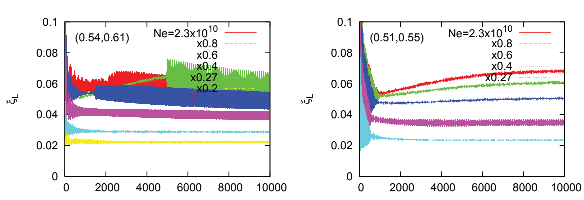

Figure 6: Evolution of the beam-beam parameter given by strong-strong simulation Left and right is given by full PIC and fitted Gaussian, respectively.

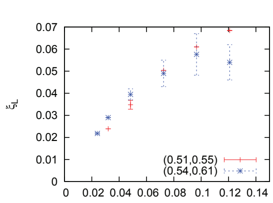

Figure 7: Achived beam-beam parameter summary for FCC-ee/Z (full PIC).

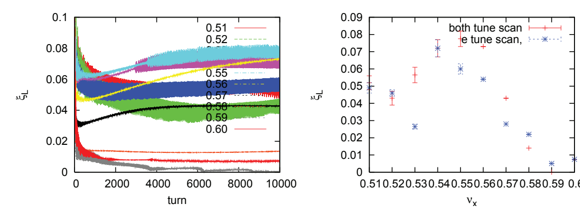

Figure 8: Tune scan for the beam-beam parameter. Right plot summerizes the beam-beam parameter.

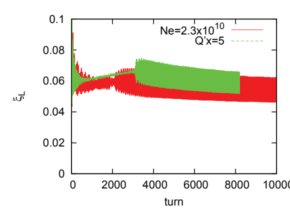

Figure 9: Evolution of the beam-beam parameter with/without chromaticity given by strong-strong simulation (full PIC).

Figure 11 shows the beam-beam parameter evolution using fitted Gaussian model. The beam-beam parameter behaves well, though it is somewhat lower than the design (0.88).

FCC-ee-Z (parameters June, 2016)

Z factory is proposed larger Piwinski angle $\sigma_z\theta_c/\sigma_x = 6$ or 10. The target beam-beam parameter is 0.13. Fitted Gaussian and full PIC options are used in the strong-strong simulation. Figure 12 shows evelution of the beam-beam parameter. Top two plots depicts for $\sigma_z\theta_c/\sigma_x = 6$. Left and right are given by full PIC and fitted Gaussan, respectively. The behaviors are somewhat different, but the beam-beam parameter is saturated far lower than the design 0.13. Bottom plot depics for $\sigma_z\theta_c/\sigma_x = 10$. The behavior seemd worse for larger Piwinski angle.

The parameter of Z factory is similar as SuperKEKB. Main difference is target beam-beam parameter is $\xi_L = 0.08$ for SuperKEKB while 0.13 for FCC-ee-Z. We compare simulated beam-beam limit of SuperKEKB and FCC-ee-Z. Choosing half intensity in FCC-ee-Z, initial beam-beam

ISBN 978-3-95450-187-8

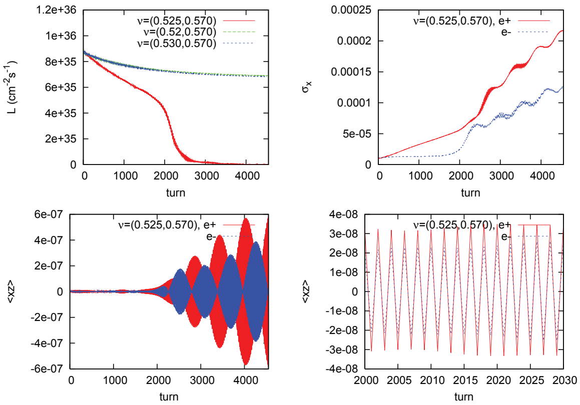

Figure 10: Evolution of luminosity, beam size, $\langle xz\rangle$ in SuperKEKB given by strong-strong simulation (cobination of PIC and rms Gaussian).

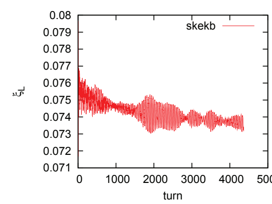

Figure 11: Evolution of the beam-beam parameter in SuperKEKB given by strong-strong simulation (fitted Gaussian).

