SIMULATIONS OF POLARIZATION LEVELS AND SPIN TUNE BIASES IN HIGH ENERGY LEPTONS STORAGE RINGS*

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Abstract

The use of resonant depolarization has been suggested for precise beam energy measurements in the 100 km long Future Circular Collider e+e-. The principle behind resonant depolarization is that a vertically polarized beam excited through an oscillating horizontal magnetic field gets depolarized when the excitation frequency is in a given relationship with the beam energy. In this paper the possibility of self-polarized leptons at 45 GeV (Z resonance) and 80 GeV (WW physics) in presence of quadrupole vertical mis-alignment is investigated.

INTRODUCTION

$e^{\pm}$ beams in a ring accelerator may become vertically polarized through the Sokolov-Ternov effect [1]. A small part of the radiation emitted by particles moving in a constant homogeneous field is accompanied by spin flip wrt the field direction. The probability of spin flip in the direction parallel to anti-parallel and from anti-paralle to parallel to the field are slightly different and this results in a polarization of 92.4 %, independently of energy. The polarization rate is given by

$$\frac{1}{\tau_{ST}} = \frac{5\sqrt{3}}{8}\frac{r_0 h}{2\pi m_0}\frac{\gamma^5}{|\rho|^3}$$

which strongly depends upon energy and radius. In actual storage rings there are not only dipoles. Quadrupoles for instance are needed for beam focusing. When a particle emits a photon it starts to perform synchro-betatron oscillations around the machine actual closed orbit experiencing extra possibly non vertical fields. The expectation value $\vec{S}$ of the spin operator obeys to the Thomas-Bargmann-Michel-Telegdi (Thomas-BMT) equation [2] [3]

$$\frac{d\vec{S}}{dt} = \vec{\Omega}\times\vec{S} \tag{1}$$

$\vec{\Omega}$ depends on machine azimuth and phase space position, $\vec{u}$. In the laboratory frame and MKS units it is given by

$$\vec{\Omega}(\vec{u};s) = -\frac{e}{m_0}\Big[\Big(a+\frac{1}{\gamma}\Big)\vec{B} - \frac{a\gamma}{\gamma+1}\vec{\beta}\cdot\vec{B}\vec{\beta} - \Big(a+\frac{1}{\gamma+1}\Big)\vec{\beta}\times\frac{\vec{E}}{c}\Big]$$

with $\vec{\beta} \equiv \vec{v}/c$ and $a = (g-2)/2$=0.0011597 (for $e^{\pm}$).

In a planar machine the *periodic* solution, $\hat{n}_0$, to Eq.(1) is vertical and, neglecting the electric field, the number of spin precessions around $\hat{n}_0$ per turn, the naive "spin tune", in the rotating frame is $a\gamma$. Photon emission results in a randomization of the particle spin directions (*spin diffusion*). Using a semiclassical approach, Derbenev and Kondratenko [4] found that the polarization is oriented along $\hat{n}_0$ and its asymptotic value is

$$P_{\rm DK} = P_{\rm ST}\frac{\oint ds < \frac{1}{|\rho|^3}\hat{b}\cdot(\hat{n}-\frac{\partial\hat{n}}{\partial\delta}) >}{\oint ds < \frac{1}{|\rho|^3}\Big[1-\frac{2}{9}(\hat{n}\cdot\hat{v})^2+\frac{11}{18}(\frac{\partial\hat{n}}{\partial\delta})^2\Big] >}$$

with $\hat{b} \equiv \hat{v}\times\dot{\hat{v}}/|\dot{\hat{v}}|$ and $\delta \equiv \delta E/E$. $\hat{n}$ the *invariant spin field* [5], i.e. a solution of Eq.(1) satisfying the condition $\hat{n}(\vec{u};s)$=$\hat{n}(\vec{u};s+C)$, C being the machine length. The <> brackets indicate averages over the phase space. The term $\partial\hat{n}/\partial\delta$ quantifies the depolarizing effects resulting from the trajectory perturbations due to photon emission.

The corresponding polarization rate is

$$\tau_p^{-1} = P_{\rm ST}\frac{r_e\gamma^5\hbar}{m_0 C}\oint < \frac{1}{|\rho|^3}\Big[1-\frac{2}{9}(\hat{n}\cdot\hat{v})^2+\frac{11}{18}\Big(\frac{\partial\hat{n}}{\partial\delta}\Big)^2\Big] >$$

In a perfectly planar machine $\partial\hat{n}/\partial\delta$=0 and $P_{\rm DK}$=$P_{\rm ST}$. In presence of quadrupole vertical misalignments (and/or spin rotators) $\partial\hat{n}/\partial\delta \neq 0$ and it is particularly large when spin and orbital motions are in resonance

$$\nu_{spin} \pm mQ_x \pm nQ_y \pm pQ_s = \text{integer}$$

For FCC-e^+e^- with $\rho \simeq 10424$ m, fixed by the maximum attainable dipole field for the hadron collider, the polarization time at 45 and 80 GeV are 256 and 14 hours respectively.

