MEASUREMENTS OF THE BETATRON FUNCTIONS AND PHASES IN RHIC

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Abstract

The Relativistic Heavy Ion Collider (RHIC) consists of two rings with six fold symmetry. The six interaction regions (IR)s are connected with twelve FODO cells. RHIC quadrupoles in the interaction regions have independent tuning capability. The betatron functions will be measured by a three methods. First, tunable IR quadrupoles will be adjusted to measure betatron functions at those locations through the change in tune. Second, sinusoidal coherent dipole oscillations will be used to measure the betatron phases and functions (as performed in LEP). Third, a correction dipole kick technique will be used (as at Fermilab). Special attention will be given to the "betatron squeeze" procedure by which the two large experiments PHENIX and STAR will achieve minimum betatron functions between 1 and 2 m.[1]

1 INTRODUCTION

This is a report about preparations for betatron and phase function measurements by different techniques in the Relativistic Heavy Ion Collider (RHIC). The RHIC commissioning is beginning in the first quarter of 1999. We will describe previously used methods as well as the hardware and software requirements necessary to successfully perform the measurements.

RHIC consists of two identical six fold symmetric rings which will provide collisions of protons and identical or different heavy ion species (typically fully stripped gold) up to the energies of 100 GeV/nucleon at six interaction regions. At two interaction regions (IRs) fully stripped gold ion bunches of the two beams will collide at two focusing points with $\beta^* \sim 1-2m$. This will allow average luminosities of $L \simeq 2 \cdot 10^{26} 1/scm^{\ 2}$. The RHIC lattice is made of six arcs with twelve standard $\sim\ 90^o$ FODO cells between the IRs. The IRs are made of almost the same FODO cells with missing dipoles, to allow for zero dispersion at collision points. The IR tunable FODO cells also allow matching of the betatron functions between the high focusing triplets and the arc FODO cells. The expected values of the lattice functions in RHIC are presented in Table 1.

Table 1: Maximum Twiss Functions in RHIC at an IR with $\beta^* = 1m$

Region	$\hat{\beta}_x$	$\hat{\beta}_y$	D_x
Triplets	1354	1336	0.585
Arcs	47	48	1.89

[1]Work performed under the auspices of the U.S. Department of Energy

The maximum values of the betatron functions are within the strong focusing triplet quadrupoles around the two low β IRs while the other values are presented within the arc FODO cells. The beam positions around the ring will be measured with a total of 334 beam position monitors (BPMs) per one ring. Almost half of the BPMs (total of 160) are dual plane monitors. Each BPM is capable of measuring and recording the turn by turn positions of the center of the beams. The transverse positions of the BPMs relative to the superconducting quadrupoles, are determined by a special "antenna". A precise relationship between the BPM position and the quadrupole center is determined by surveying the antenna at three different offsets the BPM and recording both its position with respect to the quadrupole center, and the response signal from the BPM plates. The quadrupole center positions were determined and recorded at the same time [1]. The BPM positions with respect to the center of the quadrupoles are electronically transfered to a SYBASE database. A betatron tune measurement will be performed by measuring the response due due to a single fast dipole kick in one of transverse plane. The maximum value of the kick to the gold ion beam, at the top energy 100 GeV/nucleon is $\sim\ 0.6\mu rad$, providing a beam center offset of $\sim\ 24\mu m$ at a BPM with $\beta \sim 50m$.

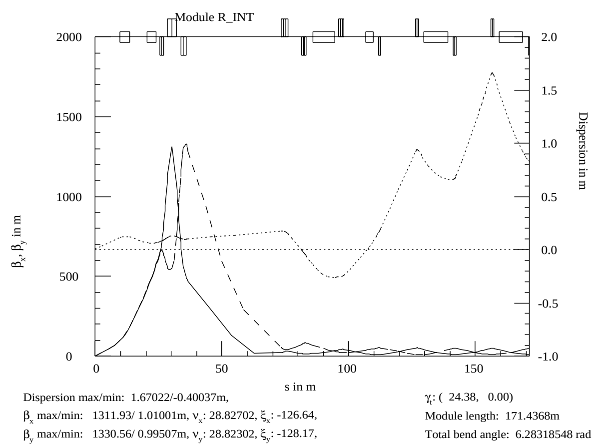

Figure 1: Interaction Region optics in RHIC with $\beta^* = 1m$.

2 BETA FUNCTION MEASUREMENTS USING SINGLE QUADRUPOLES

There are ten quadrupoles on each side of each collision point with adjustable strength as presented in Figure 1. Some of the quadrupoles have an additional *trim* quadrupole with a separate power supply, while others have a shunt power supply in addition to one of two main quadrupole buss circuits. The betatron functions at each adjustable quadrupole are determined by measuring the tune shift [2] due to a change of strength *l k*:

$$\Delta\nu \simeq \frac{1}{4\pi}\int_l \beta(s)\Delta k ds, \quad (1)$$

$$\beta_Q \simeq \frac{4\pi\Delta\nu}{l\Delta k}, \quad (2)$$

where β_Q is the average function of the beta function at the quadrupole. The tune shift will be measured by a betatron tune measurement system described above. This will allow betatron functions measurements at every quadrupole within the interaction region.

