FIELD MEASUREMENT OF THE ELETTRA CAVITY HIGH ORDER MODES

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Abstract

The longitudinal shunt impedance of the monopole Higher Order Modes (HOMs) of the ELETTRA cavity has been measured using the perturbative method and have been compared with MAFIA simulations. The data will be used for more precise calculations of Coupled Bunch growth rates. The measurement set-up allows a precise identification of the polarization plane for the dipole.

1 INTRODUCTION

The Slater theorem [1] establishes that a small object displaced into a cavity induces a resonant frequency shift which is proportional to the cavity field distribution:

$$\omega_p^2 = \omega_0^2 \left(1 + \frac{\int_\tau \left(\mu_0 \left|\vec{H}\right|^2 - \varepsilon_o \left|\vec{E}\right|^2 \right) dV}{\int_V \left(\mu_0 \left|\vec{H}\right|^2 + \varepsilon_o \left|\vec{E}\right|^2 \right) dV} \right)$$

where ω_p is the perturbed frequency, ω_0 the resonant mode unperturbed frequency, V is the cavity volume, τ is the object volume, $\vec{H}$ and $\vec{E}$ are the electromagnetic fields. For a small perturbation the Slater theorem can be written:

$$\frac{\omega_p - \omega_0}{\omega_0} = \frac{1}{4U} \int_\tau \left(\mu_0 \left|\vec{H}\right|^2 - \varepsilon_0 \left|\vec{E}\right|^2 \right)$$

where U is the cavity stored energy. The field integral is solved under the assumption that the object volume is small in order to neglect the local field perturbation with respect to the total cavity unperturbed field. Moreover the field is considered constant along the object shape so that the electrostatic field approximation can be applied. For metallic rotationally symmetric objects Slater and Maier [2] found the following expression:

$$\frac{\omega_p - \omega_0}{\omega_0} = \frac{\tau}{4U} \left[k_z^m \mu_0 \vec{H}_z^2 + k_\perp^m \mu_0 \vec{H}_\perp^2 - k_z^e \varepsilon_0 \vec{E}_z^2 - k_\perp^e \varepsilon_0 \vec{E}_\perp^2 \right]$$

where $k_{i=z,\perp}^{j=m,e}$ are form factors strongly dependent on the object shape and orientation with respect to the field. The frequency shift depends on the object volume, its form factor and on the cavity fields. For oblate ellipsoids k_z^e is greater then the other form factors and this property is used to measure the longitudinal component of the electric field. Metallic spherical objects are not selective with respect to field orientations; the frequency shift is simply due to the total $\vec{E}$ and $\vec{H}$ fields:

$$\frac{\omega_p - \omega_0}{\omega_0} = \frac{\tau}{4U} \left[\frac{2}{3} \mu_0 \vec{H}^2 - 3\varepsilon_0 \vec{E}^2 \right]$$

Dielectric objects are also used as perturbation objects. The frequency shift due to a dielectric sphere is [3] :

$$\frac{\omega_p - \omega_0}{\omega_0} = \frac{\tau}{2U} \left[k^m \mu_0 \vec{H}^2 - k^e \varepsilon_0 \vec{E}^2 \right]$$

$$k^m = 3\frac{\mu_r - 1}{\mu_r + 2} \qquad k^e = 3\frac{\varepsilon_r - 1}{\varepsilon_r + 2}$$

For dielectric material with $\mu_r = 1$ the bead perturbs only the electric field.

The longitudinal shunt impedance can be measured by means of the perturbation method since it depends on the electric field distribution along the cavity z-axis:

$$\frac{R_{eff}}{Q_0} = \frac{\left| \int_z E_z \, e^{jhz} dz \right|^2}{2\omega U} \text{, where } h = \frac{\omega}{\beta c}.$$

The frequency perturbation along the z-axis caused by a metallic needle is proportional to the longitudinal field of the monopole modes:

$$\frac{\omega_p - \omega_0}{\omega_0} = -\frac{\tau}{4U} k_z^e \varepsilon_0 \vec{E}_z^2$$

The shunt impedance is then evaluated accordingly:

$$\frac{R_{eff}}{Q_0} = \frac{1}{\tau \, k_z^e} \frac{4\pi}{\varepsilon_0 \omega_0^2} \left| \int_z \pm\sqrt{\left|\omega_0 - \omega_p\right|} \, e^{jhz} dz \right|^2$$

2 MEASUREMENT SET-UP

The shunt impedance measurements have been carried out with a shortened syringe stainless steel needle having length=10 mm and ∅=1mm. The transverse mode identification has been performed with a dielectric PVC sphere ∅=9 mm.

