IDEAS FOR SIBERIAN SNAKES AND SPIN ROTATORS IN VERY HIGH ENERGY e^+e^- RINGS

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Abstract

The high value of the radiated power in synchrotron radiation in very high energy e^+e^- storage rings presents unique challenges for the design of Siberian Snakes and spin rotators in such machines. This paper presents some ideas which may lead to a feasible design of such devices. The idea is to employ solenoids interleaved with the arc dipoles, to yield a set of noncommuting spin rotations, which can rotate an initially vertical spin to any desired direction. The solenoids should be (approximately) optically transparent, and can be 'spin matched' to the ring using known procedures. Preliminary numerical studies indicate the design may be feasible.

SPIN ROTATOR AND SNAKE SCHEMATIC

For a general review of spin dynamics in accelerators and for a review of Siberian Snakes and spin rotators in accelerators, we direct the reader to [1] and [2], respectively. The concern here is e^+e^- rings of very high energy, where the very high synchrotron radiation (SR) radiated power places serious constraints on the design of Siberian Snakes and spin rotators. The subject has of course been studied by others already, see, e. g. [3]. The present work also draws on an analysis by the author for FCC-ee [4]. The prototype ring below is FCC-ee, although the essential ideas apply to any very-high energy e^+e^- collider. The subject of beam polarization in such rings falls naturally into two sections: (i) transverse polarization, for energy calibration, and (ii) longitudinal polarization at the interaction point (IP), for tests of the electroweak theory and other tests/searches for new physics. We discuss these in turn.

For energy calibration via the resonant depolarization (RD) of transversely polarizaed beams, it is envisaged that a set of 'pilot' non-colliding e^+ and e^- bunches will be circulated in each ring. (The colliding bunches for HEP will be unpolarized.) The polarized bunches will be produced at low energy in polarized electron and positron sources and then accelerated to high energy for injection into the main ring. Hence the polarized bunches must be accelerated across a large number of intrinsic and imperfection depolarizing spin resonances. To avoid depolarization via the Froissart-Stora formula [5], one or more Siberian Snakes are required in the booster ring. The basic scenario for FCC-ee is to employ full-energy injection, and this is likely to be case for other rings also. The booster ring will occupy the same tunnel as the main ring, i. e. it will have the same circumference and top energy. Hence the designs of Snakes or spin rotators will apply equally to the booster ring and the main ring.

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Longitudinally polarized colliding beams are usually envisaged at the Z^0 peak, to test the electroweak theory, However, they could be useful to explore physics at other energies. It is also possible that, at a later stage, a hadron ring may be installed for an $e-p$ collider option. In such a case, a longitudinally polarized lepton beam would be a natural choice. Longitudinally polarized colliding beams require the e^+ and e^- bunches to be polarized *in situ* in the main rings. This almost certainly requires the use of polarization wigglers, which will increase the SR radiated power. Some details of polarization wigglers for FCC-ee were analyzed in [4]. Since the polarization must be preserved during storage, Siberian Snakes will be required in the main ring. In addition, longitudinally polarized colliding beams require the use of spin rotators.

Hence we are led to consider designs for Siberian Snakes and spin rotators in the booster and/or main ring. We treat a particle of charge e, mass m, with velocity $\vec{v}$, Lorentz factor $\gamma = 1/\sqrt{1-v^2/c^2}$ and spin $\vec{s}$. The magnetic moment anomaly will be denoted by $a = \frac{1}{2}(g-2)$. The externally prescribed electric and magnetic fields of the accelerator are denoted by $\vec{E}$ and $\vec{B}$, respectively. The Thomas-BMT (Bargmann, Michel and Telegdi) equation [6, 7] is given by $d\vec{s}/dt = \vec{\Omega}\times\vec{s}$. Treating only motion in magnetic fields, the spin precession vector is

$$\vec{\Omega} = -\frac{e}{\gamma mc}\left[(\gamma a+1)\vec{B}_\perp + (a+1)\vec{B}_\parallel\right]. \qquad (1)$$

Here $\vec{B}_\parallel = (\vec{B}\cdot\vec{v})\vec{v}/v^2$ and $\vec{B}_\perp = \vec{B}-\vec{B}_\parallel$ are the components parallel and orthogonal to the particle velocity, respectively. Because of eq. (1), Snakes and rotators built using transverse bending fields have traditionally been considered as the superior choice for use at high energies.

However, the above conclusion is derived by considering only the spin rotation angle in a magnet and neglects the synchrotron radiation generated. The above view is therefore applicable to hadron rings, where synchrotron radiation is negligible, but must be reexamined for very high energy e^+e^- colliders. It was argued in [4] that Snakes and rotators built from transverse field dipoles produce an unacceptable SR load, or else entail unacceptably large beam orbit excursions. This includes the so-called 'Steffen Snakes' (see [2] for details) and the Derbenev-Grote Snake/rotator design [8]. The use of helical field Snakes and rotators was analyzed in [4], where it was argued that the closed orbit beam excursions might be tolerable. However, helical field Snakes also generate SR, which is a disadvantage of the design.