3 BETA FUNCTION AND PHASE MEASUREMENTS BY THE COHERENT BETATRON RESONANCE EXCITATION

Coherent betatron oscillations do not induce the emittance growth if they are adiabatically introduced at a tune outside the incoherent spectrum [3]. The closed orbit of a beam disturbed by a single constant dipole kick θ_c is[2]:

$$x = \frac{\theta_{x,c}}{2sin(\pi\nu_x)}\sqrt{\beta_x(s)\beta_x(c)}\ cos(|\Delta\psi(s)| - \pi\nu_x), \quad (3)$$

where $|\Delta\psi(s)|$ is absolute value of the betatron phase difference between the dipole kick position and the point s of observation. The betatron function and phase at the position of the dipole kick c are $\beta_x(c)$ (in the vertical plane $\beta_y(c)$, while at the point of observation s the betatron functions are $\beta_{x,y}(s)$ and $\phi_{x,y}(s)$. The dipole kick in the horizontal plane is proportional to the integrated dipole field $\theta_{xc} = B_{yc}\ell/B\rho$, where ℓ is the dipole length, B_{yc} is the vertical magnetic field, and $B\rho$ is the *magnetic rigidity*. The dipole field is horizontal $B_{x,c}$ for the vertical transverse plane excitation. Coherent betatron oscillations occur when the dipole field perturbation oscillates [3] with a tune ν_m:

$$B_{yc} = B_M cos(2\pi t\nu_m), \quad (4)$$

where B_M is the maximum amplitude of the dipole field, t is the number of revolutions around the accelerator, and ν_m is the modulation tune. It is convenient to consider the particle's phase space vector [3] in the rotating frame at the frequency $2\pi\nu_m$, where the angle of rotation each turn is $2\pi\delta$ with:

$$\delta = \nu_{x,y} - (k - \nu_m), \quad (5)$$

where *k* is an integer, and $\nu_x = 28.19$ and $\nu_y = 29.18$ are the horizontal and vertical tunes in RHIC, respectively. The time average of the horizontal kick $\langle\theta_{x,yc} = X'\ or\ Y'\rangle$ in the rotating frame is [3]:

$$\langle X'\rangle = \frac{1}{2}\frac{B_M\ell}{B\rho}, \quad (6)$$

while the fixed point distance $X_{coh}(orY_{coh})$ in the rotating frame is [3]:

$$2\pi\delta X_{coh} = \frac{1}{2}\beta_c\frac{B_M\ell}{B\rho}. \quad (7)$$

From the relationship between the transverse beam size during coherent particle oscillations, called the *"measured"* beam size [3], with the previous undisturbed beam size σ_o the magnitude of the integrated field in RHIC, was estimated to be $B_M\ell = 0.030Tm$ (or $300Gm$).
When the coherent oscillations of the beam are established, the BPMs around the RHIC will record the signals on each turn. This is going to be synchronous with the coherent dipole excitation because they are *self triggered* by the beam arrival. The phases at each BPM are obtained by the *Fourier analysis* of at least 1024 turns. As described in detail by the *"Phase and betatron measurements in LEP"* [4], these signals provide very accurate information on the phases between any two monitors $\Delta\phi_{k+1,k} = \phi_{k+1} - \phi_k$. The signals of the BPM positions and phases from the three BPMs are enough to obtain all three betatron functions β, ν and α by using a transfer matrices *A, B, C* between them:

$$-A-> \qquad -B-> \quad (8)$$

$$BPM_1 \; -- BPM_2 \; -- BPM_3 \quad (9)$$

$$---C---> \quad (10)$$

The phases between each pair of BPMs are $\phi_{1,2},\ \phi_{1,3}\ , \phi_{2,3}$ (are obtained by the 1024 turn by turn measurements and the *Fourier* spectrum analysis), and from a connection between the design transfer matrices and the measured BPM positions at each monitor [4]:

$$\beta_1^{meas.} = \frac{a_{12}c_{12}}{b_{12}}\left(cot\phi_{12}^{meas.} - cot\phi_{13}^{meas.}\right) \quad (11)$$

where the a_{12}, c_{12} and b_{12} are the transfer matrix elements from the design lattice. More detail about this derivation is presented in [4]. The other betatron functions are determined by the same procedure [4].