Each bead has been fixed on a nylon thread moved by a stepper motor along the z-axis of the cavity. The measurements are made when the bead has come to a complete stop. Direct frequency measurements as well as phase measurements have been performed by means of the Network Analyzer HP 8510. The system is fully controlled by a personal computer as shown in figure 1. The measurement program has been written in the LABVIEW 3.0 environment. The measurement set-up has been tested by performing a perturbation measurement on a known field with a small silver sphere with ∅=6.15 mm. For this purpose a stainless steel pill-box cavity has been built with the TM_{010} mode resonating at 1065 MHz. The correspondent frequency shift is [3]:

$$\frac{f_p - f_0}{f_0} = -\frac{2\pi\, r_{bead}^{\;3}}{\pi\, La^2 J_1^2(x_{01})}$$

where a is the cavity radius, L its length and J_1 is the first order Bessel function evaluated in x_{01} = 2.405. The theoretical frequency shift is 173 10^{-6}, the measured value is 174 ± 2 10^{-6}, in good agreement with the theoretical prediction.

The form factor k_z^e of the needle has also been measured with the perturbation of the pill-box TM_{010} mode on the z-axis. The form factor formula is [2]:

$$k_z^e = \frac{R^3}{\beta^2(S-R)}, \quad R = \sqrt{1-\beta^2} \quad, S = \frac{1}{2}\ln\frac{1+R}{1-R}, \quad \beta = \frac{b}{a},$$

where b and a are respectively the minor and the major semi-axis of the rotational ellipsoid. It should taken into account that the volume of the oblate ellipsoid is $\tau = \frac{4\pi}{3}a^3\beta^2$.

The theoretical value for the needle is $k_z^e = 49.3$, the measured value is $k_z^e = 49.8$.

The pill-box cavity has been also used to check the PVC dielectric ball perturbation. The pill-box TM_{110} mode is not influenced by the dielectric ball moved on the z-axis, where there is only a transverse magnetic field, but the TM_{010} mode is shifted accordingly, giving a measured dielectric constant $\varepsilon_r = 2.28$ to be compared with the $\varepsilon_r = 2.25$ as given by the data sheet for the PVC.

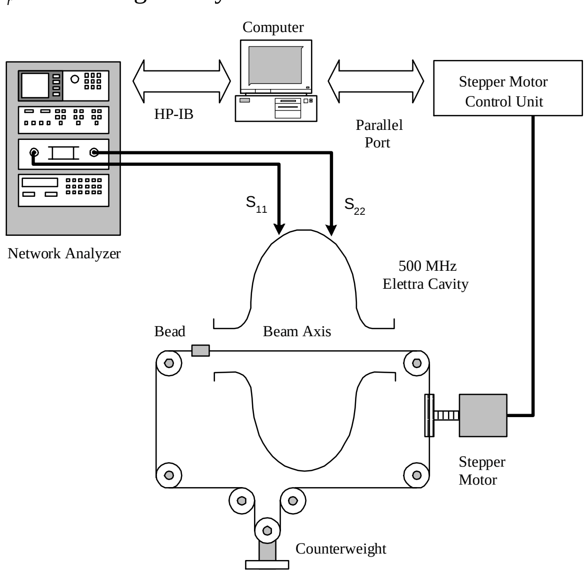


Figure 1: Field measurement set up.

3 LONGITUDINAL MODES

The measurements of the longitudinal shunt impedance of the ELETTRA spare cavity have been performed in air, at a temperature of 25 °C to ensure good thermal cavity stability. The stainless steel needle bead of length 10 mm has been used. The total cavity axial length is 452 mm, including the flanges. The measurement has been performed over a path of 500 mm subdivided in 200 steps, so that the measurements with the bead outside the cavity volume are included. This furnishes an indication of the measurement noise. The measurement of the L9 mode has been done with the installation of an additional beam pipe of length 300 mm on the cavity axial port. As is shown in the fig 2 the L9 electric field propagates into the beam pipe and the overall length, cavity plus pipe, should be taken into account to measure the whole field as seen by the beam.

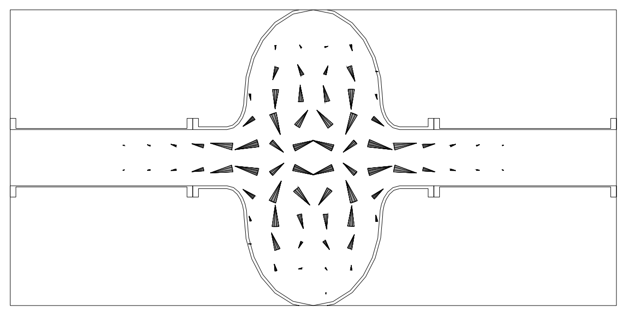


Figure 2: MAFIA electric field pattern of the L9 mode in the cavity and the additional beam pipes.

The measurements have been made over all the cavity length in order to detect any possible asymmetry between the two cavity half cells, but no asymmetry could be measured. Two different measurement procedures have been performed: direct frequency shift measurement of the cavity reflected signal and phase measurement of the cavity transmitted signal.

The frequency measurements have been carried out with either capacitive or inductive loops, depending on the mode field pattern. The loops should be critically coupled to the mode field pattern. It should be noted that the cavity has several pickup pipes installed only on the equatorial plane, so that the L8 mode could not be excited with the requested coupling factor. Moreover the L7 and L9 modes should be excited on the beam axis. The measurements have been performed only on half of the cavity axis to avoid cross talk between pickup and bead. The frequency measurement lasts 25 minutes. The phase measurement is faster, it lasts 10 minutes, but in this case the pickup probe of the transmitted signal has been strongly undercoupled.