Hence in this note, we consider the possibility of employing solenoids to design Snakes and spin rotators. Solenoids do not add to the SR power load. They also have the advantage that they are optically transparent and techniques

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are known for 'spin matching' to the optics of the ring [9]. Quadrupoles are placed between two solenoids in series and the entire 'Snake system' is both optically transparent and spin transparent. However, solenoids have the disadvantage that their spin rotation axis points along the reference axis only. Hence, for example, a solenoid spin rotator will rotate the polarization from vertical to radial (and vice-versa). A spin rotation through 90° in the horizontal plane is required to attain longitudinal polarization at the interaction point (e. g. see [3, Fig. 3-127]). For example, for FCC-ee at the Z^0 peak, this requires an orbit bend of $(\pi/a)(m_e/M_Z) \simeq 0.015$ rad between a solenoid spin rotator and the IP. The ring geometry must be altered to accomodate this bend. The situation is further complicated if there are vertical bends between the rotator and the IP, which is likely to be the case at FCC-ee. There is the additional undesirable feature that the above angle of 0.015 rad is specific to the energy of the Z^0 peak, and the ring geometry must be modified to operate at other beam energies. For Siberian Snakes, if an even number of solenoids are employed, the fractional spin tune is zero, hence the spin motion is on resonance. If an odd number of solenoid Snakes are employed, the fractional spin tune is $\frac{1}{2}$, *but* the stable polarization direction lies in the *horizontal* plane. A circulating beam polarization in the horizontal plane is more susceptible to depoalrizing effects than a vertically polarized beam. On the other hand, a pair of diametrically opposed Snakes with orthogonal spin rotation axes yields a fractional spin tune of $\frac{1}{2}$ *and* the stable polarization is vertical in the arcs, up in one arc and down in the other. See [2] for details.

For all of the above reasons, we are led to consider if it is possible to design a spin rotator using solenoids, which can rotate an initially vertical spin to *any* desired direction in three dimensions. Clearly this cannot be accomplished using only one solenoid. However, *three* solenoids, interleaved with arc dipoles, *do* have the requisite number of degrees of freedom to rotate an initially vertical spin to an arbitrary final direction. Arc dipoles are employed so that the SR power load is not increased by inserting additional dipoles in the ring. The schematic configuration is displayed below (note that the 'horizontal bends' could be several dipoles in series)

$$\text{sol 1} - B - \text{sol 2} - B - \text{sol 3}$$

The solenoid spin rotation angles are $\psi_{1,2,3}$ and the dipole spin rotation angle is ϕ. The bends are considered to be part of the lattice (arc dipoles) hence ϕ is not a free parameter. The free parameters are $\psi_{1,2,3}$. We employ cordinate axes $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ where $\vec{e}_1$ points radially outwards, $\vec{e}_2$ points along the reference axis and $\vec{e}_3$ points vertically upwards. A positive rotation is counterclockwise. Then the overall spin rotation matrix is

$$M = e^{-i\psi_3\sigma_2/2} e^{-i\phi\sigma_3/2} e^{-i\psi_2\sigma_2/2} e^{-i\phi\sigma_3/2} e^{-i\psi_1\sigma_2/2} . \qquad (2)$$

Let the spin rotation matrix be parameterized by

$$M = \xi_0 - i\vec{\xi} \cdot \vec{\sigma} . \qquad (3)$$

Then the matrix M can be simplified to yield

$$\xi_0 = \cos\frac{\psi_1 + \psi_3}{2} \cos\frac{\psi_2}{2} \cos\phi - \sin\frac{\psi_1 + \psi_3}{2} \sin\frac{\psi_2}{2} , \qquad (4a)$$

$$\xi_1 = -\cos\frac{\psi_2}{2} \sin\frac{\psi_1 - \psi_3}{2} \sin\phi , \qquad (4b)$$

$$\xi_2 = \sin\frac{\psi_1 + \psi_3}{2} \cos\frac{\psi_2}{2} \cos\phi + \cos\frac{\psi_1 + \psi_3}{2} \sin\frac{\psi_2}{2} , \qquad (4c)$$

$$\xi_3 = \cos\frac{\psi_2}{2} \cos\frac{\psi_1 - \psi_3}{2} \sin\phi . \qquad (4d)$$

Preliminary numerical tests indicate that if we set $\psi_1 - \psi_3 = 0$ or $\psi_1 - \psi_3 = \pi$, we can find values of ψ_1 and ψ_2 to rotate an initially vertical spin to any desired final direction. This is of course a preliminary finding, and further study is required to validate the feasibility of this design. However, it does hold the *tentative* promise that, for transverse polarization, Snakes with orthogonal spin rotation axes can be designed for use in the booster synchrotron. For longitudinal polarization at the interaction point, it may be possible to design spin rotators which can deliver longitudinally polarized colliding beams at any beam energy, *without* modification of the ring geometry.

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