4 BETA FUNCTION MEASUREMENTS USING THE CORRECTION DIPOLE MAGNETS

If the betatron functions are known at two dipole correctors the betatron functions and phases β and ϕ at every BPM can be found [5]. A closed orbit is perturbed by a corrector dipole by a kick θ. A value of the kick is known from a dipole current excitation and the transfer function of the specific corrector. The transfer functions - defined as a relationship between the integral dipole field $\int Bds = B\ell$ to the current excitation - of the RHIC correction dipoles were measured with a precision of $\sim 10^{-4}$. Two closed orbits due to two separate perturbations by the two correctors, make two beam positions at each BPM. The two simultaneous equations of the beam offsets at each BPM have two unknowns: β and $\delta\phi$ (shift of the BPM phase with respect to the design value ϕ_{BPM}) [5]:

$$\frac{2sin(\pi\nu)}{\theta_{C1}\sqrt{\beta_{C1}}}\,x_1 = \sqrt{\beta}\,cos(\theta_{C1}+\delta\phi) \qquad (12)$$

$$\frac{2sin(\pi\nu)}{\theta_{C2}\sqrt{\beta_{C2}}}\,x_2 = \sqrt{\beta}cos(\theta_{C1}-\delta\phi), \qquad (13)$$

where:

$$\theta_1 = \pi\nu + \phi_{C1} - \phi_{BPM}, \qquad (14)$$

$$\theta_2 = \phi_{BPM} - \phi_{C2} - \pi\nu, \qquad (15)$$

where x_1 and x_2 are the BPM measurements at the two correctors excitations. This idea can be expanded to many successive corrector excitations followed by a series of BPM measurements. The recorded values of the BPM positions around the ring, due to each specific corrector excitation, can be used to calculate the betatron functions and phases around the two RHIC rings by an iterative procedure. The starting values of the phases and betatron functions during the iterations are the design values.

4.1 Results from the Beta Function Measurements in the Fermilab TEVATRON

A global betatron function measurements were performed during the commissioning in 1995 of the upgraded Low Beta inserts in the Tevatron. To improve the procession of the measurement, the correction dipole was ramped providing a position change in the BPM verses the corrector strength. This data was analyzed to give the maximum position change and the error. The position error in the BPM system was estimated to be $x, y_{rms} \sim 60\mu$ and the maximum displacement due to the corrector dipole excitation was $\Delta x, y \sim 5cm$. This allowed measurements of the betatron functions at all BPMs with a random error of approximately 5 $\%$. A measurement comparison with the design values showed a a 10% systematic error in the BPMs/correction dipole. The first observation in the measurement result was a large betatron phase error at each BPM with respect to the expected value. The measured data were then fitted to the design values with an assumption of 0.3 $\%$ quadrupole gradient errors within the low beta triplet. An error of 80 $\%$ in the $\delta\beta/\beta$ wave in both the horizontal and vertical planes was observed. The β wave was removed by the gradient adjustment within the triplet quadrupoles. After the correction a betatron function error was $\delta\beta/\beta \leq 10\%$ which corresponds to the estimated systematic measurement error.

This method can be also be used to measure and locate the source and size of the quadrupole roll errors. The beam oscillation is observed within a transverse plane orthogonal to the plane of a single dipole excitation. The orbit error within the orthogonal plane is recorded. The analysis is very simple if error in the orthogonal plane is caused by a single rolled quadrupole. However, the method could also be used to measure the quadrupole skew errors around the machine. In the TEVATRON a location of the skew quadrupole error of $\sim 2mrad$ had been determined.

5 CONCLUSIONS

This report shows a part in preparation for the RHIC commissioning and operation. We showed three methods of the betatron function measurements planned to be used in RHIC. The single quadrupole excitation a the IRs will allows us to measure the betatron functions at each quadrupole within the IRs. Coherent dipole excitations will allow the phase and betatron function measurements of both rings within few seconds. This application will be possible when the hardware is built and installed. The third method of measuring the betatron functions at the BPMs by the correction dipole excitations will be possible to explore even during the commissioning period and experience from the TEVATRON measurements are very encouraging.

6 REFERENCES

[1] D. Trbojevic, P. Cameron, G.L. Ganetis, M.A. Goldman, R. Gupta, M. Harrison, M.F. Hemmer, F.X. Karl, A. Jain, W. Louie, S. Mulhall, S. Peggs, S. Tepikian, R. Thomas, and P. Wanderer, “Alignment and Survey of the Elements in RHIC”, Proceedings of the 1995 PAC and International Conference on High Energy Accelerators, (1995), Dallas, Texas, TAA009, pp. 2099-2101.

[2] E.D. Courant and H.S. Snyder, “Theory of Alternating Gradient Synchrotron”, Annals of Physics, Vol. 3, (1958), pp. 1-48.

[3] M. Bai, S.Y. Lee, J.W. Glenn, H. Huang, L. Ratner, T. Roser, M.J. Syphers, and W. van Asselt, “Experimental Test of Coherent Betatron Resonance Excitations”, Phys. Rev. E, Vol. 56, No. 5, November 1997, pp. 6002-6007.

[4] P. C. Garcia, “Luminosity and beta function measurements at the electron-positron collider ring LEP”, Doctoral Thesis, May 17, 1996, Burjassot, Valencia, pp. 49.

[5] M. Harrison and S. Peggs, “Global Beta Measurements from two Perturbed Closed Orbits”, Proceedings from the IEEE Particle Accelerator Conference in Washington D.C. , March(1987), pp.1105-1107.