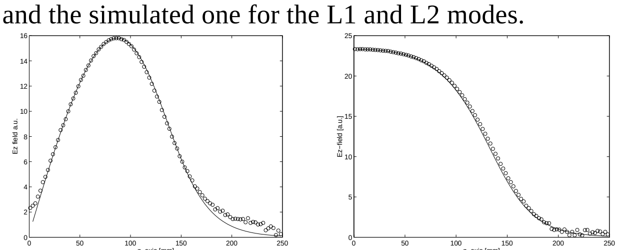
Mode	Simulation			Measurement		I	II
	f (MHz)	Q	Reff/Q (Ω)	f (MHz)	Q	Reff/Q (Ω)	Reff/Q (Ω)
L1	949.23	46900	29.3	948.262	44000	27.9 ± 1.1	29.5 ± 1.0
L2	1066.26	66600	0.7	1054.466	45800	0.98 ± 0.09	0.92 ± 0.10
L3	1423.17	57800	5.0	1419.875	41800	5.4 ± 0.2	5.3 ± 0.3
L4	1517.46	63500	4.8	1510.586	48700	5.0 ± 0.3	4.9 ± 0.3
L5	1621.90	79100	9.8	1595.581	45900	11.6 ± 0.4	10.5 ± 0.5
L6	1874.74	59000	0.4	1876.562	32000	0.31 ± 0.07	0.31 ± 0.05
L7	1953.46	83600	1.6	1945.329	60300	1.72 ± 0.20	1.73 ± 0.10
L8	2088.41	63400	0.0	-	-	-	-
L9	2129.57	53700	7.8	2116.804	21000	-	7.34 ± 0.09

Table 1 : Electromagnetic parameters of the ELETTRA cavity: simulated and measured. Column I lists the frequency measurements, Column II lists the phase measurements.

The relationship between the frequency shift and phase is:

$$\frac{f_p - f_0}{f_o} = \frac{1}{2Q_{load}} tg\phi$$

The phase and frequency measurements match and the comparison with simulation are good, as shown in table 1. Figures 3a and 3b show the measured electric field and the simulated one for the L1 and L2 modes.



a b

Figure 3: Electric field for the L1 mode (a) and L2 mode (b) over half cavity length; measured, dots, and MAFIA, line.

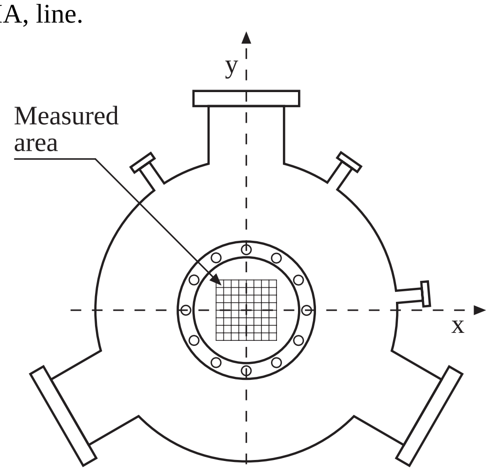
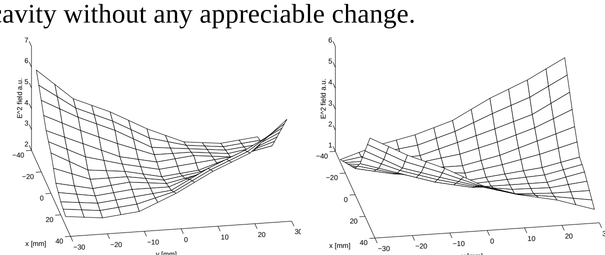


Fig. 4: Middle plane of the cavity.

4 TRANSVERSE MODES

The measurement set-up has been used to identify the polarization angle of the transverse modes with respect to the reference plane xy, normal to the beam z-axis. To detect the actual polarization angle a squared area in the middle of the cavity, normal to the z-axis, has been scanned with the dielectric bead, see figure 4. The measured perturbation is then proportional to the total electric field. Preliminary data on the TE-like modes, such as the D1 mode, show that the field is almost uniform over the investigated area so measurements are not yet conclusive. The TM-like modes, such as the D2 mode, change sign crossing the z-axis, thus their polarization can be easily measured. The result for the D2 mode is shown in fig 5 a and b. Due to the finite size of the dielectric ball, the perturbation reaches a minimum, but it is always present. The two polarizations are clearly oriented at 45° with respect to the xy plane. The same measurement has been repeated with the HOMFS movable plunger [4] fully inserted into the cavity without any appreciable change.



a b

Fig 5: D2: the total electric field (squared values) over the scanned area for the first (a) and second (b) polarization.

Further measurements are required to confirm this polarization angle in view of the transverse impedance measurement and of the evaluation of multibunch transverse instabilities.

REFERENCES

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[4] M. Svandrlik et all, “Improvements in curing coupled bunch instabilities...”, in *Proceedings of the 1997 Particle Accelerator Conference*, Vancouver.