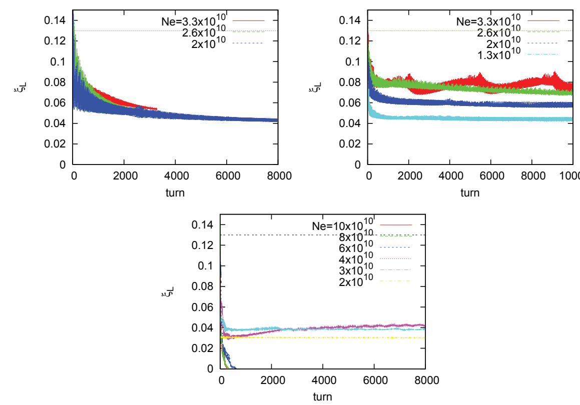

Figure 12: Evolution of the beam-beam parameter given by strong-strong simulation. Top left and right is given for Piwinski angle of $\sigma_x\theta_c/\sigma_z$=6 using Full PIC Gaussian fitting, respectively. Bottom plot depics for $\sigma_z\theta_c/\sigma_x = 10$.

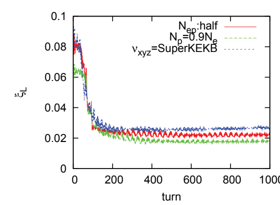

Figure 13: Evolution of beam-beam parameter for half intensity, detuning of transparency condition and SuperKEKB operating point.

parameter is similar level as SuperKEKB. Tune is slightly different, thus the same tune is adopted in FCC-ee-Z as SuperKEKB. In SuperKEKB, beam strength is not transparent: tune shift for two beam is differnt 0.08 and 0.088. Therefore one beam in FCC-ee-Z is decreased 10%. In both case beam-beam parameter goes dwn from 0.08 to 0.02 as shown in Figure 13 in contrast with Figs.10 and 11.

Horizontal beam size of SuperKEKB and TLEP-ee-Z is similar, but emittance and β_x are different. They are $\varepsilon_x = 3$ nm, $\beta_x = 0.03$ m for SuperKEKB, while $\varepsilon_x = 0.09$ nm, $\beta_x = 0.5$ m for FCC-ee-Z. The horizontal tune shift is 0.03 for FCC-ee-Z and 0.0028 for SuperKEKB. We now change emittance and $\beta_x$with keeping the horizontal beam size .$\varepsilon_x = 0.9$ nm, $\beta_x = 0.05$ m. Figure **??** shows evolutions of beam-beam parameter, beam sizes and $\langle xz\rangle$ for two cases, ..$\varepsilon_x = 0.9$ nm, $\beta_x = 0.05$ m (blue) and $\varepsilon_x = 0.09$ nm, $\beta_x = 0.5$ m (red) The behavior is remarkably different and is consistent with SuperKEKB result.

SIMPLIFIED MODELS

For qualitative understanding of the coherent instbility, two simplified models were examined. One is one particle-airbag interaction model. One particle (e^+) with the horizontal size $\Sigma_x = \sqrt{\sigma_x^2 + \theta^2\sigma_z^2}$ interacts with an airbag beam (e^-) which consists of a number of micro bunches. Figure 15 shows the schematic view of the model. We consider linear force between one particle and airbag. 1,000 micro-bunches are used to represent the airbag beam.

Figure 16 shows a simulation result for the model. Top plot depicts typical horizontal amplitude on longitudinal phase space. The mode number for synchrotron motion is high $m \sim 10$ in the figure. Bottom plot depicts horizontal amplitude at t=1000 turn as function of horizontal tune. Three lines are given for beam-beam tuneshift $\xi = 0.03, 0.05, 0.07$. The oscillation is stable at ν_x chose to half integer, while it