Here it is assumed that beam polarization of about 10% is sufficient for an accurate depolarization measurement. The time, $\tau_{10\%}$, needed for the beam to reach this polarization level is given by

$$\tau_{10\%} = -\tau_p \times \ln(1-0.1/P_\infty)$$

At 80 GeV it is $\tau_{10\%}$=1.6 hours, but $\tau_{10\%}$= 29 hours at 45 GeV.

At low energy the polarization time may be reduced by introducing properly designed wiggler magnets i.e. a sequence of vertical dipole fields, $\vec{B}_w$, with alternating signs.

FCC-e^+e^- maximum synchrotron radiation power is set to 50 MW per beam and the beam current at the various energies as been scaled accordingly. This limits the integrated wiggler strength. Moreover the wiggler increases the beam energy spread for which the effect on polarization must be investigated.

At 80 GeV wigglers are not needed. However the energy dependence of the spin motion makes the attainable polarization level more sensitive to machine errors.

Preliminary studies for a FCC-e^+e^- by using a"toy" ring [6] have shown that even in presence of quadrupole vertical

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misalignments and BPMs errors, useful level of polarization may be obtained at 45 GeV with wigglers and at 80 GeV without.

In this paper a FCC-e^+e^- optics by K. Oide [7] with β_y^*=1 mm is considered. MAD-X is used for simulating quadrupole mis-alignments and closed orbit correction. The lattice with errors and corrections is dumped to a file which can be read by the SITROS package [8] used for polarization calculations.

THE OPTICS

The optics contains 2 Interaction Points (IPs) based on quadrupole doublets. In the version used in this paper it is β_x^*=0.5 m and β_y^*=1 mm. The arcs are based on FODO cells with 90 degrees phase advance in both planes. The large β_y (10 km) at the strong IR quadrupoles makes the closed orbit very sensitive to their vertical misalignment and generates large chromaticity which correction requires strong sextupoles.

The expected rms orbit is given by

$$< z_{rms} >= F\ \delta^Q_{rms}$$

with

$$F \equiv \frac{1}{2\sqrt{2}|\sin \pi Q_z|}\sqrt{< \beta_z >}\sqrt{\Sigma_{i=1}^{NQ}\beta_{z,i}(k\ell)_i^2}$$

(z=x or y). The orbit response to vertical misalignments for FCC-e^+e^- is summarized in Table 1 for q_y=0.2 and δ^Q_{rms}=200 μm.

Table 1: Orbit Sensitivity to Misalignments

	F	$< y_{rms} >$ (mm)
all quads	613	123
w/o doublets	141	28

The value of 200 μm for δ^Q_{rms} may be conservative; in particular one may expect that it will be possible to get a better alignment for the IPs doublets.

For simulating the effect of quadrupole random vertical misalignments and their correction one Beam Position Monitor (BPM) and one vertical corrector (CV) were introduced close to each vertical focusing quadrupole and doublet quadrupoles. The fractional part of the betatron tunes were set to q_x=0.2 and q_y=0.3 for keeping the vertical tune far from the integer and sextupoles were turned off. Neverthless it is not possible to get a stable machine when 200 μm rms random offsets are introduced at once. In order to evaluate the achievable polarization for the *already corrected* machine, the sextupoles were switched off and the errors were added in steps of 1 μm for each of the doublet quadrupoles and of 10 μm for all the other quadrupoles at once. By each step the orbit due to each of the doublet quadrupoles was corrected by using the single CV close by, while 500 CVs selected by the MICADO algorithm were used for correcting the orbit due to the other quadrupoles. Evidently such tricks cannot be played in practice. However the initial machine set-up will take place starting with a more “relaxed” optics and a number of countermeasures can be deployed for establishing a starting closed orbit which analysis is beyond the scope of this paper.

It turned out that for 3 over 13 seeds the MAD-X Twiss module fails right when the sextupoles are turned on at the very end of the procedure.