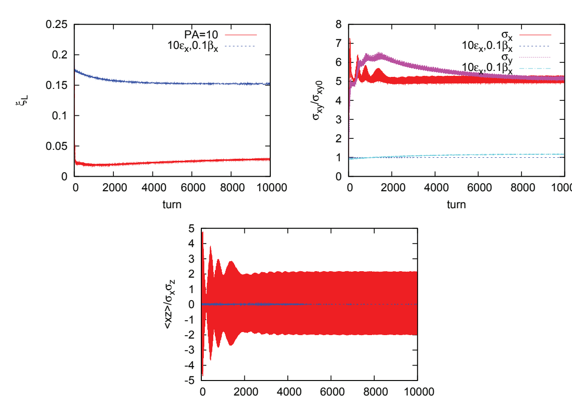

Figure 14: Evolutions of beam-beam parameter, beam sizes and $\langle xz\rangle$ for two cases, $\varepsilon_x = 0.9$ nm, $\beta_x = 0.05$ m (blue) and $\varepsilon_x = 0.09$ nm, $\beta_x = 0.5$ m (red).

ISBN 978-3-95450-187-8

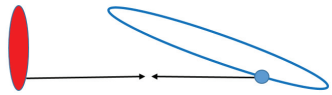

Figure 15: Schematic view of one particle-airbag interaction model.

is unstable leave from half integer. One-two particle model had been studied in Ref. [5]. The one-two particle model take into account only lowest head-tail mode $m = 1$. The model was stable. Now airbag considering higher mode showed contrasted results shown above.

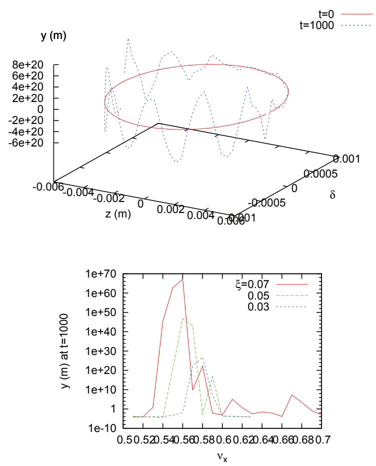

Figure 16: Simple mode simulation. Top and bottom plots are typical horizontal amplitude on longitudinal phase space and horizontal amplitude at t=1000 turn as function of horizontal tune, respectively

Figure 17 shows schematic view of the second model; two-airbug interaction model. Each beam is represented by airbag, which consists of a number of micro-bunches. Figure 18 shows bunch shape of airbag after 5, 20, 30, 50 collisions.

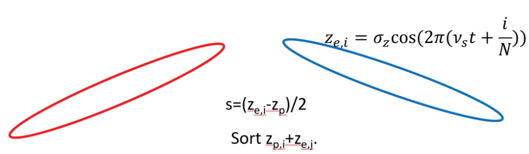

Figure 17: Schematic view of two-airbug interaction model.

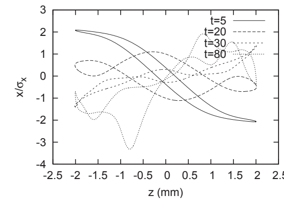

Figure 18: Bunch shape after 5, 20, 30, 50 airbag collision.

SUMMARY

Various kind of strong-strong beam-beam simulation has been performed for FCC-ee. Every simulations show strong coherent beam-beam instability in head-tail mode. The mode is high $m \approx 10$ in simplified mode. Strong-strong simulations also.showed complex head-tail motion. The coherent instability seems serious in FCC-ee. Squeezing β_x^* helps the instability. To check feasibility of a design using crab waist collision, strong-strong simulation is inevitable.

The author thanks fruitful discussions with Drs. Y. Cai, K. Oide, D. Shatilov, F. Zimmermann.

REFERENCES

[1] P. Raimondi, proceedings of the 2nd SuperB workshop, Frascati, March 2006.

[2] K. Ohmi, "Simulated Beam-beam Limits for Circular Lepton and Hadron Colliders", in *Proc. IPAC2016*, Busan, Korea, May 2016, paper MOZA01, pp. 27–32,

[3] K. Ohmi, *Phys. Rev.*, **E 62**, 7287 (2000).

[4] K. Ohmi, M. Tawada, Y. Cai, S. Kamada, K. Oide, and J. Qiang, *Phys. Rev. ST-AB*, **7**, 104401 (2004).

[5] K. Ohmi and A. W. Chao, *PRST-AB*, **5**, 101001 (2002)

ISBN 978-3-95450-187-8