The reason for this seems to be the relatively large skew quadrupoles created by the SYL and SYR sextupoles at each side of the IPs. The phase advance between SY1L and SY1R and SY2L and SY2R is 180^0 degrees. As the strengths of the sextupoles on the left side of the IPs have the opposite sign of those on the right side, if the beam offsets in such sextupoles are anty-simmetric, which is likely due to the phase advance, they generate a coupling wave which may be strong enough to cause the optics to become unstable.

POLARIZATION SIMULATIONS

The 45 GeV Case

4 LEP-like wigglers [9] with B^+=0.7 T were introduced in dispersion free sections with $\beta_x \simeq$ 50-80 m. The time needed to reach 10% beam polarization is about 2.9 h. The horizontal emittance increases from 0.088 nm to 0.5 nm. By using a larger number of poles should be possible to get a smaller emittance increase.

In the absence of BPM errors, after orbit correction it is y_{rms}=0.05 mm and the rms value of the polarization axis distortion, $|\delta\hat{n}_0|_{rms}$, is 0.4 mrad. The resulting polarization vs. $a\gamma$ is shown in Fig. 1 for orbital tunes q_x=0.1, q_y=0.2 and q_s=0.1.

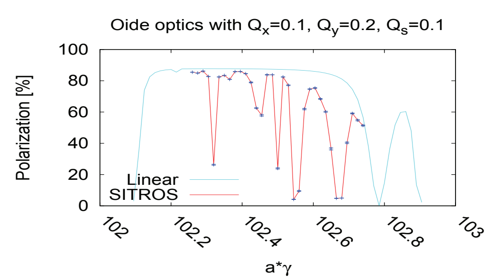

Figure 1: Polarization vs. $a\gamma$ after closed orbit correction for the ring with 4 wigglers; BPMs errors not included.

The 80 GeV Case

The same error realization at 80 GeV results in $|\delta\hat{n}_0|_{rms}$=2 mrad. The corresponding polarization is shown in Fig. 2.

Reducing $\delta\hat{n}_{0,rms}$ to 1.5 mrad with harmonic bumps [10] gives some improvement (see Fig. 3).

The harmonic bumps increase ϵ_y from 12.8 pm to 19.5 pm and the polarization related to the vertical betatron motion alone is somewhat reduced (see Fig. 4 and 5), indicating

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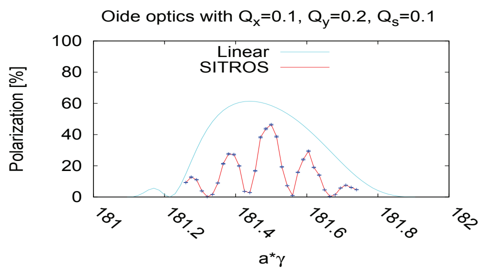

Figure 2: Polarization vs. $a\gamma$ after closed orbit correction for the ring w/o wigglers; BPMs errors not included.

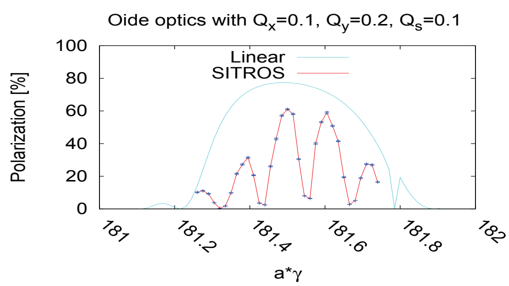

Figure 3: Polarization vs. $a\gamma$ after closed orbit and $\delta\hat{n}_0$ correction for the ring with 4 wigglers; BPMs errors not included.

that there is may be space for improvements in the harmonic bump scheme used.

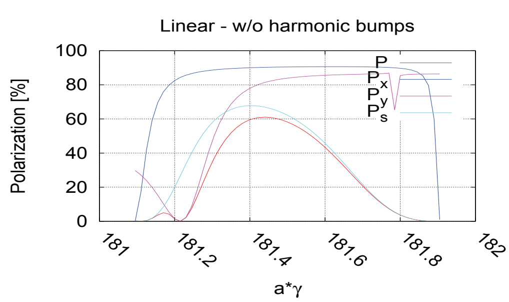

Figure 4: Polarization vs. $a\gamma$ in linear spin motion approximation after closed orbit correction for the ring with 4 wigglers. The blue, magenta and cyan lines show the polarization when only horizontal, vertical or longitudinal motion is considered respectively.

ENERGY MEASUREMENT BIASES

In addition of proving that useful polarization levels may be reached, it must be proved that the required energy measurement precision (better than 100 keV) may be achieved. Some issues such as beamstrahlung limited beam lifetime,

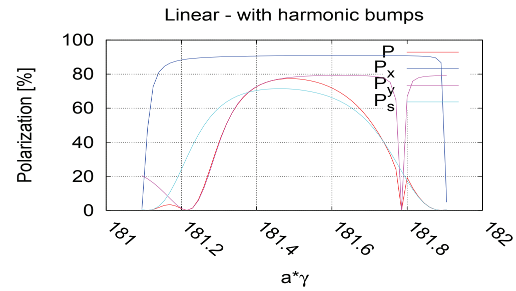

Figure 5: Polarization vs. $a\gamma$ in linear spin motion approximation after closed orbit and $\delta\hat{n}_0$ correction for the ring with 4 wigglers. The blue, magenta and cyan lines show the polarization when only horizontal, vertical or longitudinal motion is considered respectively.

energy sawtooth and synchrotron radiation power budget, set constraints on number of needed measurement stations, measurement scenario and wigglers operation [11]. In addition the relationships $\nu_{spin} = a\gamma$ strictly holds for a purely planar ring.

The effect of closed orbit distortion has been evaluated for LEP by using a simplified model by R. Assmann [12] who found that for half-integer ν_s^0 it is $\Delta\nu_s$=0 in first and second order in the extra-spin rotations. For $\nu_s^0 \neq 0.5$ it is

$$< \Delta\nu_s >= \frac{\cot \pi\nu_s^0}{8\pi}(a\gamma)^2\left[< \Sigma_q(K\ell)_q^2 y_q^2 > + < \Sigma_k \theta_k^2 >\right]$$

y_q being the *effective* beam position at the quadrupoles.

The corresponding energy error for FCC-e^+e^- for 10 error realizations is shown in Fig. 6.

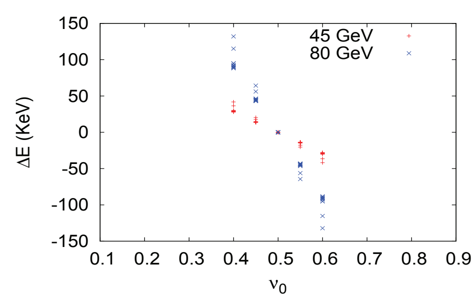

Figure 6: Energy error vs. nominal spin tune for 10 different error seeds.

Analytical expressions (see [13]) shall be implemented and used for comparison.

Finally the energy calibration error due to the angle between RF electric field and beam trajectory at the accelerating cavities [14] is shown in Table 2 where y'_{rms} is the rms vertical slope in mrad.

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Table 2: Calibration Error Sensitivity to Orbit in the RF Cavities.

	ΔE (KeV)
45 GeV	$2 \times y'_{rms}$
80 GeV	$16 \times y'_{rms}$

With

$$< y'_{rms} > \simeq \sqrt{\frac{< \gamma_y >}{< \beta_y >}} < y_{rms} > \simeq 0.1 < y_{rms} >$$

the contribution from the RF electric field should be small.

CONCLUSION

Preliminary studies for the 45 GeV and 80 GeV case have been presented for the current β_y^*=1 mm FCC-e^+e^- optics. The large sensitivity of the orbit to vertical misalignment of quadrupoles makes the orbit correction difficult. In particular we learned that the orbit at the SYL and SYR sextupoles on the left and right side of the IPs must be well controlled.

However the goal of this study was to assess the feasibility of self-polarization for energy calibration once a stable closed orbit has been established.

At 80 GeV, $\delta\hat{n}_0$ due to misalignments increases and although the energy spread is the same as at 45 GeV with wigglers, polarization is lower. Large harmonic bumps for correcting $\delta\hat{n}_0$ may cause a vertical emittance increase. With the toy ring it was shown that it is possible, for instance by using dispersion-free 5-coils bumps, to correct $\delta\hat{n}_0$ w/o spoiling the vertical emittance.

The present study has shown that self-polarization for energy calibration should be possible in the β_y^*=1 mm FCC-e^+e^- optics. However here only quadrupoles vertical misalignments have been considered and BPMs errors have not been included. The rms misalignment of 200 μm is conservative, a smaller value could be expected in particular for the doublet quadrupoles. The exercise on the "toy" ring had shown the importance of the BPMs errors on the orbit correction quality; however the 10% calibration error there assumed was conservative, 2%-3% should be achievable.

Various additional corrections aiming to preserve the small goal vertical emittance (1 pm at 45 GeV beam energy) have been considered by other contributors [15] [16] at this workshop. Their effect on polarization must be of course studied.

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